Econometric Specification Analysis — An Application to the Aggregate Demand for Money in the Netherlands

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1. Introduction

Econometric analysis of money market relationships has nowadays its own tradition in applied econometrics. In order to get a flavour of the results that are available, the reader can look at the summary and the list of references given by Fase/Kuné [1974], [1975] and by Laidler [1969]. Among the money market equations, the demand function has received special attention by econometricians, perhaps partly because the specification of a money demand function requires less institutional knowledge than e.g. the specification of a money supply equation. Many of the recent empirical studies in econometrics can be characterized as attempts to statistically verify and test some of the assumptions underlying the econometric model. The demand for money function is not an exception. The recent study by Hendry/Mizon [1978] illustrates how one can ascertain an empirically verified money demand equation.

Our aim in this paper is similar, that is, we examine the validity and reliability of some assumptions underlying an econometric model. The starting point is an aggregate demand function for money specified and estimated by Fase/Kuné [1974] using Dutch quarterly data for the period 1952–1971. This model consists of a demand for money equation explaining desired holding of money by expected income and price level, an interest rate and a business cycle indicator. The assumptions of partial adjustment of nominal money balances to desired ones and of adaptive expectations for the real income and price variables complete the model. One interesting feature of the model is that the authors incorporate assumptions which are plausible from the point of view of economic theory. Nevertheless, it is worthwhile to investigate whether the specification of this theoretically meaningful model is in agreement with the information in the data. Therefore, we shall formally test several assumptions concerning the lag structure, the disturbance serial correlations and the restrictions on the structural parameters, using the same data as Fase/Kuné [1974]. We shall also investigate whether the model remains valid for the

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\(^2\) The authors are indebted to Professor M.M.G. Fase, De Nederlandsche Bank, who has kindly provided them with the data used in this study and with helpful comments on a preliminary version of this paper, to Professors D.F. Hendry, G.E. Mizon and H. Visser for the useful suggestions. Some of the work reported in this paper was done by the second author during the Warwick Summer Workshop 1979.
years 1972—1976. In this way, we hope to get more insight into what we consider as being one of the major building-blocks of a monetary model and to end up with a specification that is theoretically meaningful and not contradicted by the information in the data.

In a recent study of the demand for money in the EEC countries, Den Butter/Fase [1979] follow a similar approach, by giving particular attention to the dynamic specification and the properties of the disturbances.

From a methodological point of view, one would ideally like to start the statistical investigation with a fully specified econometric model. In this way, the problem of "measurement without theory" [see Koopmans] is avoided and the statistical properties of the econometric methods used can be ascertained. However, as economic theory, especially on an aggregate level, does not yield a fully specified econometric model, it is necessary to make ad-hoc assumptions (e.g. the correlation properties of the disturbances). Empirical verification of some of these assumptions dominates the strategy of looking for escape in somewhat shaky and non-tested suppositions. Thereby we recognize the possibility that the main feature of the final model specification may be that of tracing the statistical regularities in the data.

In the approach adopted in this paper, we start with a fairly general specification, assumed to be invariant wherein the variables are included on grounds of considerations from economic theory and the structural equations of the model represent economic laws. Although in this paper we basically stick to a single equation model, the approach can be applied to a general linear simultaneous equation model (SEM). However, the SEM requires additional carefulness with respect to the choice of a parametrization and its implications for the estimation and testing procedures to be used. The models considered in this paper are log-linear in the variables. Any functional form is an approximation of the reality and may strongly influence the results of the empirical analysis. Of course, the functional form of a model can be tested. Recently, much work on comparing alternative functional forms has been done [see e.g. Pesaran/Deaton] and it is to be expected that the choice of the functional form will get growing attention in applied econometrics. With respect to the non-linearity in the variables, we should like to state that we agree with Morgenstern [1963, p. 94] that "Non-linearity is a great complication and is, therefore, best avoided as much as possible."

The dynamics of the starting model, i.e. the number of lags of the endogenous and exogenous variables, should be specified whenever possible on the basis of theoretical considerations. For instance, the partial adjustment and adaptive expectations' schemes determine entirely the dynamics of the model by Fase/Kuné [1974]. It will often be advisable to make the lag structure for the endogenous and exogenous variables in the starting model sufficiently large so as to guarantee the invariance of the maintained hypothesis over the sample period and to make the white noise assumption of the disturbances appropriate. This facilitates the estimation, testing and interpretation of the model. The white noise assumption on the disturbances of the starting model should be tested, using misspecification tests based on the residual autocorrelations, such as e.g. the Box-Pierce test or the Lagrange multiplier test. If the white noise assumption is rejected, the number of lags has to be increased or the possible autocorrelation in the disturbances has to be modelled explicitly (e.g. a moving average disturbance term). Several authors, see
e.g. Hendry/Mizon [1978], Sims [1980] among others, advocate, partly for different reasons, to start with a general model. A major objective is to minimize the risk of misspecification due to the exclusion of variables and/or sufficiently long lags when there is little knowledge about the correct specification.

In the next step, overidentifying restrictions on the parameters of the model will be tested. One kind of restrictions are the common factor restrictions proposed by Sargan [1975] and tested by Hendry/Mizon [1978], [1980]. The advantage of this kind of restrictions is that they imply a model with simplified dynamics and autoregressive disturbances which can be easily estimated. Although restrictions such as the common factors lead to a statistically simplified model, the economic interpretation of the parameters under the restrictions is not always straightforward. Therefore, we prefer to consider first restrictions which have an interpretation in terms of economic behaviour. Restrictions implied by expectation schemes, e.g. adaptive expectations [see Faese/Kuné, 1974], the long run properties of the model [see e.g. Davidson et al.], the direction of causality will have to be investigated in many applications. In doing so, it will usually not be possible to formulate the tests of these restrictions as a uniquely ordered nest of tests as is done in the common factor approach. Therefore, the statistical properties of the sequence of tests applied to the initial model are not easily established, although those of a test in the sequence taken separately are known – at least in large samples.

Restrictions which are not rejected and which have a plausible economic interpretation are imposed on the parameters of the model. Then the restricted model is checked using criteria such as its goodness of fit, the significance and plausibility of the restricted parameter estimates, the outcome of misspecification tests applied to the residuals, the stability of the specification over subperiods and the predictive performance. The implications of the dynamic specification for the properties of the transfer function form of a multi-equation linear model and of the final equations of a model in which the exogenous variables have an autoregressive-integrated-moving average representation have to be checked along the lines proposed by Zellner/Palm [1974] and by Wallis [1977] among others. Valuable insight can be obtained by trying to explain the results found by other investigators using similar data [see e.g. Davidson et al.]. In this way, we hope to end up with a model that is satisfactory from a theoretical point of view and in accordance with the information in the data.

