Some Indicators of Regional Labour-Market Equilibrium

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1. Introduction

The need for a new measure for labour-market equilibrium emerged from the discussion around a fundamental study of labour and the labour market, carried out by the Netherlands Economic Institute. It became clear during this discussion that the traditional indicators of the situation on the labour market, such as unemployment, open demand, etc., are insufficiently precise in describing the actual situation. The reasons why will be given below.

Labour-market equilibrium is generally defined in terms of demand for and supply of positions. The condition for equilibrium is then \( D = S \) (demand for labour (supply of positions) equals supply of labour (demand for positions); equilibrium ex ante), or \( D = S = E \) (demand and supply equal the number of filled positions; in other words, there is no unemployment and there are no vacancies; equilibrium ex post). The degree to which the actual situation on the labour market deviates from the equilibrium position is usually expressed in a percentage of the working population.

When these figures are handled on a very high level of aggregation (e.g., the national level) it appears extremely difficult to get a good insight into the actual situation of the labour market. This can be improved by differentiation of the indicators according to regions,

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1 The authors gratefully acknowledge the assistance of Mrs. A. C. A. Elderson in editing the English of this paper.
sectors, professions, or other separate markets. Seasonal and frictional elements are then still left out of consideration, but maybe more important is the problem that even after disaggregation the figures indicate registered supply and registered demand rather than actual supply and actual demand. Particularly as far as the supply of positions is concerned, these figures may deviate very considerably from reality. To come nearer to reality, the concepts of "disguised unemployment" and "disguised vacancies" have been introduced, rightly so, though these concepts seem very difficult to estimate. Furthermore, additional insight into the regional situation may be gained from figures about commuting and migration, though it must not be forgotten that these volumes of commuting and migration are also determined by other factors than the employment situation, and cannot, therefore, be looked upon as indicators of the deviation from the equilibrium situation of the labour market.

But even when all the above considerations have been accounted for, it is still questionable if such factors as "vacancies" and "unemployment" cover all the elements that play an essential role in the situation of a specific labour market. In the conventional equilibrium concepts, factors like commuting distance, wage level, work conditions and labour performance are left out of consideration, yet they relate directly to the labour and production situations, and an understanding of the tensions existing in connection with them is essential in appreciating whether or not the actual situation is one of equilibrium.

2. Some fundamental considerations

It seems appropriate to start the search for a proper measure of labour-market equilibrium from the basic concept of well-being. Our well-being rests not only on the goods we possess and can effectively enjoy, but also, and especially, on the ratio between them and all the goods that we feel could contribute to our well-being. In other words, it is not the quantity of goods in our possession, but that quantity seen in relation to the quantity we desire, that determines our degree of well-being. The degree of well-being is the ratio between the goods

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we have to satisfy our needs and those needs themselves, or, formulated in yet another way, it is the degree to which our needs can be satisfied by the goods that are available to us.

A similar definition is given by Pigou. "A man’s welfare then consists in his satisfactions. But what does satisfaction mean? (...) It might seem that, when his desire attitude is given, his satisfaction depends straightforwardly on the extent to which his desires are fulfilled." ¹

To translate this definition of well-being in terms of labour and the labour market, it seems useful to define the labour market somewhat more precisely than, as is usually done, as the functional place where labour is supplied and demanded. Within the framework of welfare economics the following definition might be more appropriate: "The labour market contains everything relevant to the efforts of workers and entrepreneurs to satisfy their needs as far as these needs can be satisfied through labour efforts." This definition implies that the degree to which needs are satisfied, i.e., the relative tension between the desired and the actual situation, forms, together with the possibility to reduce this tension for both workers and entrepreneurs by adjusting either the desires or the work conditions, the central problem of the labour market. Essential, though difficult to measure, are the preference functions of individual workers with respect to the various aspects of positions to be filled — such as income to be earned, profession to be carried on, circumstances under which the work is to be performed — as well as the preference function of the entrepreneurs with regard to the number and nature of the positions to be created, the wages they are prepared to pay, and the working conditions they are prepared to offer. The confrontation of these functions with the actual situation constitute the essence of the labour market problem.

Two discrepancies may now be distinguished in relation to those demanding positions and those supplying positions:

1. The ratio (or the difference) between the desires of the workers and the actual situation as far as it is relevant to them with regard to, e.g., income, profession and journey-to-work distance.

2. The ratio (or the difference) between the desires of the entrepreneurs and the actual situation as far as it is relevant to them with regard to, e.g., wages to be paid and work performance.

