Forward foreign exchange rates and expected future spot rates

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This paper explores whether knowledge of the time-series properties of the premium in the pricing of forward foreign exchange can be usefully exploited in forecasting future spot exchange rates. Signal-extraction techniques, based on recursive application of the Kalman filter, are used to measure the premium. Predictions using premium models compare favourably with those obtained from the use of the forward rate as a predictor of the future spot rate. The results also provide an interesting description of the time-series properties of the premium.

I. INTRODUCTION

Forward discount bias is a phenomenon that has been studied extensively in the literature. In addition to forward exchange unbiasedness being rejected, it is generally found that the change in the future exchange rate is negatively related to the forward discount. A prominent explanation for the rejection of forward rate unbiasedness is the existence of a time-varying risk premium. Other explanations involve peso problems, irrationality of expectations, learning behaviour and market inefficiency. Useful surveys of the empirical findings in this area are provided by Hodrick (1987), Lewis (1995) and Engel (1996). In this paper we will remain agnostic on the exact source of forward exchange bias and refer to the 'premium', which is not necessarily the same as a risk premium, but can also capture elements of the other explanations. The same vocabulary is used in Fama (1984) and Clarida and Taylor (1987).

There exists a substantial literature on the presence or absence of premia (possibly time-varying) in the pricing of forward contracts for foreign exchange. Conditional on the hypothesis that the foreign exchange market is efficient or rational, the existence of time-varying risk premia has been documented in the literature, among others, by, Hansen and Hodrick (1983), Fama (1984), Hodrick and Srivastava (1984), Korajczyk (1985) and Wolff (1987a). Deviations from rational expectations were documented, among others, by Froot and Frankel (1989) and Cavaglia et al. (1994).

In this paper we will explore whether knowledge about the time series properties of premia can be usefully exploited in forecasting future spot exchange rates. In order to identify such premia, we follow Wolff (1987a) and Nijman et al. (1993) in taking a signal-extraction approach, based on recursive application of the Kalman filter. This approach provides a natural framework for modelling the premium as an unobservable component in order to generate forecasts that take advantage of the time series properties of the premium.

The paper is organized as follows. In Section II some evidence on the properties of current forward and spot exchange rates as predictors of future spot rates is presented. In Section III the methodology that is used in this paper is explained: here the Kalman filter is described in some detail. In Section IV the prediction results that were obtain on the basis of the premium models that are constructed in the previous section are presented. Section V offers some conclusions.

II. CURRENT SPOT AND FORWARD RATES AS PREDICTORS OF FUTURE SPOT RATES

A number of researchers have investigated the properties of forward exchange rates as predictors of future spot rates,
The $t$-statistic of the random walk forecast is one by definition.

Table 1. Summary statistics on the forecasting performance of current forward and spot exchange rates as predictors of one-month-ahead future spot rates

<table>
<thead>
<tr>
<th>Rate</th>
<th>ME$^a$</th>
<th>MAE$^a$</th>
<th>RMSE$^a$</th>
<th>$U$-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$/$pound</td>
<td>-0.22</td>
<td>1.98</td>
<td>2.57</td>
<td>1.021</td>
</tr>
<tr>
<td>$/$mark</td>
<td>-0.31</td>
<td>2.38</td>
<td>3.13</td>
<td>1.017</td>
</tr>
<tr>
<td>$/$yen</td>
<td>-0.15</td>
<td>2.13</td>
<td>2.94</td>
<td>1.034</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spot rate</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$/$pound</td>
<td>-0.43</td>
<td>1.92</td>
<td>2.52</td>
<td>1.000$^b$</td>
</tr>
<tr>
<td>$/$mark</td>
<td>0.00</td>
<td>2.31</td>
<td>3.08</td>
<td>1.000$^b$</td>
</tr>
<tr>
<td>$/$yen</td>
<td>0.06</td>
<td>1.98</td>
<td>2.84</td>
<td>1.000$^b$</td>
</tr>
</tbody>
</table>

Note: $^a$ The Me, MAE, and RMSE are approximately in percentage terms.