In this context, it is important to emphasize what Morgenstern [1963, 94–95] writes about the process of modelling and the sources of errors inherent in it:

"It must be understood that any such process, whatever its ultimate possibility, involves four steps: (1) the initial data, (2) the model, (3) the computations, and (4) the comparison of the numerical results with reality. Each one of them has its own sources of errors: the initial data are available only with a certain degree of accuracy (which it may be impossible to determine), the model is an idealization of reality, the computations can produce errors that are added to those existing at the start. The numerical result with all its cumulative errors will be compared with, and "checked" against, a "reality" that is again only revealed up to an (unknown) error factor. This is then hopefully called "verification"."

The four types of errors described by Morgenstern possibly affect the outcome of the specification analysis. Also, we should not expect the data to discriminate well between
hypotheses which are not formulated with sufficient sharpness. On the other hand, testing of the model specification is possible to the extent that the empirical results are not falsified by errors of measurement in the data used to estimate the model or to check its predictive performance. The presence of seasonals or incorrect elimination of them may also have serious consequences for the modelling process. Treating ordinal data as if they were cardinal measurement can lead to difficulties when testing restrictions on the model and cause problems with the interpretation of the results of the specification analysis. While having these possibilities in mind, we shall analyse a demand for money function along the lines described above, using the same data as Fase/Kuné [1974].

In Section 2, we present the model used by Fase/Kuné [1974] and discuss the underlying assumptions. Section 3 is devoted to the results of the econometric analysis of the model for the period 1952–76. We report results on estimation, on tests of theoretically meaningful restrictions and on residual analysis. Finally, in Section 4 we give the major conclusions of our analysis and point of problems that will be analysed in future work.

2. A Model for the Aggregate Demand for Money in the Netherlands

Fase/Kuné [1974], hereafter F & K, have specified and estimated several models for the demand for money. The most general specification in their study will be presented here. In order to facilitate the interpretation of their model, we rewrite it as follows (all variables are in natural logarithm)

\[ M_t^* = \alpha_0 + \alpha_1 Y_{t+1}^* + \alpha_2 R_t + \alpha_3 P_{t+1}^* + \alpha_4 C_t \] (2.1)

where

\[ M_t^* \] = desired amount of liquidities (nominal) in period t

\[ Y_{t+1}^* \] = expected income for period \( t + 1 \), with the expectations formed in period t

\[ R_t \] = an interest rate

\[ P_{t+1}^* \] = the expected price level for period \( t + 1 \)

\[ C_t \] = a business cycle indicator in period t.

For the unobservable variables \( M^*, Y^*, \) and \( P^* \), F & K postulate a partial adjustment equation for \( M_t^* \)

\[ M_t - M_{t-1} = \theta (M_t^* - M_{t-1}), \quad 0 < \theta \leq 1, \] (2.2)

and an adaptive expectations' mechanism for \( Y^* \) and \( P^* \), which we rewrite as

\[ Y_{t+1}^* - Y_t^* = \lambda (Y_t - Y_t^*), \quad 0 < \lambda \leq 1, \] (2.3)

and

\[ P_{t+1}^* - P_t^* = \kappa (P_t - P_t^*), \quad 0 < \kappa \leq 1. \] (2.4)

We can write the expressions (2.2), (2.3) and (2.4) using the lag operator \( L \), defined as

\[ L x_t = x_{t-1}, \]

\[ \theta M_t^* = M_t - (1 - \theta) L M_t \] (2.5)
\[ Y_{t+1}^* = \lambda \sum_{i=0}^{\infty} (1 - \lambda)^i L^i Y_t = \frac{\lambda Y_t}{1 - (1 - \lambda) L} \]  
(2.6)

\[ P_{t+1}^* = \kappa \sum_{i=0}^{\infty} (1 - \kappa)^i L^i P_t = \frac{\kappa P_t}{1 - (1 - \kappa) L} \]  
(2.7)

and substitute the results for the unobserved variables in (2.1), which yields

\[ M_t = (1 - \theta) L M_t + \alpha_0 \theta + \frac{\alpha_1 \theta \lambda Y_t}{1 - (1 - \lambda) L} + \frac{\alpha_2 \theta R_t}{1 - (1 - \kappa) L} + \frac{\alpha_3 \theta \kappa P_t}{1 - (1 - \kappa) L} + \alpha_4 \theta C_t. \]  
(2.8)

Premultiplying expression (2.8) by the polynomials in \( L \) in the denominator gives us the
final expression, which F & K write as

\[ \tilde{M}_t = \alpha_0 \theta \lambda \kappa + (1 - \theta) \tilde{M}_{t-1} + \alpha_1 \theta \lambda Y'_t + \alpha_2 \theta \tilde{R}_t + \alpha_3 \theta \kappa P'_t + \alpha_4 \theta \tilde{C}_t, \]  
(2.9)

where a variable (say \( X'_t \)) with a (\( \sim \)) is defined as

\[ \tilde{X}_t = [1 - (1 - \lambda) L] [1 - (1 - \kappa) L] X_t, \]
\[ Y'_t = [1 - (1 - \kappa) L] Y_t \] and
\[ P'_t = [1 - (1 - \lambda) L] P_t. \]  
(2.10)

Finally F & K add a disturbance term \( u_t \) to the equation (2.9). They implicitly assume
that \( u_t \) has expectation zero, constant variance, zero serial correlations, an assumption
which they relax later on in favour of a first order autoregressive process for \( u_t \), and that
the disturbance and the explanatory variables in (2.9) are independent.

At this stage, the model deserves some comments.

1) Along with many authors, F & K assume that the model is log-linear. Economic theory
has little to say about the functional form of a demand for money equation. In the
present study we assume a log-linear functional form, an assumption that could be
formally tested.

2) Economic theory of the demand for money generally explains the demand for real
balances [see e.g. Friedman]. F & K specify their relationship in terms of nominal
balances. Provided the price variable measuring the real value of money is preter-
mined, we can transform the model into a relationship for real balances without
affecting the statistical properties of the model. Den Butter/Fase [1979] investigate
demand functions for the nominal and real money balances in EEC countries and
conclude that the latter are to be preferred — for empirical reasons — to the results
for nominal balances.