If the needs of workers (or potential workers) are indicated by $P_s$ and the actual situation by $R$, the discrepancy $d_1$ can be written as:

$$T_s = P_s - R;$$

the discrepancy $d_2$, analogously, equals

$$T_d = P_d - R. \quad (1)$$

The greater the "distance" between the needs or desires and the actual situation, the less satisfactory these situations are. Workers and entrepreneurs are fully satisfied only if both $T_s = 0$ and $T_d = 0$.

It may be assumed that the abovementioned discrepancies will cause both workers and entrepreneurs to try and adjust $R$ in order to reach a more satisfactory situation. An attempt to improve the situation could be that workers start negotiations with entrepreneurs to accomplish an adjustment. A worker could also try to improve his relative position by looking for work with another firm. The discrepancies can become so great in certain cases that workers start looking for positions with a character completely different from that of the one they are holding, that is to say, for a different profession.

The remaining discrepancy may give rise to a number of phenomena, such as training and re-training, sickness, frustration, and decreased work performance.

Entrepreneurs might seek to improve the situation by decreasing the number of positions to be filled (by means of mechanization or otherwise), adjusting the nature of positions in such a way that they can be filled by workers who are actually available, etc. For those demanding labour the situation will never become quite satisfactory either.

Although we shall return to this point later, it seems appropriate to indicate already here that the situation of the parties on the labour market seen from the angle set out above is completely similar to the position of an individual in welfare theory, who is also trying to improve his relative position, but will never be completely satisfied.

J. Kornai indicates that the equilibrium concept is closely connected with the concept of "rest". He defines an economic equilibrium as a situation in which none of the participants in the economic process is interested in changing his behaviour and consequently destroying the

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1 Obviously these discrepancies can also be expressed as ratios. In that case they are, respectively:

$$T_s = P_s/R,$$
$$T_d = P_d/R.$$
equilibrium situation. Applying this definition to the labour market with respect to both workers and entrepreneurs, equilibrium will exist if there is no discrepancy, either for workers or for entrepreneurs. That means that $T_w = 0$, and $T_d = 0$; then

$$P_w = P_d = R$$

will hold. In fact, in practice there will never be a situation of equilibrium in this sense. For that reason we will rather concentrate on the search for a measure that indicates the deviation from equilibrium, and the way in which that deviation can be decreased. Obviously, this measure will be of a complex nature, and will have to be built up gradually to become operational in labour-market policy.

3. A measure of discrepancy

3.1. The spatial element in equilibrium

We will start the construction of a discrepancy coefficient by considering the spatial element in the labour market.

Denote the attractiveness of a position in region $i$ for a worker living in $h$ by $a_{hi}$. Assume that the attractiveness of this position is a decreasing function of the distance between $h$ and $i$ ($d_{hi}$). We may then write

$$a_{hi} = e^{-\alpha d_{hi}}$$

The coefficient $\alpha$ represents the degree to which the attractiveness of the position decreases with distance. Since this distance has to be interpreted as generalized transportation costs (money costs, time and effort), its value depends among other things on the means of transportation available to the worker.

Obviously the minimum value of $a_{hi}$ equals zero. This value is reached for $d_{hi} \to \infty$. The maximum value of the attractiveness of the position (ceteris paribus, of course) equals unity (for $d_{hi} = 0$).

In principle the coefficient will be different for different individuals. It will, however, be possible to define social groups (social positions, age groups, sex) within which the $\alpha$'s differ relatively little. It will be assumed in the following that $\alpha$ is equal for all individuals.

1 J. Kornai, _Anti-equilibrium, On economic systems theory and the tasks of research_, Amsterdam/London, 1971, pp. 24 and 35.
The supply discrepancy can now be defined as the ratio between the ideal situation and the actual situation. If the actual situation is indicated by \( a_{hi} \), reached for a distance \( d_{hi} \), the discrepancy coefficient for an individual in this case equals

\[
T_h^* = \frac{(a_{hi})_{\text{max}}}{a_{hi}}
\]  

(2)

Since \((a_{hi})_{\text{max}} = 1\) (commuting distance zero), the supply-discrepancy coefficient equals

\[
T_h^* = \frac{1}{a_{hi}} = e^{\alpha d_{hi}}
\]  

(3)

\(a_{hi}\) thus represents the partial welfare position of an individual living in \( h \) and working in \( i \), that is, the degree of satisfaction he gains from his present position in relation to his ideal position. Considering that policy makers do not aim at an ideal labour situation but focus their policy on feasible objectives, one might envisage discrepancy coefficients in which \((a_{hi})_{\text{max}}\) is replaced with \((a_{hi})_{\text{acceptable}}\). In what follows we shall not pursue this idea, however.