The $U$-statistic of the random walk forecast is one by definition.


Table 1 presents summary statistics on the one-month-ahead forecasting performance of the forward rate and the once lagged spot rate for three important exchange rates involving the US dollar: the dollar/pound, dollar/mark and dollar/yen exchange rates. For reasons of comparison, the original dataset that was employed by Wolff (1987a) and Nijman et al. is studied (1993) to develop the unobserved components models. Spot exchange rates and 30-day forward rates are taken from the Harris Bank Database that is supported by the Center for Studies in International Finance at the University of Chicago. The rates are logs of US dollars per unit of foreign currency. There are 148 observations covering the period 6 April 1973 to 13 July 1984.

Forecasting accuracy is measured by four summary statistics that are based on standard symmetric loss functions: the mean error (ME), the mean absolute error (MAE), the root mean squared error (RMSE) and the $U$-statistic. Theil's $U$-statistic is the ratio of the RMSE to the RMSE of the random walk forecast. Because we are looking at the logarithm of the exchange rate, the ME, MAE and RMSE are unit-free (they are approximately in percentage terms) and comparable across currencies. By comparing predictors on the basis of their ability to predict the logarithm of the spot exchange rate, we circumvent any problems arising from Jensen's inequality. Because of Jensen's inequality the best predictor of the level of the spot exchange rate expressed as units of currency $i$ per unit of currency $j$ may not be the best predictor of the level of the spot exchange rate expressed as units of currency $i$ per unit of currency $j$. Table 1 shows some basic forecasting results for our dataset. Consistent with the previous literature the results in Table 1 indicate that the current spot rate is a better predictor of the future spot rate than the current forward rate, both in terms of mean absolute error and root mean squared error.

In Table 2 autocorrelations of the forecast error $F(t, t + 1) - S(t + 1)$, the log of the thirty-day forward rate, observed at month $t$ for a contract maturing at $t + 1$, minus the log of the corresponding spot rate observed around the maturity date are presented. Significant autocorrelations are reported in a number of cases, in particular at lag one. These autocorrelations give us information about the time series properties of the underlying premium terms. In Section III a hypothesis concerning the autocorrelation patterns that are observed in Table 2 is advanced.

### III. MODELLING THE PREMIUM

The model

The forward exchange rate can be conceptually divided into an expected future spot rate and a premium term:

$$ F(t, t + 1) = E[S(t + 1)|t] + P(t) $$

(1)

where $E[S(t + 1)|t]$ is the rational or efficient forecast of the log of the spot rate at time $t + 1$, based on all information available at time $t$, and $P(t)$ is the risk premium. Different asset pricing models give different expressions for the premium term. Subtracting $S(t + 1)$ from both sides in Equation 1, we obtain

$$ F(t, t + 1) - S(t + 1) = E[S(t + 1)|t] - S(t + 1) + P(t) $$

(2)

and if we define $v(t) = (E[S(t + 1)|t] - S(t + 1)$, we have

$$ F(t, t + 1) - S(t + 1) = P(t) + v(t) $$

(3)

where $[v(t)]$ is an uncorrelated, zero mean sequence, given our assumption of rational expectations. Equation 3 states...
that the forecast error of the forward rate as a predictor of the future spot rate consists of a premium component and a white noise error term due to the arrival of new information concerning the spot rate between times \( t \) and \( t + 1 \). If we have information about the time series properties of the premium term, this knowledge could potentially be usefully combined with the forward rate in generating forecasts of future spot exchange rates. Note that this methodology is not necessarily restricted to identifying risk premia: if persistent peso problem terms, market inefficiencies, or terms related to learning behaviour are present, they can be ‘picked up’ in the same manner.

From Equation 3 it follows that the autocorrelations that were presented in Table 2 in the previous section are also the autocorrelations of the combined time series process \([P(t) + \nu(t)]\). It is convenient to refer to the premium component \( P(t) \) as the signal that we would like to characterize and to \( \nu(t) \) as noise that is added to the signal. The problem that we face can thus be characterized as extracting a signal from a noisy environment.

