The Stability of the Covariances of International Property Share Returns

Piet M. A. Eichholtz*

Abstract. This paper looks at the covariance structure of international property share returns. Portfolio models, which are used to generate efficient international asset allocations, require estimates of a covariance structure of asset returns as input. Usually, the realized structure is used as a proxy, but that is only valid if this structure is stable. We test for this stability. We find covariances of international property share returns to be unstable, while correlations are stable between some time-periods, and unstable between others. The results cast some doubts on the use of standard portfolio models for the allocation of international real estate portfolios.

Introduction

International investments can reduce the risk of investment portfolios since asset returns in different countries are not perfectly correlated. For stocks, the extent of this risk reduction has been investigated in numerous studies, of which Grubel (1968), Levy and Sarnat (1970), and Grubel and Fadner (1971), were among the first. For real estate, this subject has received less attention. Nevertheless, the characteristics of real estate investments outside of the United States would be an important research topic. Ibbotson, Siegel and Love (1985) estimate this asset category's value to be 37% of total world wealth in 1984.

Only a few attempts have been made to examine the risk reduction possibilities of international real estate investments. This is mainly due to lack of data. Sweeney (1989) determines the optimal international real estate allocation for a British investor by using a Markowitz (1952) approach. She uses international rental growth rates as a basis for her analysis. Giliberto (1990a) and Giliberto and Testa (1990) use the returns on property shares from different countries to determine the influence of international diversification on the risk of the real estate portfolio of an American investor. Gordon (1992) calculates efficient frontiers for investments in United States and United Kingdom real estate. As inputs for his portfolio model he uses appraisal-based returns indices. All four studies find that real estate returns in different countries are weakly correlated and that international diversification can lower real estate portfolio risk. Consequently, they find that portfolio allocations to international real estate are substantial regardless of whether the investor is British or American.

Determining the optimal allocation of international real estate portfolios has been more rare. Of the referenced work, only Sweeney (1989), Giliberto (1990a) and Giliberto and Testa (1990) determine the composition of efficiently diversified international real

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estate portfolios. As inputs for this they use the realized covariance matrix of their return series. The generated efficient portfolio compositions are counterintuitive: the allocations to important cities and countries are zero or negligible, while other cities and countries receive allocations of up to 100% of the portfolio.

Obtaining portfolio weights using Markowitz models requires estimates of the international covariance structure of property returns. In the international real estate investment literature, this is done by using the realized covariance figures. This, however, is only justified if the international covariance structure of asset returns is stable in time. If this is not the case, then using the realized covariance structure results in suboptimal portfolio compositions.

However, even if realized covariances would be the best possible predictor of future covariances, we still would have a problem. As we already stated, the weights of optimal portfolios depend on international covariances. If these covariances are not stable, neither will be portfolio weights. A constantly changing covariance structure will therefore require constant rebalancing of the international property portfolio. This implies that even a so-called passive strategy aimed at optimal diversification would require active management to keep the portfolio efficient. Thus, the stability of the variances and of the correlations of international property share returns are essential to the construction of international real estate portfolios.

Only very few authors have investigated the stability of the covariance structure of international asset returns. Most papers so far published investigate international common stock returns. Kaplanis (1988) investigates for ten countries the stability of the international correlation and covariance matrices of monthly stock market returns for the period 1967 through 1982. She compares matrices estimated over four equal subperiods using a test developed by Jennrich (1970). The null hypothesis that the correlation matrix is stationary over any two time periods—adjacent or not—could not be rejected at the 85% confidence level. On the other hand, the null hypothesis of a stationary covariance matrix could be rejected at the 95% confidence level for most subperiods compared. Longin and Solnik (1993) test the stationarity of correlation and covariance matrices with the same test, but use a longer time period: 1960 through 1990. They find that the correlation matrix is stable between some time periods, and unstable between others. The covariance matrix, on the other hand, was found to be unstable for all time periods. In both papers, the test statistics for consecutive periods tend to be somewhat lower than for nonconsecutive periods. This indicates a gradual change of the true covariance matrix, which implies that using realized covariance matrices as inputs for portfolio models only yields reliable results for the short-term optimal portfolio composition. Moreover, it implies that the covariance matrices used for portfolio optimization should be estimated over relatively short historic return series. The question is whether this also holds for real estate portfolios.