If the number of people living in \( h \) who are willing to fill a position is indicated by \( S_h \) (working population), the total attractiveness of the existing situation to these group equals

\[
S_h a_h
\]

If from this group \( E_{hi} \) individuals work in \( i \), then the total attraction felt by workers in \( h \) towards filling positions in all regions equals:

\[
\sum_{i} E_{hi} a_{hi} = \sum_{i} E_{hi} e^{-\alpha d_{hi}}
\]

(4)

The remaining part of the population is unemployed \((U_{hi})\). To express their situation mathematically, it is assumed that their work is located at an infinite distance from \( h \). Total attractiveness of work for residents in \( h \) willing to work is thus

\[
S_h a_{hi} = \sum_{i} S_{hi} e^{-\alpha d_{hi}} = \sum_{i} \left( U_{hi} + E_{hi} e^{-\alpha d_{hi}} \right) = \sum_{i} E_{hi} e^{-\alpha d_{hi}}
\]

(5)

Since \((S_h a_{hi})_{\text{max}} = S_h\) and \(T_h^* = \frac{(S_h a_{hi})_{\text{max}}}{S_h a_{hi}}\), it follows that

\[
T_h^* = \frac{1}{a_h} = \frac{U_h + \sum_{i} E_{hi}}{\sum_{i} E_{hi} e^{-\alpha d_{hi}}}
\]

(6)
Similarly for the whole labour market (all regions \( h \)), it may be stated that

\[
T^* = \frac{\sum_{h} S_h}{\sum_{h} \sum_{i} E_{hi} e^{-\alpha_d h_i}} = \frac{\sum_{h} U_h + \sum_{h} \sum_{i} E_{hi}}{\sum_{h} \sum_{i} E_{hi} e^{-\alpha_d h_i}} \tag{7}
\]

Obviously the discrepancy \( T^* \) increases with increasing distances, and reaches a minimum value when all \( d_{hi} = 0 \) and all \( U_h = 0 \). In this case everybody willing to work finds employment in his immediate neighbourhood; \( T^* \) then equals unity. If the measure for the discrepancy between supply equilibrium and the actual situation is defined in such a way that it reaches a zero value in case of equilibrium, we may write:

\[
\gamma^* = 1 - \frac{1}{T^*} \tag{8}
\]

It may be assumed that entrepreneurs are indifferent to the commuting distances of their workers as long as these do not influence their work performance. Under this assumption we may derive the demand discrepancy as follows.

The attractiveness of the labour market situation will reach a maximum for the entrepreneur in \( i \) when all positions offered (\( D_i \)) are occupied. The actual attractiveness can be derived from the actual number of positions filled (\( \sum_{h} E_{hi} \)), if the attractiveness of unfilled positions is put at zero. The demand discrepancy with regard to region \( i \) can then be written as

\[
T^d_i = \frac{D_i}{\sum_{h} E_{hi}} = \frac{V_i + \sum_{h} E_{hi}}{\sum_{h} E_{hi}} \tag{9}
\]

in which \( V_i \) indicates the number of vacancies. \( T^d_i \) will decrease when the number of positions actually filled in respect of the supply of positions increases. The minimum value is reached when \( V_i = 0 \); \( T^d_i \) then equals unity.

Similarly for the whole labour market we obtain

\[
T^d = \frac{\sum_{i} D_i}{\sum_{i} \sum_{h} E_{hi}} \tag{10}
\]
As a measure for the discrepancy between demand equilibrium and the actual situation we may define, analogously to the discrepancy relating to supply equilibrium,

$$\gamma^d = I - \frac{I}{\tilde{I}}$$  \hfill (11)

The discrepancy between a state of equilibrium and the actual situation on the labour market as a whole equals

$$\gamma = w_d \gamma^d + w_s \gamma^s$$  \hfill (12)

in which $w_d$ and $w_s$ represent the weights given to the demand side and the supply side of the labour market ($w_d + w_s = 1$).

3.2 The profession element in equilibrium

The preceding section was limited to the spatial aspects of the labour market. In this section the profession element will be introduced. It will be shown that basically the same approach is possible after the introduction of the concept of "distance" between professions. It is difficult to give a precise definition of this concept; we shall use as a preliminary definition that the distance between two professions represents the reciprocal of the propensity to shift from one profession to another, all other things being equal.