In this section we take a Kalman-filtering approach to signal-extraction. In order to apply the Kalman filter, some assumptions about the time series properties of the premium must be made. On the basis of the autocorrelation patterns that we reported in Table 2, we conjecture that the premium process \([P(t)]\) may be adequately described by an autoregressive time series process of order one:

\[
P(t) = \alpha P(t-1) + \xi(t)
\]

(4)

where \( \alpha \) is a constant autocorrelation coefficient and \([\xi(t)]\) is an uncorrelated, zero mean sequence. It can be shown that, under this assumption together with the assumption that \(|\alpha| < 1\), the combined process \([P(t) + \nu(t)]\) has autocorrelations \( \rho_j \) for \( j = 1, 2, 3, \ldots \), where \( \rho_j \) is defined as:

\[
\rho_j = \alpha^j - \alpha^j / [(\sigma_\xi^2 / \sigma_\nu^2) / (1 - \alpha^2) + 1] = \alpha^j - \alpha^j / [[(\sigma_\xi^2 / \sigma_\nu^2) / (1 - \alpha^2) + 1]
\]

(5)

Here \( \sigma_\xi^2 \) and \( \sigma_\nu^2 \) denote the variances of \( \xi(t) \) and \( \nu(t) \), respectively. For instance, if \( \alpha = 0.9 \) and \( \sigma_\xi^2 / \sigma_\nu^2 = 0.05 \), then we obtain \( \rho_1 = 0.188 \), \( \rho_2 = 0.169 \), \( \rho_3 = 0.152 \), etc. Thus, we expect to find fairly small positive autocorrelations that decay exponentially from the starting value \( \rho_1 \). Given the fact that autocorrelations of such small magnitude are hard to detect in finite samples, we argue that the results in Table 2 are roughly consistent with our hypothesis concerning the time series properties of \( P(t) \).

In the forecasting experiment that is undertaken in the next section, the two-equation model given by Equations 3 and 4 is considered. In addition, the following properties are assumed:

a. \( E[\nu(t)] = 0 \), \( \text{var}[\nu(t)] = \sigma_\nu^2 \)

b. \( E[\xi(t)] = 0 \), \( \text{var}[\xi(t)] = \sigma_\xi^2 \)

c. \( [\nu(t)] \) is an independent sequence

d. \( [\xi(t)] \) is an independent sequence

e. \( \nu(t) \) and \( \xi(r) \) are independently distributed for all \( r, t \)

f. \( [\nu(t), \xi(t)] \) and \( P(r) \) are independently distributed for all \( r, t \).

With these assumptions, the system Equation 3-4 is easily recognized as a state-space model, which can be recursively estimated by means of the Kalman filter (see e.g. Anderson and Moore, 1979). Equation 3 is the observation equation and Equation 4 is the state-transition equation.

**The Kalman Filter**

It is assumed that the error terms \( \nu(t) \) and \( \xi(t) \) are normally distributed and that the premium \( P(t) \) has a prior distribution with mean \( P(0|0) \) and variance \( \Sigma(0|0) \). At every point in time \( t \) after the history of the process \( R(t) = [F(r, t), \ldots, F(r, t+1)] \) was observed, we want to revise our prior distribution of the unknown state variable \( P(t) \). The Kalman Filter allows us, given knowledge of \( P(0|0), \Sigma(0|0) \) and the ratio \( \sigma_\xi^2 / \sigma_\nu^2 \), to compute recursively the mean and variance of \( P \) for each subsequent period.