So far, researchers of international real estate investments were not able to test for the stability of the international covariance structure, due to lack of property data with a long enough history. We have monthly property company returns data over twenty years, which allows us to test the stability of the covariance and correlation matrix in a reliable way.¹

In real estate markets, return-influencing events tend to be of a relatively local nature. Hartzell, Shulman and Wurtzebach (1987) and Mueller (1993) have shown that regional real estate markets are influenced by dynamics in regional economies. This causes non-
simultaneous developments in different real estate markets. On the other hand, real estate markets are becoming more and more open and international property-related capital flows are growing (Lizieri and Denham, 1993). This suggests an increasing interdependence of international real estate markets.

In this paper we investigate the stability of the international correlation and covariance structure of property company returns. The paper proceeds as follows. In section two we describe the international property company data we use. Section three explores the existing research concerning stability of the variance and covariance in international asset returns. Also in this section, we present some tests of this stability for property share returns, and their results. In section four we discuss some of the consequences of the results of our analysis. Section five concludes the paper.

Data

We use the national property company indices from Datastream. These indices reflect market-weighted total returns on the stocks of publicly listed property companies. With a few exceptions, the property companies included in the indices invest in real estate in their own country. Included are companies investing in various kinds of real estate and in various regions within their country. Appendix A provides the companies included in each country index. For Belgium, France, Italy, the United Kingdom, Australia, Japan, Singapore, Canada, and the United States, these indices have sufficient length for our purposes. For all nine countries except Canada, the time series run from February 1973 through May 1993. This period is long enough to divide it into subperiods of sufficient length to make reliable inferences regarding the stability of the covariance structure.

Sample statistics in local currency and market values in United States dollars of the Datastream property share indices are included in Exhibit 1. For the full sample period, the average logarithmic monthly returns range from 0.51% for the United States to 1.34% for Australia. The returns differ substantially over subperiods. From the same table one can see that property shares have been quite risky investments in the sample period. The standard deviations of monthly returns are between 4% and 14% for the full period and all subperiods.

Exhibit 2 gives the full sample correlation matrix of the Datastream property share returns. The diversification potential of international property share investments has been substantial in our sample period. Although all correlations are positive, they are not high, ranging from 0.07 between Italy and Japan to 0.44 between Singapore and Australia. If we compare these correlations to Giliberto's (1990a) matrix, which is based on property share data from January 1985 through December 1989, we see differences between individual corresponding correlations. Moreover, for the same countries we investigate, Giliberto's correlations range more widely, namely from −0.08 to 0.61.

The Stability of Correlations, Variances and Covariances

Sharpe's oft quoted dictum says that historic returns may tell us something about covariance, less about risk and almost nothing about expected return. From Exhibit 1, we indeed see that property share returns and standard deviations differ substantially from subperiod to subperiod. We will now see whether Sharpe's observation also holds for correlations and covariances of property share returns.
### Exhibit 1