The distance between professions $j$ and $m$ can be written as $\delta_{jm}^s$, in which the superscript $s$ indicates that the distance is considered from the workers' point of view.

Write

$$a_{jm} = e^{-\delta_{jm}^s}$$  \hfill (13)

in which $a_{jm}$ represents the attractiveness of working in profession $m$ for somebody skilled in profession $j$. Since the distance between professions has not a priori a definite dimension, no coefficient need be introduced. Obviously, the value of $a_{jm}$ lies between zero and unity.

Combining the two aspects of space and profession, we may write

$$a_{jm}^l = e^{-\delta_{jl}^s} \cdot e^{-\delta_{jm}^s} = e^{-\delta_{jm}^s \left( \frac{\alpha}{\alpha} + d_{hl} \right)}$$  \hfill (14)

The term $\frac{\delta_{jm}^s}{\alpha} + d_{hl}$ is the sum of the elements $\frac{\delta_{jm}^s}{\alpha}$ and physical distance.
This implies that $\frac{\delta_{im}}{\alpha}$ had the dimension of physical distance.

The supply discrepancy for region $h$ and profession $m$ is now defined as

$$
(T^s)_h^m = \frac{S_h^m}{\sum_j \sum_i E_{ji}^m e^{-\alpha d_{hi} - \delta_{im}}} = \frac{U_h^m + \sum_j \sum_i E_{ji}^m}{\sum_j \sum_i E_{ji}^m e^{-\alpha d_{hi} - \delta_{jm}}} 
$$

Supply equilibrium is defined as the situation in which $d_{hi} = 0$, $\delta_{im} = 0$, and $U_h^m = 0$. In this case everybody working in profession $m$ and living in region $h$ finds work in his own profession and in his own region.

The supply discrepancy for all professions $m$ in region $h$ equals

$$
T_h^s = \frac{S_h}{\sum_j \sum_i \sum_m E_{hi}^m e^{-\alpha d_{hi} - \delta_{im}}} 
$$

Total supply discrepancy, consequently, equals

$$
T^s = \left( \sum_h \sum_m S_h \right) / \left( \sum_h \sum_m \sum_i \sum_j E_{hi}^m e^{-\alpha d_{hi} - \delta_{jm}} \right) 
$$

For the demand for labour a similar function can be written. Like before, the commuting distance will not play a role, but the distance between the profession required and the actual profession of the worker will. Indicating the attractiveness of this situation for an entrepreneur in $i$ by $c_{im}^d$, we may write

$$
c_{im}^d = e^{-\delta_{im}} 
$$

This time the distance has a superscript $d$ indicating that the distance between professions $j$ and $m$ is judged from the point of view of the entrepreneur.

The demand discrepancy for profession $m$ in region $i$ can thus be written as

$$
(T^d)_{i}^m = \frac{D_i^m}{\sum_j \sum_k E_{ki}^m e^{-\delta_{jm}}} 
$$
For the sake of simplicity no distinction is made between sectors. Total demand discrepancy can be written, analogously to total supply discrepancy:

\[ T^d = \frac{\sum_i D_i}{\sum_{i} \sum_{m} \sum_{j} E_{ji}^{lm} e^{-\delta_{jm}}} \]  

(20)

### 3.3 The significance of the results

The significance of the foregoing results is to be found in the fact that the formulas express tensions existing on the labour market that are not reflected in the conventional definitions of labour-market equilibrium. They show that considerable discrepancies in terms of either commuting distances or profession distances may exist even in a situation where there is a formal equilibrium on the labour market (no unemployment, no vacancies). It appears that the new approach has considerable advantages over the conventional ones, since the latter's definitions will never contribute to the explanation of the adaptation processes that result from the discrepancies. As such we have already mentioned migration, training and re-training; we may add that failure to adapt may lead to sickness and frustration. It seems that the definitions of employed, unemployed and vacancies leave too much undefined that is of basic importance for the explanation of actual behaviour of participants in the labour market.

### 4. Empirical verification of the functions

The authors realize that measuring the coefficients as well as the variables themselves in the functions indicated above will be extremely difficult. In the following some suggestions will be made, which are no more than pointers in a certain direction, waiting to be elaborated further.

The basic assumption is that the attractiveness of a given movement or shift manifests itself in the relative number of movements or shifts from one position to another.