3) F & K analyse the specification (2.9) as a single regression equation. Therefore, identi-
fication and simultaneity are only incidentally considered. Within the framework of
the monetary approach to balance of payments [see e.g. Johnson], one can substanti-
ate the assumption of independence between the explanatory variables in (2.9) and the
disturbance term. Under a regime of fixed exchange rates, a small country’s price and
interest rate levels are assumed to be closely related to world prices and interest rates.
and therefore predetermined. Changes in the real sector, e.g. in real income and the
business cycle indicator \( C_t \) in (2.9), are assumed to be exogenous to the monetary
sector, while the supply of money is instantaneously adjusted to the money demand,
implying that \( \theta = 1 \). The sample used by F & K largely covers a period of fixed ex-
change rates. The transition to flexible exchange rates implies a structural change and
offers a possible explanation as to why the model F & K does not necessarily remain
valid for the recent years. The assumptions made in the monetary approach to the bal-
ance of payments determine the direction of the causality in the money demand func-
tion and give theoretical support to considering the money demand function as a
reduced form and also as a transfer function equation.

4) In a final stage, the disturbance term is added to equation (2.9). Assuming that one or
several of the behavioural equations (2.1) – (2.4) are stochastic would lead to serially
correlated disturbances in the final expression (2.9), so that the correlations of the
disturbances would be functions of the structural parameters. In any case, estimation
and statistical testing in the presence of serial error correlations and lagged endogenous
variables require special carefulness, a point of which Fase/Kuné are also aware. In
the next section we shall pay attention to the specification of the disturbances.

5) The specification of the model (2.1) – (2.4) implies six restrictions on the parameters
of equation (2.9). We can write equation (2.9) as a regression equation with 13 ex-
planatory variables

\[
M_t = \beta_0 + \beta_1 M_{t-1} + \beta_2 M_{t-2} + \beta_3 M_{t-3} + \beta_4 Y_t + \beta_5 Y_{t-1} + \beta_6 R_t
\]
\[
+ \beta_7 R_{t-1} + \beta_8 R_{t-2} + \beta_9 P_t + \beta_{10} P_{t-1} + \beta_{11} C_t + \beta_{12} C_{t-1}
\]
\[
+ \beta_{13} C_{t-2} + \epsilon_t
\]

(2.11)

where the 14 parameters \( \beta_i \) are assumed to be nonlinear functions of the 8 structural
coefficients \( \alpha_0, \alpha_1, \ldots, \alpha_4, \theta, \lambda \) and \( \kappa \).

\[
\beta = \begin{bmatrix}
\beta_0 \\
\beta_1 \\
\beta_2 \\
\beta_3 \\
\beta_4 \\
\beta_5 \\
\beta_6 \\
\beta_7 \\
\beta_8 \\
\beta_9 \\
\beta_{10} \\
\beta_{11} \\
\beta_{12} \\
\beta_{13}
\end{bmatrix} = \begin{bmatrix}
\alpha_0 \theta & \lambda & \kappa \\
3 - \kappa - \lambda - \theta \\
-(1 - \kappa)(1 - \lambda) - (2 - \kappa - \lambda)(1 - \theta) \\
(1 - \lambda)(1 - \kappa)(1 - \theta) \\
\alpha_1 \theta & \lambda \\
-\alpha_1 \theta \lambda (1 - \kappa) \\
\alpha_2 \theta \\
-\alpha_2 \theta (2 - \kappa - \lambda) \\
\alpha_2 \theta (1 - \kappa)(1 - \lambda) \\
\alpha_3 \theta & \kappa \\
-\alpha_3 \theta \kappa (1 - \lambda) \\
\alpha_4 \theta \\
-\alpha_4 \theta (2 - \lambda - \kappa) \\
\alpha_4 \theta (1 - \lambda)(1 - \kappa)
\end{bmatrix}
\]

(2.12)
This implies 6 restrictions on the $\beta_i$'s written as

$$
\phi(\beta) = \begin{bmatrix}
\beta_{13}\beta_6 - \beta_8\beta_{11} \\
\beta_{11}\beta_7 - \beta_8\beta_{12} \\
\beta_{10}\beta_4\beta_{11} + \beta_5\beta_9\beta_{11} - \beta_9\beta_4\beta_{12} \\
\beta_3\beta_6\beta_7 - \beta_5\beta_6\beta_8 - \beta_5^2 \\
\beta_4\beta_8\beta_9 - \beta_5\beta_6\beta_{10} \\
\beta_1\beta_9\beta_4\beta_8 + \beta_{10}\beta_4\beta_8 + \beta_5\beta_9\beta_8 - \beta_3\beta_6\beta_4\beta_9
\end{bmatrix} = 0
$$

(2.13)

where $\phi$ is a $6 \times 1$ vector of implicit functions of the $\beta_i$'s. F & K choose several pairs of values for $\lambda$ and $\kappa$.

For each pair, they compute the variables $\tilde{M}_t, \tilde{R}_t, \tilde{C}_t, Y'_t$ and $P'_t$ defined in (2.9) and regress $\tilde{M}_t$ on the four remaining variables. As can be seen from (2.9), there is a one-to-one relationship between the coefficients of this linear regression equation and the structural parameters $\alpha_0, \ldots, \alpha_4$ $\theta$. In this way, F & K impose the restrictions (2.13) on the $\beta_i$'s in (2.11) for given values of $\lambda$ and $\kappa$. In the next section we shall investigate which values for $\lambda$ and $\kappa$ maximize the likelihood function under normally distributed disturbances and fixed initial values. For the model (2.9), maximizing the likelihood function is equivalent to maximizing the $R^2$. We shall also test the set of restrictions (2.13) imposed by F & K on their model. The assumption of normally distributed disturbances, which we do not test in this paper, is handy when testing for restrictions and for mis-specification in an econometric model. Furthermore, due to the presence of the lagged endogenous variable and other stochastic regressors, we have to rely on the asymptotic distributions of the test statistics to be used in the next section.

3. Results of the Empirical Analysis

In this section we present some estimation results of the F & K model (2.9) and its unrestricted version (2.11), and analyse the results in the light of the expected sign, the significance of the parameters, the stability of the parameter estimates over the sample period and the values of the elasticities. We test the restrictions formulated in (2.13) and analyse the residuals of the models (2.9) and (2.11) with the aim to empirically validate the demand function for money.