**Datastream Property Share Indices, Logarithmic Monthly Returns, Local Currency**

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>73/2-93/5</th>
<th>73/2-78/2</th>
<th>78/3-83/3</th>
<th>83/4-88/4</th>
<th>88/5-93/5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>263</td>
<td>.81</td>
<td>.41</td>
<td>.51</td>
<td>1.96</td>
<td>.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(6.45)</td>
<td>(4.43)</td>
<td>(6.87)</td>
<td>(6.73)</td>
<td>(7.26)</td>
</tr>
<tr>
<td>France</td>
<td>2,664</td>
<td>1.02</td>
<td>-.25</td>
<td>1.70</td>
<td>1.85</td>
<td>.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(6.86)</td>
<td>(5.12)</td>
<td>(5.76)</td>
<td>(9.25)</td>
<td>(6.36)</td>
</tr>
<tr>
<td>Italy</td>
<td>931</td>
<td>.98</td>
<td>-1.17</td>
<td>3.10</td>
<td>.81</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(8.55)</td>
<td>(7.78)</td>
<td>(11.61)</td>
<td>(7.79)</td>
<td>(5.24)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>10,294</td>
<td>.95</td>
<td>.29</td>
<td>1.50</td>
<td>1.77</td>
<td>.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(8.94)</td>
<td>(14.06)</td>
<td>(6.22)</td>
<td>(6.73)</td>
<td>(6.02)</td>
</tr>
<tr>
<td>Australia</td>
<td>2,722</td>
<td>1.34</td>
<td>.17</td>
<td>1.69</td>
<td>2.77</td>
<td>.73</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(9.51)</td>
<td>(12.63)</td>
<td>(6.56)</td>
<td>(11.07)</td>
<td>(5.73)</td>
</tr>
<tr>
<td>Japan</td>
<td>30,831</td>
<td>.57</td>
<td>.05</td>
<td>.67</td>
<td>2.35</td>
<td>-.79</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(8.18)</td>
<td>(6.83)</td>
<td>(4.13)</td>
<td>(10.41)</td>
<td>(9.52)</td>
</tr>
<tr>
<td>Singapore</td>
<td>4,140</td>
<td>.64</td>
<td>-1.80</td>
<td>3.06</td>
<td>.02</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(13.23)</td>
<td>(17.31)</td>
<td>(12.58)</td>
<td>(13.30)</td>
<td>(7.25)</td>
</tr>
<tr>
<td>Canada</td>
<td>1,337</td>
<td>.60</td>
<td>na</td>
<td>2.87</td>
<td>1.26</td>
<td>-2.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(11.27)</td>
<td>na</td>
<td>(14.05)</td>
<td>(8.13)</td>
<td>(9.83)</td>
</tr>
<tr>
<td>United States</td>
<td>5,093</td>
<td>.51</td>
<td>-1.45</td>
<td>1.77</td>
<td>.92</td>
<td>.81</td>
</tr>
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<td></td>
<td></td>
<td>(6.66)</td>
<td>(8.46)</td>
<td>(5.22)</td>
<td>(6.29)</td>
<td>(5.78)</td>
</tr>
</tbody>
</table>

*Notes:* For Canada, the time period is March 1978–May 1993. Market values at December 1992, in millions of US dollars.

### Exhibit 2

**Correlation Matrix of Property Share Returns, Logarithmic Monthly Returns, Local Currency, 1973/2–1993/5**

<table>
<thead>
<tr>
<th></th>
<th>bg</th>
<th>fr</th>
<th>it</th>
<th>uk</th>
<th>au</th>
<th>jp</th>
<th>sg</th>
<th>cn</th>
</tr>
</thead>
<tbody>
<tr>
<td>bg</td>
<td>.26 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fr</td>
<td>.23 ***</td>
<td>.20 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>it</td>
<td>.15 ***</td>
<td>.22 ***</td>
<td>.18 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>uk</td>
<td>.16 ***</td>
<td>.26 ***</td>
<td>.23 ***</td>
<td>.32 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>au</td>
<td>.10 *</td>
<td>.30 ***</td>
<td>.07</td>
<td>.16 ***</td>
<td>.17 ***</td>
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<tr>
<td>jp</td>
<td>.11 **</td>
<td>.18 ***</td>
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</tr>
<tr>
<td>cn</td>
<td>.14 **</td>
<td>.33 ***</td>
<td>.10 *</td>
<td>.22 ***</td>
<td>.36 ***</td>
<td>.20 ***</td>
<td>.37 ***</td>
<td>.13 **</td>
</tr>
</tbody>
</table>

*Notes:* Correlations for Canada are calculated for the period March 1978–May 1993. bg = Belgium, fr = France, it = Italy, uk = United Kingdom, au = Australia, jp = Japan, sg = Singapore, cn = Canada, us = United States.

*, **, *** denote significance at the 10%, 5%, 1% levels.
We perform tests on both the covariance matrix and the correlation matrix of property share returns, using the Jennrich (1970) test. More information about this test is included in Appendix B. We present the results of the analysis of returns expressed in local currency, and in nominal terms. These are the returns earned by an investor who is fully hedged against currency risk, but not hedged against inflation risk. We also did stability tests when the covariance matrix was expressed in United States dollars, and/or was based on real returns. Although these covariance matrices differed slightly from the local currency case, the results of the stability tests were very similar and are therefore not presented here.