The attractiveness for a person living in \( h \) skilled in profession \( j \) of working in \( i \) in profession \( m \) was given by

\[ a_{hi}^{lm} = e^{-\delta_{jm}} - \pi_{dhi} \]  

(21)
Since \( d_{hl} \) is known, we need estimates of \( \alpha \) and \( \delta_{jm}^l \) in order to be able to estimate \( a_{hl}^m \), the basic element of the supply discrepancy. Once all \( a_{hl}^m \) are known, the supply discrepancy can be calculated.

Now assume

\[
n_{jm} = n_j n_m e^{-\delta_{jm}^l} \quad (22)
\]

in which \( n_{jm} \) is the number of people shifting from profession \( j \) to profession \( m \), \( n_j \) the number of workers in profession \( j \) at the beginning of the period, and \( n_m \) the number of workers in profession \( m \) at the same point of time. As the scale by which \( \delta_{jm}^l \) is measured is irrelevant, (22) is presented without a scale coefficient.

Write

\[
\begin{align*}
\frac{n_{jm}}{n_j} &= n_m e^{-\delta_{jm}^l}, \text{ or} \\
\gamma_{jm} &= n_m e^{-\delta_{jm}^l} \\
\delta_{jm}^l &= \ln n_m - \ln \gamma_{jm}
\end{align*}
\]

In this equation \( n_m \) is supposed to be known. \( \gamma_{jm} \) can be derived from the transition matrix \( \Gamma \)

\[
\Pi_{t+1} = \Gamma \Pi_t \quad (24)
\]

which represents the fractions of the number of workers in one profession shifting to another during the period to which the matrix applies. Provided adequate information is available, it would be best to base oneself on professional shifts within one single region, so as to minimize the disturbing effect of differences in commuting distance.

The foregoing leads to an estimate of \( \delta_{jm}^l \).

The estimation of \( \alpha \) might proceed as follows.

Assume

\[
n_{hl} = \frac{A_{hl} e^{-\alpha d_{hl}}}{\sum_i A_{ih} e^{-\alpha d_{ih}}} n_h \quad (25)
\]

in which \( n_{hl} \) is the number of people commuting from \( h \) to \( i \), \( A_{hl} \) is the attractiveness of the labour market in \( i \) for workers in \( h \), to be calculated on the basis of the knowledge of all \( \delta_{jm}^l \)'s gathered in the first stage. Similarly, we may write

\[
n_{kl} = \frac{A_{kl} e^{-\alpha d_{kl}}}{\sum_i A_{ik} e^{-\alpha d_{ik}}} n_k \quad (26)
\]
From (25) and (26) we can derive

\[
\frac{n_{h1}}{n_k} = \frac{A_{h1}}{e} e^{-\alpha (\delta_{h1} - \delta_{k1})} \frac{n_h}{n_k}
\]  

(27)

from which \( \alpha \) can be estimated.

It appears that the method is relatively simple and straightforward. However, the search for a better method, in which the \( \delta \)'s and \( \alpha \) are estimated simultaneously, should continue.

For the demand side the situation is in fact simpler, though probably more difficult to measure from general data. Surveys within firms will very likely have to provide the necessary data. Theoretically, however, the procedure is quite simple. If

\[
\frac{\partial \varphi}{\partial n_{1j}} = p_{1j}
\]

is the marginal productivity of a worker skilled in profession \( j \) and working in the same profession, and

\[
\frac{\partial \varphi}{\partial n_{mj}} = p_{mj}
\]

is the productivity of a worker skilled in profession \( m \) but working in profession \( j \), then the attractiveness of employing the latter rather than the former is

\[
\frac{\partial \varphi}{\partial n_{mj}} / \frac{\partial \varphi}{\partial n_{1j}} = p_{mj}/p_{1j}
\]

(30)

Since the attractiveness of employing an m-worker in a j-profession has been defined as

\[
c_{mj} = e^{-\delta_{mj}^d}
\]

(31)

we easily find

\[
\delta_{mj}^d = -\ln \frac{p_{mj}}{p_{1j}}
\]

(32)

5. Final remark

It is not unlikely that there are differences between the values that \( \alpha \) and even the \( \delta \)'s take for different social groups, and it seems \textit{a priori}
important to focus studies on these differences. Age groups, and the worker's sex, will also have to be taken into account.

Moreover, though the two factors covered in the preceding sections will constitute the body of the analysis, it may be assumed that other factors, too, are of importance. The two authors will in the near future be involved in studies in these fields which they feel might give some insight into labour-market problems, problems that seem more important now than ever before.

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