F & K assign a priori values to the parameters $\lambda$ and $\kappa$, and make a final choice on the basis of two selection criteria, maximizing $R^2$ and a price elasticity of about one. This procedure leads to the values $\lambda = 0.5$. Estimation results of the model (2.9) for $\lambda = 0.5$ by OLS using seasonally adjusted data (except for the interest rate) are reported in Table 1. The figures between brackets are $t$-values (in absolute value). The estimate of the constant in model 1 differs from the result reported by Fase/Kuné [1974]. This is due to a different choice of units. For the period 1952–76, III the model has been estimated by Blommestein/van Dijk [1978] (see model 3 in Table 1). We also report estimates for the period 1972–76, III (see model 2). The long-term elasticities reported in Table 1 measure the percentage variation in equilibrium money holdings with respect to a percentage variation in an exogenous variable. The equilibrium situation in a stationary economy is characterized by the equality of realized values to desired and expected values, i.e. $M_t = M^*_t, Y_t = Y^*_t$ and $P_t = P^*_t$. 
### Table 1: Ordinary least squares estimates of equation (2.9)

<table>
<thead>
<tr>
<th>Period</th>
<th>$\lambda$</th>
<th>$\kappa$</th>
<th>constant</th>
<th>$\tilde{M}_{t-1}$</th>
<th>$Y_{t-1}'$</th>
<th>$\tilde{R}_{t}$</th>
<th>$P_{t-1}'$</th>
<th>$C_{t}$</th>
<th>D.W.</th>
<th>$R^2$</th>
<th>DF</th>
<th>ln$L$</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$\delta_3$</th>
<th>nr.</th>
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</thead>
<tbody>
<tr>
<td>1952, I -</td>
<td>.5</td>
<td>.5</td>
<td>-.20</td>
<td>.69 (8.70)</td>
<td>.13 (2.41)</td>
<td>.06 (1.68)</td>
<td>.17 (2.26)</td>
<td>.03</td>
<td>2.31</td>
<td>.99</td>
<td>71</td>
<td>248.1</td>
<td>.84</td>
<td>-.20</td>
<td>1.09</td>
<td>1</td>
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<td>1971, IV</td>
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</tr>
<tr>
<td>1952, I -</td>
<td>.9</td>
<td>.7</td>
<td>-.31</td>
<td>.91 (23.2)</td>
<td>.08 (1.75)</td>
<td>-.04 (1.95)</td>
<td>.09 (1.67)</td>
<td>.02</td>
<td>1.92</td>
<td>.99</td>
<td>71</td>
<td>254.7</td>
<td>1.03</td>
<td>-.46</td>
<td>.14</td>
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<tr>
<td>1972, I -</td>
<td>.5</td>
<td>.5</td>
<td>1.14</td>
<td>.64 (3.64)</td>
<td>-.30 (1.51)</td>
<td>.93 (1.77)</td>
<td>.53 (3.53)</td>
<td>-.24</td>
<td>1.5</td>
<td>.96</td>
<td>12</td>
<td>59.4</td>
<td>-.66</td>
<td>2.55</td>
<td>2.94</td>
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<td>1976, III</td>
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<tr>
<td>1952, I -</td>
<td>.5</td>
<td>.5</td>
<td>.02</td>
<td>.68 (7.91)</td>
<td>.06 (1.5)</td>
<td>.02 (4.8)</td>
<td>.22 (3.53)</td>
<td>.01</td>
<td>2.41</td>
<td>.99</td>
<td>90</td>
<td>288.6</td>
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</table>

$\ln L =$ the value of the log-likelihood

### Table 2: Ordinary least squares estimates of equation (2.11)

<table>
<thead>
<tr>
<th>Period</th>
<th>Constant</th>
<th>$M_{t-1}$</th>
<th>$M_{t-2}$</th>
<th>$M_{t-3}$</th>
<th>$Y_{t-1}$</th>
<th>$R_{t}$</th>
<th>$R_{t-1}$</th>
<th>$R_{t-2}$</th>
<th>$P_{t}$</th>
<th>$P_{t-1}$</th>
<th>$C_{t}$</th>
<th>$C_{t-1}$</th>
<th>$C_{t-2}$</th>
<th>D.W.</th>
<th>$R^2$</th>
<th>DF</th>
<th>ln$L$</th>
<th>nr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1952, I -</td>
<td>-.245</td>
<td>1.34</td>
<td>-.554</td>
<td>-.162</td>
<td>.072</td>
<td>-.042</td>
<td>-.073</td>
<td>-.096</td>
<td>-.056</td>
<td>.054</td>
<td>.064</td>
<td>.006</td>
<td>-.001</td>
<td>2.02</td>
<td>.99</td>
<td>63</td>
<td>257.3</td>
<td>4</td>
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<tr>
<td>1971, IV</td>
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<td>(.43)</td>
<td>(10.2)</td>
<td>(2.7)</td>
<td>(1.3)</td>
<td>(.61)</td>
<td>(1.8)</td>
<td>(1.6)</td>
<td>(1.3)</td>
<td>(.48)</td>
<td>(.62)</td>
<td>(.37)</td>
<td>(.066)</td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>1952, I -</td>
<td>-.160</td>
<td>1.35</td>
<td>-.443</td>
<td>-.004</td>
<td>.039</td>
<td>.020</td>
<td>.005</td>
<td>.040</td>
<td>-.047</td>
<td>-.011</td>
<td>.126</td>
<td>-.009</td>
<td>.016</td>
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The parameters $\alpha_i$ in equation (2.9) are thus long-run elasticities. The compounded parameters in equation (2.9) are short-run elasticities. Conditionally on $\lambda = \kappa = .5$, F & K test for a structural change in model 1. The null hypothesis of no structural change could not be rejected by means of the Chow test: $F = .627, \nu_1 = 6$ and $\nu_2 = 65, T_1 = 38$ and $T_2 = 39, T_1 + T_2 = 78$. F & K conclude that their model is satisfactory as the estimated coefficients are significant and have the expected sign, the price elasticity is close to one, $R^2$ is high and the D.W. value is acceptable. A D.W.-statistic close to two is not surprising as it is biased towards non-detection of possible autocorrelation in the disturbances when lagged endogenous variables are present. The Farrar-Glauber test indicates the presence of correlation among the explanatory variables, mainly caused by the significant dependence between $\tilde{M}_{t-1}$ and $P_t'$. We take this as a warning that the data probably will not give us very precise information on the parameter values. According to the criteria used by F & K, the estimates for the periods 1952-76, III and 1972-76, III are quite unsatisfactory. For model 3 (Table 1) the coefficients of $\tilde{R}_t$ and $\tilde{C}_t$ are not significantly different from zero. The estimated coefficient of $\tilde{R}_t$ does not have the expected sign, although the 90 per cent confidence interval for the coefficient of $\tilde{R}_t$, $[-.0468, .0868]$, covers negative values. For period 1972-76, III, the parameters are estimated with greater precision, but the estimates of the coefficients for $Y_t, \tilde{R}_t,$ and $\tilde{C}_t$ have a “wrong” sign. Also for the models 2 and 3, the Farrar-Glauber test indicates significant multicollinearity. The long-term price elasticity $\alpha_3$ does not differ significantly from one. An estimate of the large sample variance of $\hat{\alpha}_3$ is given by $\text{var}(\hat{\alpha}_3) = 1.529$ (using a first order Taylor series approximation). Conditionally on $\lambda = \kappa = .5$, the hypothesis of no structural change for the periods 1952-71 and 1972-76, III has to be rejected (the Chow test is $F = 4.64, \nu_1 = 6, \nu_2 = 84$). This finding is not surprising as the second part of the sample covers a period of flexible exchange rates, of great changes in international money markets, high inflation and interest rates ... Recently, Kuipers/Wilpstra [1978, p. 1932] investigated the stability of the demand function for money using quarterly data on narrowly and broadly defined nominal balances for the period 1952, I - 1976, IV. One of their conclusions is that the post-war demand function for money in the Netherlands should be unstable.