Except for the Canadian time series of property share returns we divide all time series into four equal subperiods of sixty-one months. This is long enough to yield reliable results (see Kaplanis, 1988). The Jennrich test allows for subperiods of unequal length, but since we are unable to identify structural breaking points in the covariance structure, we use four equal periods. The four periods are February 1973 through February 1978, March 1978 through March 1983, April 1983 through April 1988, and May 1988 through May 1993. The Canadian time series is divided into three subperiods beginning and ending simultaneously with the second through fourth subperiod of the other time series. In Exhibit 3 we present results of the Jennrich test for stability of the correlation matrix and the covariance matrix. When comparing the first period matrix to other matrices, we exclude the Canadian correlations.

We first test the null hypothesis that the correlation matrix is stable. The Jennrich \( \chi^2 \) statistics in the third column of Exhibit 3 indicate that stability of the correlation matrix is not rejected at conventional significance levels, except when comparing the first and second periods to the third period. The statistics reported in the fourth column of Exhibit 3 indicate that the covariance matrix is unstable regardless of the time periods we compare. Kaplanis (1988) and Longin and Solnik (1993) have obtained similar results for common stock returns, although the instability of the covariance matrix we observe is more significant. This can be explained by the more local nature of property markets

Exhibit 3
Stability of the Correlation and of the Covariance Matrix, Results of the Jennrich Test

<table>
<thead>
<tr>
<th>Periods Compared</th>
<th>Correlations Jennrich Chi-Square</th>
<th>Covariances Jennrich Chi-Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb 73/Feb 78</td>
<td>33.72</td>
<td>121.95**</td>
</tr>
<tr>
<td>Mar 78/Mar 83</td>
<td>56.18*</td>
<td>124.58**</td>
</tr>
<tr>
<td>Apr 83/Apr 88</td>
<td>40.72</td>
<td>83.37**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feb 73/Feb 78</td>
<td>46.72**</td>
<td>125.05**</td>
</tr>
<tr>
<td>Mar 78/Mar 83</td>
<td>44.84</td>
<td>115.33**</td>
</tr>
<tr>
<td>Feb 73/Feb 78</td>
<td>33.21</td>
<td>148.08**</td>
</tr>
</tbody>
</table>

Note: *, ** denote significance at 5%, 1% levels.
relative to the stock markets. We do not find lower test statistics for consecutive periods than for nonconsecutive periods, which suggests that the correlation and covariance matrices do not change gradually over time. A priori, we can therefore make no judgement about the length of the estimation period for the calculation of international correlation and covariance matrices of real estate returns.

These findings imply that the variances of international property share returns are less stable than their correlations. This agrees with intuition. The correlation between asset returns on different capital markets is a measure for the degree of integration of these markets, and integration is not likely to change suddenly. Rather, with increasing international capital flows, financial markets become more integrated relatively slowly. Even if governments abolish all restrictions on international capital movements from one day to the next, cross-border capital flows will not emerge immediately on a large scale. On the other hand, variances may change while correlations remain constant, if variabilities of international property returns change simultaneously and equivalently. Events affecting the international real estate markets in the same way can have such consequences. An example of such an event could be the oil crisis.

The question remains whether the observed instability of the covariance matrix is structural or merely caused by a uniform change in the individual variances and covariances. Therefore, we examine the stationarity of the variances of national property share returns, also using the Jennrich $\chi^2$ statistic. The results are presented in Exhibit 4. For all countries, variances are nonstationary between most of the subperiods. This suggests structural changes of the covariance matrix rather than just a uniform change in the level of the covariances. However, this result should be treated carefully since it could simply reflect the fact that there are less true changes of covariances than of correlations.