Denote the conditional distribution of \( P(t) \) given \( R(t) \) by \( p[P(t)|R(t)] \). Given our normality assumptions, \( p[P(t)|R(t)] \) and \( p[P(t+1)|R(t+1)] \) are also normal and completely characterized by their first two moments. If we denote the mean and variance of \( P[P(t)|R(t)] \) by \( P(t|t) \) and \( \Sigma(t|t) \) respectively, and those of \( P[P(t+1)|R(t)] \) by \( P(t+1|t) \) and \( \Sigma(t+1|t) \), then the Kalman Filter recursions for \( t = 0, 1, 2, \ldots \), are given by Equations 6-10:

\[
P(t+1|t) = P(t|t)
\]

(6)

\[
\Sigma(t+1|t) = \alpha^2 \Sigma(t|t) + \sigma_\xi^2
\]

(7)

\[
P(t+1|t+1) = P(t+1|t) + k(t+1)
\]

\[
\times [F(t+1, t+2) - S(t+2) - P(t+1|t)]
\]

(8)

\[
\Sigma(t+1|t+1) = \Sigma(t+1|t) - k(t+1) \Sigma(t+1|t)
\]

(9)

with

\[
k(t+1) = [\Sigma(t+1|t) + \sigma_\nu^2]^{-1} \Sigma(t+1|t)
\]

(10)

Without the normality assumptions, the above results hold for best linear unbiased prediction rather than conditional expectations.

**IV. PREDICTION RESULTS ON THE BASIS OF PREMIUM MODELS**

In this section the Kalman Filter is employed for recursive signal-extraction and spot rate prediction. Predicted values
for the spot exchange rate at time \( t + 1 \), given the information that is available at time \( t \), can be generated as

\[
E[S(t+1)|t] = F(t, t+1) - E[P(t)|t]
\]

\[= F(t, t+1) - \alpha E[P(t-1)|t] \quad (11)\]

Thus, the forward rate as a predictor of the future spot rate is essentially 'corrected' for a premium term.

In order to be able to apply the Kalman Filter, two parameters need to be specified: the state transition parameter \( \alpha \) and the variance ratio \( \sigma_P^2/\sigma_P^2 \). For convenience, we will define \( \lambda \equiv \sigma_P^2/\sigma_P^2 \). Since information about the magnitudes of \( \alpha \) and \( \lambda \) is difficult to obtain, a search procedure is implemented: we search over a plane of values for the parameters \( \alpha \) and \( \log_{10}(\lambda) \). Given a combination of values for \( \alpha \) and \( \log_{10}(\lambda) \), predictions for future spot rates can be generated from Equations 3 and 4 on a period by period basis and summary statistics on the model's forecasting performance can be calculated. Values of \( \alpha \) in the range 0.00–1.10 (grid size 0.01) and values of \( \log_{10}(\lambda) \) in the range -10.0–0.0 (grid size 0.1) are tried.

To be able to start off the Kalman filter we also need to specify the starting values \( P(0|0) \) and \( \Sigma(0|0) \). In order to obtain reasonable values for \( P(0|0) \) and \( \Sigma(0|0) \), we estimated Equation 12 by ordinary least squares (OLS):

\[
F(t, t+1) - S(t+1) = \text{constant} + \text{error term} \quad (12)
\]

using the first 25 observations of our sample. The estimated intercept and its estimated variance were then used as prior mean and variance for the state variable \( P(t) \). The Kalman filter was then started off for the 26th observation and run for two periods before forecasts were generated. The model's ability to predict one-month-ahead spot exchange rates is judged mainly on the basis of the root mean square forecast error (RMSE). In both the \( \alpha \) and \( \lambda \) dimensions distinct global minima are found in terms of the RMSE criterion, for all three exchange rates.

In Figs 1–3 the model's forecasting performance for different values of the autocorrelation coefficient \( \alpha \) is shown for the three exchange rates. \( U \)-statistics are reported in the figures, i.e. RMSEs are scaled by the RMSEs of corresponding random walk forecasts. The \( U \)-statistics in Figs 1–3 are calculated for the values of \( \lambda \) for which overall minimum RMSEs are attained. For comparison, levels of the \( U \)-statistics that result from the use of the forward rate and the lagged spot rate as predictors of the four-week-ahead spot exchange rate are indicated in the graphs.