Next, we look more closely at the choice of the values for $\lambda = \kappa = .5$. A first question is whether this pair of values gives an overall maximum of the likelihood function if we assume that the disturbance to be added to (2.9) is a normally distributed white noise. The shape of the log-likelihood function for the periods 1952-71 and 1952-76, III for different combinations of $\lambda$ and $\kappa$ is given in Figures 1 and 2. For the first sample period the likelihood function attains its maximum at $\lambda = .9$ and $\kappa = .7$ (see model 1', Table 1), while for the entire period the ML-estimates are $\lambda = 1.$ and $\kappa = .5$. These values are higher than the values $\lambda = \kappa = .5$ retained by F & K, suggesting that the expectations with respect to the income and the price level adapt quickly. A value $\lambda = .5$. indicates that the expected income is equal to the realized income. We should also notice that the equality $\lambda = \kappa$ implies that there is a common factor $(1 - \lambda \kappa)$ in equation (2.9). We can now test the hypothesis $H_0: \lambda = \kappa = .5$ against $H_1: \lambda \neq .5$ and/or $\kappa \neq .5$, using a likelihood ratio test with 2 ln of the likelihood ratio = 13.16 for the years 1952, I - 1971, IV and equal to 13.128 for the period 1952, I - 1976, III. These likelihood ratio test statistics are $\chi^2$-distributed in large samples with two degrees of freedom (= the number of restrictions.
imposed in $H_0$) so that for both periods we have to reject the null hypothesis at a 1% level of significance.\(^3\)

Fig. 1: Contours of the log-likelihood function for the period 1952, I – 1971, IV

Fig. 2: Contours of the log-likelihood function for the period 1952, I – 1976, III

\(^3\) The use of the Bonferroni inequality in multiple comparisons implies smaller levels of significance for the individual comparison. We leave it to the reader to choose his own significance level.
It is also interesting to investigate whether the restrictions (2.13) formulated by F & K through the specification in (2.1) – (2.4) are in accord with the information in the data. For this reason, we estimate the unrestricted model (2.11). The results for the unrestricted model are reported in Table 2 (nr. 4 and 5). As expected, they are not very satisfactory. Few coefficients are significantly different from zero. The estimates do not always have the sign expected on the basis of theoretical considerations. As the restrictions have not been imposed on the models 4 and 5, there are several ways to compute “estimates” for λ and κ. We now test the null hypothesis \( H_0 : \phi(\beta) = 0 \), where \( \phi(\beta) \) is the vector of 6 nonlinear restrictions (2.13) on the regression coefficients of the model (2.11), against the alternative hypothesis \( H_1 : \phi_i(\beta) \neq 0 \), some \( i = 1, 2, \ldots, 6 \). The test statistic, 2 ln of the likelihood ratio, is equal to 5.2 for the years 1952–71 and 8.506 for the years 1952–76, III. It is \( \chi^2 \)-distributed in large samples with six degrees of freedom. For the first and second sample, we cannot reject \( H_0 \) at a 10% level of significance. It is worthwhile to mention that the WALD-test [see e.g. Sargent],

\[
W = T \phi(\hat{\beta})^\top V^{-1}(\hat{\beta}) \phi(\hat{\beta}) \quad \text{with} \quad V(\hat{\beta}) = \left. \frac{\partial \phi}{\partial \beta} \frac{\partial \phi}{\partial \beta} \right|_{\beta = \hat{\beta}} \quad \text{and} \quad V(\beta)
\]

being the large sample covariance matrix of \( \hat{\beta} \), has the same asymptotic properties as the likelihood ratio test provided the \( \hat{\beta} \)'s are ML estimates of the unrestricted model. The Wald-test only requires estimates of the unrestricted model (2.11). Therefore, it is especially appropriate for testing hypotheses for which it is cumbersome to compute the maximum of the restricted likelihood function or when it is difficult to get fully efficient estimates of the unrestricted model. The Wald-statistic, which is approximately \( \chi^2 \)-distributed with 6 degrees of freedom under \( H_0 : \phi(\beta) = 0 \) in (2.13), is equal to 1.935 for the years 1952, I – 1971, IV, and 1.8340, for the period 1952, I – 1976, III, so that for the period 1952–76 we cannot reject the hypothesis at conventional levels of significance. Notice that the OLS estimator for model (2.11) is also the maximum likelihood estimator.

Now we shall pay attention to the structure of the disturbances. For the models 1 and 3, the assumption of spherical disturbances tested by means of the Goldfeld-Quandt test is not rejected at a 5% level of significance. However the empirical evidence in the residuals is not in favour of the assumption of zero error correlations.

In Table 3 we report the estimated residual autocorrelation (ACF) and partial autocorrelation (PACF) functions for several regression models\(^4\). Estimated large sample standard errors for the ACF and the PACF are given in the column SE [see e.g. Box/Jenkins]. It is obvious that there are significant correlations in the residuals of models 1, 3 and 4. Given that the significant residual autocorrelations for these models are positive, one might suspect that the formula \( s^2(X'X)^{-1} \) underestimates the variance of the OLS estimator, implying that the t-values for the regression coefficients are overestimated. For model 6, the third and eight autocorrelation coefficients and the third and ninth partial autocorrelation coefficients are significantly different from zero at a 5% level. Simple autoregressive moving average (ARMA) models have been fitted to the residuals of the regression models 1 and 3 using a nonlinear least squares estimation method.