### Exhibit 4

**Stability of the Variances, Results of the Jennrich Test**

<table>
<thead>
<tr>
<th>Countries</th>
<th>a to b</th>
<th>b to c</th>
<th>c to d</th>
<th>a to c</th>
<th>b to d</th>
<th>a to d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>10.41***</td>
<td>.03</td>
<td>.35</td>
<td>9.55***</td>
<td>.18</td>
<td>12.79***</td>
</tr>
<tr>
<td>France</td>
<td>.84</td>
<td>11.89***</td>
<td>7.81***</td>
<td>17.23***</td>
<td>.60</td>
<td>2.80**</td>
</tr>
<tr>
<td>Italy</td>
<td>8.84***</td>
<td>8.79***</td>
<td>8.67***</td>
<td>.00</td>
<td>26.73***</td>
<td>8.63***</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>27.58***</td>
<td>.37</td>
<td>.75</td>
<td>24.01***</td>
<td>.07</td>
<td>29.06***</td>
</tr>
<tr>
<td>Australia</td>
<td>20.15***</td>
<td>14.05***</td>
<td>20.30***</td>
<td>1.05</td>
<td>1.10</td>
<td>26.42***</td>
</tr>
<tr>
<td>Japan</td>
<td>13.17***</td>
<td>32.37***</td>
<td>.49</td>
<td>9.70***</td>
<td>28.48***</td>
<td>6.25**</td>
</tr>
<tr>
<td>Singapore</td>
<td>5.82**</td>
<td>.19</td>
<td>17.93***</td>
<td>4.06**</td>
<td>15.36***</td>
<td>30.06***</td>
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<tr>
<td>Canada</td>
<td>na</td>
<td>15.13***</td>
<td>2.13</td>
<td>na</td>
<td>7.18***</td>
<td>na</td>
</tr>
<tr>
<td>United States</td>
<td>12.26***</td>
<td>2.06</td>
<td>.42</td>
<td>5.07**</td>
<td>.64</td>
<td>8.03***</td>
</tr>
</tbody>
</table>


*, **, *** denotes significance at the 10%, 5%, 1% levels.
Alternatives

Kaplanis (1988) further compares alternative ways to forecast the correlation and covariance structures of international asset returns. Unfortunately, the forecasting techniques she tests hardly perform better than the simple historic model, which is to use the terms of the matrix of the previous subperiod. Our correlations and covariances are even less stable than those found by Kaplanis, which leads one to expect that predictions regarding them are even harder to make.

The inability to forecast the international covariance matrix of real estate returns casts further doubt on the use of portfolio models to generate the optimal international real estate portfolio allocation. An alternative would be to adjust these models for the possibilities of time-varying covariances. However, even then they do not yield satisfactory results. Engel and Rodrigues (1989, p. 134) test the international CAPM with time-varying covariances and conclude that "the model does not get a shred of support."

A second alternative to achieve the optimal international allocation of the real estate portfolio is to use the composition of the world real estate portfolio. This method would build on work that has been done for the United States by Miles et al. (1991) and Hartzell, Pitman and Downs (1994). In this work, the market portfolio is treated as the base portfolio, from which investors can deviate according to their expectations of future performance, but at the cost of adding diversifiable risks to market risk. The basic idea behind this approach is also used by stock index investors. Due to data limitations, this method has so far not been applied to international real estate portfolios. Especially estimating market values of real estate in different countries can be difficult. Besides, even if the weights of the world real estate portfolio would be known, they would not be the weights of the optimal international real estate portfolio. As Solnik (1991, p. 40) notes, "in a fully efficient integrated, international capital market, buying the world market portfolio would be the natural strategy." However, international real estate markets are segmented due to currency risk, differences in taxation and barriers to capital flows. Therefore, the market portfolio's weights have to be corrected for the effects of this segmentation. All this, however, remains for future research.

Conclusions

In this paper, we have investigated whether correlations, variances and covariances between international property share returns are stable. Our findings indicate that correlations, although not always stable, are more stable than variances and covariances. Nevertheless, the instability of the covariances limits the use of standard portfolio models to determine the allocation of international real estate securities investments.

The market values of real estate in different countries can serve as an alternative aide to determine that allocation. Therefore, future research in this area should focus on estimating the value of the real estate wealth in countries other than the United States, and on the way these values have to be adjusted for the effects of international capital markets segmentation in order to be used as international real estate portfolio weights.

Another interesting extension of this research would be to explain the changing correlations and covariances in terms of the growing real estate-related international capital flows and/or the decreasing number of restrictions to free capital movements.
Appendix A
Composition of Datastream Property Share Indices, May 1993