Similarly, in Figs 4–6 the model's forecasting performance is shown for different values of \( \lambda \) (for the values of \( \alpha \) for which overall minimum RMSEs are attained). Several comments are in order with regard to these figures. First, the premium models do better than the forward rate over wide ranges of parameter values. Useful information concerning the future values of spot exchange rates is being
Foreign exchange rates

Fig. 3. U-statistics on the forecasting performance of the premium model for the dollar/yen exchange rate for different values of the state transition parameter $\alpha$. ($\log_{10}(\sigma_f^2/\sigma_r^2) = -0.94$).

Fig. 5. U-statistics on the forecasting performance of the premium model for the dollar/mark exchange rate for different values of $\log_{10}(\sigma_f^2/\sigma_r^2)$ ($\alpha = 0.98$).

Fig. 4. U-statistics on the forecasting performance of the premium model for the dollar/pound exchange rate for different values of $\log_{10}(\sigma_f^2/\sigma_r^2)$ ($\alpha = 0.88$).

Fig. 6. U-statistics on the forecasting performance of the premium model for the dollar/yen exchange rate for different values of $\log_{10}(\sigma_f^2/\sigma_r^2)$ ($\alpha = 0.63$).
Table 3. Parameter values of the premium models that lead to minimum RMSE one-step-ahead forecasts and summary statistics on these models’ forecasting performance

<table>
<thead>
<tr>
<th></th>
<th>$/pound</th>
<th>$/mark</th>
<th>$/yen</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.88</td>
<td>0.98</td>
<td>0.63</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.038</td>
<td>0.004</td>
<td>0.115</td>
</tr>
<tr>
<td>SNR(^a)</td>
<td>0.169</td>
<td>0.103</td>
<td>0.191</td>
</tr>
<tr>
<td>ME(^b)</td>
<td>-0.10</td>
<td>-0.24</td>
<td>-0.17</td>
</tr>
<tr>
<td>MAE(^b)</td>
<td>2.02</td>
<td>2.22</td>
<td>2.39</td>
</tr>
<tr>
<td>RMSE(^b)</td>
<td>2.68</td>
<td>3.03</td>
<td>3.20</td>
</tr>
<tr>
<td>U-stat.</td>
<td>1.003</td>
<td>1.010</td>
<td>1.008</td>
</tr>
</tbody>
</table>

Note: \(^a\) SNR is signal-to-noise ratio  
\(^b\) Approximately in percentage terms.

picked up by our empirical characterization of the time series properties of the risk premia. Second, the premium models never do better than the random walk forecast: U-statistics smaller than one are not attained. Third, the graphs all show distinct global minima in terms of the RMSE criterion.

The locations of these minima provide us with information about the premium processes in the sense that the best empirical description of the premia can be expected to give the best empirical description of the premia. However, in keeping with earlier literature on the forecasting performance of exchange rate models (see Meese and Rogoff, 1983 and Wolff, 1987b), the models that we develop are unable to outperform the simply random walk forecasting rule in a prediction experiment.

Finally, the methodology that we employ in this article is fairly general in the sense that it can be straightforwardly applied to other financial markets, such as futures markets and markets for government debt instruments.

V. CONCLUSIONS

This paper investigated whether knowledge about premia in forward exchange rates can be usefully exploited in order to forecast future spot rates. On the basis of signal-extraction models in which premia are assumed to follow AR(1) processes, we find for a wide range of parameter values that ‘correcting’ the forward rate for a premium term leads to one-step-ahead forecast errors that compare favorably with those obtained from the forward rate without correction. The premium models that are most successful in predicting future spot rates imply that 9-16% of the variance in the forward forecast errors is due to variation in the premia. However, in keeping with earlier literature on the forecasting performance of exchange rate models (see Meese and Rogoff, 1983 and Wolff, 1987b), the models that we develop are unable to outperform the simply random walk forecasting rule in a prediction experiment.

REFERENCES