\(^4\) The computations were performed using a computer program developed by C.R. Nelson, University of Washington, Seattle.
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Tab. 3: Estimated autocorrelation (A) and partial (P) autocorrelation for the residuals

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<td>-193 (1.719)</td>
<td>-.243 (2.166)</td>
<td></td>
<td></td>
<td>17.2</td>
<td>10</td>
<td>27.6</td>
<td>22</td>
<td>38.4</td>
<td>34</td>
</tr>
<tr>
<td>1</td>
<td>52-71</td>
<td>(2,1)</td>
<td>.00659</td>
<td>74</td>
<td>.000089</td>
<td>-.721 (3.37)</td>
<td>-.372 (3.39)</td>
<td>-.593 (2.71)</td>
<td></td>
<td>12.9</td>
<td>9</td>
<td>20.9</td>
<td>21</td>
<td>31.6</td>
<td>33</td>
</tr>
<tr>
<td>3</td>
<td>52-76</td>
<td>(2,0)</td>
<td>.08996</td>
<td>94</td>
<td>.000956</td>
<td>1.887 (1E+6)</td>
<td>-.885 (19.56)</td>
<td></td>
<td></td>
<td>72.0</td>
<td>10</td>
<td>94.2</td>
<td>22</td>
<td>101.9</td>
<td>34</td>
</tr>
<tr>
<td>3</td>
<td>52-76</td>
<td>(2,1)</td>
<td>.01267</td>
<td>93</td>
<td>.000136</td>
<td>-.813 (2.554)</td>
<td>-.283 (2.772)</td>
<td>-.599 (1.847)</td>
<td></td>
<td>10.6</td>
<td>9</td>
<td>15.7</td>
<td>21</td>
<td>21.4</td>
<td>33</td>
</tr>
</tbody>
</table>

Tab. 4: Estimated ARMA models for the residuals from the regression models
The results are given in Table 4. Values for the $Q$-statistic associated with ARMA models are also reported where $Q(k) = T \sum_{r=1}^{k} r^{2} \tilde{e}$, with $r_{r} \tilde{e}$ being the estimated $r$-th autocorrelation coefficient of the residuals for the ARMA $(p, q)$ models. In regression models with ARMA $(p, q)$ disturbances and no lagged endogenous variables among the regressors, $Q$ is $\chi^{2}$-distributed with $k-p-q$ degrees of freedom and independent of the explanatory variables in large samples [see Pierce]. Here we take high values for $Q$ as a warning against a possible misspecification. The figures in Table 4 are not consistent estimates of non-zero disturbance autocorrelation parameters because there are lagged endogenous variables in the regression equations, but it ought to be clear that the evidence in the residuals is not supporting the white noise assumption for the disturbances. The empirical results in Table 3 suggest that imposing the restriction $\lambda = \kappa = .5$ strengthens the residual autocorrelation. A different picture of the serial correlation structure of the residuals is obtained by looking at the spectra. The estimated spectra\(^5\) for the residuals of the models 1 and 4 are given in Figures 3 and 4 respectively. Although the residual autocorrelations of the two models are quite different, the residual spectra have a similar shape with a minimum at the frequency of .25 (a period of one year) and peaks at the frequencies .04, .20, .30. The spectrum for $\Delta M$, where $\Delta$ is the difference operator, given in Figure 5 has a very similar shape at and around the seasonal frequencies. The minimum at the frequency of .25 and the two peaks at the frequencies of .16 and .30 are probably the result of the seasonal adjustment of the series.

In conclusion, the model specified by F & K does not seem to be entirely in agreement with the information in the data. Also adding a white noise disturbance to equation (2.1) with its implication of a second order moving average error term in equation (2.9) and restrictions between the transfer function parameters and those of the error process does not yield satisfactory results. The unrestricted version of the F & K model (2.11) is acceptable as a maintained hypothesis by residual serial correlation standards. We select it as a starting point for our strategy which consists in specifying a model that is sufficiently general to be a safeguard against missing explanatory variables and autocorrelated errors and then to look for restrictions on the parameters that will not be rejected by the data.

First one may question the presence of the business cycle indicator $C$ in equation (2.11). The coefficients of the variable $C$ are not significantly different from zero in the unrestricted models 4 and 5. Part of its effect on the demand for money is expected to be explained by variations in the variable $Y_t$. Excluding the business cycle indicator from the unrestricted specification (2.11) does not lead to a substantial loss of explanatory power of the equation (see the models 6 and 7 in Table 2). The value of the likelihood ratio is 7.968 for 1952, I – 1971, IV, and 7.398 for 1952, I – 1976, III (number of degrees of freedom = 3), so that at a 5% level we reject the null hypothesis for the first period but not for the entire sample period. As the restrictions (2.13) imposed by F & K and their empirical finding that $\lambda = \kappa$ imply one common factor restriction on the model, we next investigate whether we can reduce the dimensions of the parameter space by im-

\(^5\) The spectra have been computed using a computer program developed by C.P.A. Bartels and E. Vogelvang, using the Tukey-Hanning window. The number of lags employed to compute the spectra is 24.
posing a common factor restriction. Along the lines of Hendry/Mizon [1978] we test for the presence of common factors in the unrestricted model (2.11) with and without the business cycle variable. Imposing one common factor on the model (2.11) leads to a specification like

\[ M_t = \gamma_0 + \gamma_1 M_{t-1} + \gamma_2 M_{t-2} + \gamma_3 Y_t + \gamma_4 R_{t} + \gamma_5 R_{t-1} + \gamma_6 P_t + \gamma_7 C_t + \gamma_8 C_{t-1} + \frac{\mu_t}{1 - \rho L}. \]  

(3.1)

As there are lagged endogenous variables and autocorrelated errors present in model (3.1), we estimate its parameters by the Hildreth-Lu method which yields ML estimates of the parameters in (3.1). The estimates are reported in Table 5. The likelihood ratio (LR) statistics for testing the presence of one common factor are given in the following table:

<table>
<thead>
<tr>
<th>Model</th>
<th>Period</th>
<th>C included</th>
<th>2 ln LR</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1952-71</td>
<td>yes</td>
<td>2.680</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>1952-76</td>
<td>yes</td>
<td>3.848</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>1952-71</td>
<td>no</td>
<td>4.058</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>1952-76</td>
<td>no</td>
<td>3.282</td>
<td>3</td>
</tr>
</tbody>
</table>
From the outcome of the test, we have to conclude that we cannot reject the common factor restriction. It is worthwhile to notice that the estimated common factor for model 6* is equal to zero and that the common factor in model 7* (** denotes that the common factor restriction has been imposed) is not significantly different from zero. Non-rejection of the common factor hypothesis leads us to a simpler model.