Belgium: Brederode, Elia AFV, Immobreal.
France: Credit Foncières France, Immeubles de France, Sogeparc.
Italy: Aedes, Immobiliare Metanopo, Risanamento.
United Kingdom: Argent Group, Bilton, Birkby, Bradford Properties, British Land, Brixton
  Estate, Burford Holding, Capital Shopping Centers, Chelsfield, CLS Holdings, Deajan Holdings,
  Evans of Leeds, Frogmore Estates, Greycoat, Great Portland Estate, Hambro Country, Hammerson,
  Land Securities, London Merchant Securities, MEPC, A&J Mucklow Group, Peel Holdings, PSIT,
  Slough Estates, Smith Estates, Town Centre Securities, Warner Estate.
Australia: Accor Asia Pacific, Burswood Property Trust, Gandel Retail Trust, Grosvenor Trust,
  Lend Lease, Westfield Holding.
Japan: Daihatsu Corporation, Daikyo, Daiwa Danchi, Daiwa Kosho Lease, Hankyu Realty, Heiwa
  Real Estate, Mitsubishi Estate, Mitsui Fudosan, Mitsui Real Estate, Pasco, Recruit Cosmostok,
  Sekisui House, Sumitomo Real Estate & Development, TOC, Toho Real Estate, Tokyo Tatemono,
  Tokyotokai, Tokyo Land, Towa Real Estate, Urban Life. 
Singapore: Bonvest Holdings, Central Properties, Centrepoint Properties, City Development,
  DBS Land, Hong Fok Corporation, Singapore Land, United Overseas Land, Wing Tai Holdings.
Canada: Cambridge Shopping Centers, Hammerson Property Investments, Markborough
  Properties, Monarch Developments.
United States: Debartolo Realty Corporation, Federal Realty Investment Trust, Hospitality
  Franchise, New Plan Realty Trust, Rouse, Simon Properties, United Dominion Realty Trust,
  Vornado Realty Trust, Weingarten Realty.

Appendix B
The Jennrich Test

For a formal test of the stability of correlation or covariance matrices over two periods, one could
simply calculate the average and standard deviation of the correlations or covariances in the first
period and in the second period, and do a t-test to see whether the difference between the
averages is significant. However, such a test would only take into account the values and the
number of pairwise correlations or covariances, while the length of the time series on which they
are based would be disregarded. The longer these time series, the more reliable is the estimation
of the correlations and the covariances. This reliability is information that should be used when
investigating the stability of correlations or covariances. Thus, a valid test for the stability of
the correlation and covariance should take into account the number of observations upon which the
matrices are based. The Jennrich $\chi^2$ test statistic for the equality of correlation and covariance
matrices does this. The test was developed to investigate the equality of two correlation or
covariance matrices. Testing for the stability of correlations and covariances is a special case. The
statistic is:

$$\chi^2 = \frac{1}{2} \text{tr}(Z^2) - \text{diag}'(Z)S^{-1}\text{diag}(Z).$$

Here, $Z = a^{12}R^{-1}(R_1 - R_2)$, in which $R = (n_1R_1 + n_2R_2)/(n_1+n_2)$ and $c = (n_1n_2)/(n_1+n_2)$, with $R_1$ and $R_2$ the
correlation matrices to be compared, and $n_1$ and $n_2$ the number of observations on which they
are based. Furthermore, $S = (\delta_i + r_i r^i)$, with $\delta_i$ the Kronecker delta, $r_i$ the elements of $R$, and $r^i$ the
elements of $R^{-1}$. The Jennrich test statistic has $p(p-1)/2$ degrees of freedom, with $p$ the
dimension of the correlation matrices. To get the Jennrich $\chi^2$ statistic for the equality of two
covariance matrices, the second term of the equation is omitted. In that case, the statistic has
$p(p+1)/2$ degrees of freedom. More information about the test can be found in Jennrich (1970).
Notes

Apart from property share returns, very little international real estate data is available. The question is whether the use of property share returns limits the applicability of the results. The debate concerning the use of property company shares as proxies for real estate investments has not yet reached a firm conclusion. Mengden and Hartzell (1986) have shown that property share returns have a higher contemporaneous correlation with the stock market than with appraisal-based indices. On the other hand, Giliberto (1990b) and Gyourko and Keim (1992) have found for the United States that property share indices are also related to other real estate indices. Eichholtz and Hartzell (1996) have confirmed this for other countries. These three studies all conclude that property share returns can predict appraisal-based returns.

For Canada, this period is March 1978 through May 1993.

\textsuperscript{3}R_1 and R_2 are now 1 by 1 matrices of variances of national property share returns in subperiods 1 and 2. In this case, the Jennrich $\chi^2$ test statistic has 1 degree of freedom.

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