The models for which the estimates are reported in Table 5 can be rewritten as relationships between variables expressed in growth rates and in levels.

For example, model 6* (Table 5) can be approximately written as

$$
\Delta M_t = -0.017 + 0.24 \Delta M_{t-1} - 0.04 M_{t-2} + 0.002 Y_t - 0.071 \Delta R_t
$$

$$
- 0.002 R_t + 0.085 P_t + \frac{\hat{u}_t}{1 - 0.2L}.
$$

<table>
<thead>
<tr>
<th>Period</th>
<th>const.</th>
<th>$M_{t-1}$</th>
<th>$M_{t-2}$</th>
<th>$Y_t$</th>
<th>$R_t$</th>
<th>$R_{t-1}$</th>
<th>$P_t$</th>
<th>$C_t$</th>
<th>$C_{t-1}$</th>
<th>$\rho$</th>
<th>$R^2$</th>
<th>D.W.</th>
<th>$T$</th>
<th>ln $L$</th>
<th>nr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1952, I–1971, IV</td>
<td>-0.432</td>
<td>1.001</td>
<td>-0.090</td>
<td>0.093</td>
<td>-0.079</td>
<td>0.050</td>
<td>0.104</td>
<td>0.010</td>
<td>0.005</td>
<td>0.30</td>
<td>0.999</td>
<td>1.944</td>
<td>77</td>
<td>255.5</td>
<td>4*</td>
</tr>
<tr>
<td></td>
<td>(2.165)</td>
<td>(8.063)</td>
<td>(7.73)</td>
<td>(1.966)</td>
<td>(2.201)</td>
<td>(1.330)</td>
<td>(1.599)</td>
<td>(7.112)</td>
<td>(3.575)</td>
<td>(2.759)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1952, I–1971, III</td>
<td>-0.098</td>
<td>1.426</td>
<td>-0.520</td>
<td>0.052</td>
<td>0.010</td>
<td>-0.010</td>
<td>0.116</td>
<td>-0.012</td>
<td>0.020</td>
<td>0.10</td>
<td>0.999</td>
<td>1.984</td>
<td>96</td>
<td>297.5</td>
<td>5*</td>
</tr>
<tr>
<td></td>
<td>(1.142)</td>
<td>(5.606)</td>
<td>(2.107)</td>
<td>(3.49)</td>
<td>(3.39)</td>
<td>(2.455)</td>
<td>(2.807)</td>
<td>(1.426)</td>
<td>(0.985)</td>
<td>(1.985)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1952, I–1971, IV</td>
<td>-0.074</td>
<td>1.204</td>
<td>-0.243</td>
<td>0.002</td>
<td>-0.071</td>
<td>0.069</td>
<td>0.085</td>
<td>-0.2</td>
<td>0.005</td>
<td>0.09</td>
<td>0.999</td>
<td>1.924</td>
<td>77</td>
<td>251.3</td>
<td>6*</td>
</tr>
<tr>
<td></td>
<td>(1.367)</td>
<td>(2.130)</td>
<td>(1.901)</td>
<td>(3.67)</td>
<td>(1.849)</td>
<td>(1.349)</td>
<td>(1.988)</td>
<td>(1.791)</td>
<td>(0.985)</td>
<td>(1.985)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1952, I–1971, III</td>
<td>0.055</td>
<td>1.443</td>
<td>-0.494</td>
<td>0.006</td>
<td>0.020</td>
<td>-0.014</td>
<td>0.085</td>
<td>-0.0</td>
<td>0.005</td>
<td>0.09</td>
<td>0.999</td>
<td>2.009</td>
<td>96</td>
<td>294.0</td>
<td>7*</td>
</tr>
<tr>
<td></td>
<td>(0.536)</td>
<td>(5.078)</td>
<td>(1.681)</td>
<td>(3.62)</td>
<td>(1.861)</td>
<td>(1.485)</td>
<td>(1.988)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tab. 5: ML estimates of the model (3.1) (one common factor)

However it is difficult to interpret the implications of these results for the demand for money along a steady state growth path. As an alternative to the models with the common factor restriction, we write the estimated regressions 6 and 7 in Table 2 in terms of the levels and the first differences of the variables. For the period 1952, I – 1971, IV we have approximately

$$
\Delta M_t = .08 + .45 \Delta M_{t-1} - .15 \Delta M_{t-2} - .03 (M_{t-1} - Y_{t-1} - P_{t-1})
$$

$$
+ .04 \Delta Y_t - .08 \Delta R_t - .03 R_{t-2} + .04 \Delta P_t + \hat{u}_t
$$

(3.2)

with a steady state solution given by

$$
M = A + Y + P - R,
$$

(3.3)

where

$$
A = 33 (.08 - .70 \Delta M + .04 \Delta Y - .08 \Delta R + .04 \Delta P),
$$

after taking antilogs

$$
\ln m = \alpha \rho^t p^y
$$

with $\ln \alpha = A$ and $m$, $r$, $p$ and $y$ being the antilogs of $M$, $R$, $P$ and $Y$ respectively. Equation (3.2) explains the growth rate of nominal money balances as a function of the lagged
money growth rates, an error learning mechanism \((M_{t-1} - Y_{t-1} - P_{t-1})\) being equal to minus the logarithm of the velocity of money in \(t - 1\) with respect to the transactions, the growth rate of expenditures in constant prices, the change in the interest rate, the lagged interest rate and the rate of change in prices. In the steady state the nominal demand for money as in (3.3) is homogeneous of degree one in nominal expenditures and of degree minus one in the interest rate. The error correction term, which has been used successfully by Hendry [1978], implies that the growth rate of money in period \(t\) decreases (increases) when money holdings in \(t - 1\) have been too high (too low) with respect to nominal expenditures, in other words, the growth rate of money depends on the past value of the transactions velocity of money. The specification in (3.2) of the demand for money function is a restricted version of the model 6. The restrictions imply a unit elasticity of money with respect to transactions in the steady state and a velocity which depends on the interest rate. Estimation of the restricted model yields the following results for the period 1952, I – 1971, IV (model 8):

\[
\Delta M_t = -0.078 + 0.441 \Delta M_{t-1} - 0.155 \Delta M_{t-2} - 0.019 [M_{t-1} - Y_{t-1} - P_{t-1}] \\
(1.215) \\
(3.712) \\
(1.264) \\
(1.057) \\
+ 0.053 \Delta Y_t - 0.057 \Delta R_t + 0.007 R_{t-2} - 0.018 \Delta P_t + \hat{\epsilon}_t \\
(.887) \\
(1.449) \\
(.781) \\
(.186)
\]

\[
\ln L = 251.3 \\
R^2 = .38, \\
D.W. = 1.95. \tag{3.4}
\]

The spectrum for the residuals is given in Figure 8.

![Fig. 7: Estimated spectrum for \(\Delta r\), 1952, I – 1976, III](image1)

![Fig. 8: Estimated spectrum for the residuals of model 8, 1952, I – 1971, IV](image2)

The estimates of the constant term and of the coefficient of \(R_{t-2}\) have signs different from those in (3.2) although the latter one is not significantly different from zero. The likelihood ratio test statistic for comparing (3.4) and model 6 is equal to 5.984 and has an asymptotic \(\chi^2\)-distribution with 3 degrees of freedom under the null hypothesis that equation (3.4) is the correct model. We cannot reject the model (3.4) at conventional levels of significance. The decrease in the \(R^2\) is partly due to the decrease in the variance of the left-hand side variable in (3.4). The increase in the residual variance of (3.4) compared to
that of model 6 is not sufficient to reject (3.4) when using a likelihood ratio test. The sample autocorrelations for the residuals of model 8 are reported in Table 3. The 3rd, 5th and 8th autocorrelations and the 3rd, 8th and 9th partial autocorrelations are significantly different from zero, still pointing towards a possible misspecification of the model. The results in (3.4) also imply a positive, possibly zero, interest rate elasticity of the demand for money in the steady state. A slightly different specification is given by

\[
\Delta M_t = -0.115 + 0.458 \Delta M_{t-1} - 0.112 \Delta M_{t-2} - 0.030 (M_{t-1} - Y_{t-1} - P_{t-1}) \\
(2.860) \\
(3.838) \\
(0.979) \\
(3.109)
\]

\[
+ 0.062 \Delta Y_t - 0.076 \Delta R_t + 0.012 \Delta R_{t-1} - 0.005 \Delta P + \hat{u}_t \\
(1.051) \\
(2.034) \\
(0.319) \\
(0.051)
\]

\[
\ln L = 251.1, \quad R^2 = .38, \quad \text{D.W.} = 1.96. \quad (3.5)
\]

It is interesting to note that the point estimates of several parameters are insensible to changes of specification, suggesting that we are able to decompose with some accuracy the constant term in the steady state money demand function but that it is more difficult to accurately determine the effects of variables such as the interest rate on the steady state transactions velocity of money. Three possible explanations are straightforward. First, besides a long-term interest short-term interest rates are relevant for explaining the level of liquidities in an economy. In a recent study using U.S. data, Heller/Khan [1979] show that one can improve the explanatory power of the demand function for money by taking into account the entire term structure of interest rates. They avoid the problem of possibly high multicollinearity between interest rates through characterizing the term structure of interest rates by a few parameters. Second, the long-term interest rate used by F & K, which is one of the variables explaining the velocity of money, is not seasonally adjusted while the remaining variables are seasonally adjusted. In a situation like that, the estimated relationship can differ substantially from the true relationship as

\[\text{Fig. 9: Estimated spectrum for the residuals of model 11, 1952, I - 1971, IV}\]

\[\text{Fig. 10: Estimated spectrum for the residuals of equation (3.11), 1952, I - 1971, IV}\]
has been illustrated by Wallis [1974], so that it can be worthwhile to investigate the relationship between seasonally unadjusted series, thereby modelling explicitly the seasonal variations in the data. Third, the quarterly data for income have partly been derived from annual figures [see e.g. Driehuis]. The estimates for the money demand equation using this income series may substantially differ from the estimates for a money demand equation using actual quarterly income.

A look at the estimated spectra for some of the variables (see Figs. 5-7) shows that the seasonality has not been entirely removed. The spectrum of $\Delta M$ has a peak at the frequency .32 implying a period of about 3 quarters, while the spectrum for $\Delta P$ shows a peak at the frequency of .25 with a period of one year. The spectrum of the difference rate series has several peaks, one of which is at the frequency of .25 (a period of one year) pointing towards the presence of seasonality in the interest rate series [see also Porsius]. For the period 1951, I - 1976, III, we can write the model 7 (Table 2) approximately as

$$\Delta M_t = .055 + .485 \Delta M_{t-1} + .015 \Delta M_{t-2} - .04 [M_{t-1} - Y_{t-1} - P_{t-1}] + .04 \Delta Y_t + .007 \Delta R_t + .04 \Delta R_{t-1} + .001 R_{t-1} + .035 \Delta P_t + .12 P_{t-1} + \hat{u}_t$$

(3.6)

with a steady state solution given by

$$M = A + Y + 4 P + .025 R,$$

where

$$A = 25 (.055 - .5 \Delta M + .047 \Delta R + .035 \Delta P + .04 \Delta Y).$$

After taking antilogs, the inverse of the steady state transactions velocity of money can be written as a function of the price level $p$ and interest rate $r$:

$$\frac{m}{py} = \text{constant} \times r^{.025} p^3.$$  (3.7)

The steady state velocity varies inversely with the interest rate and price level, a finding that one would not expect from theoretical considerations. Re-estimating the restricted specification (3.6) yields the following result

$$\Delta M_t = -.074 + .482 \Delta M_{t-1} -.056 \Delta M_{t-2} - .003 [M_{t-1} - Y_{t-1} - P_{t-1}] + .015 \Delta R_t + .041 \Delta R_{t-1} - .002 R_{t-1} - .103 \Delta P_t + .017 P_{t-1} + .006 \Delta Y_t + \hat{u}_t$$

(4.449)  (4.78)  (.167)

(.411)  (1.258)  (.158)

(1.466)  (.099)

$$R^2 = .45 \quad \text{D.W.} = 2.04.$$  (3.8)

The likelihood ratio test statistic for comparing model (3.8) with the unrestricted model
7 is equal to 3.128 and has an asymptotic $\chi^2$-distribution with one degree of freedom under the null hypothesis that (3.8) is the correct specification. The point estimates of (3.8) imply that the steady state velocity varies inversely with the price level. They also differ from those obtained in (3.4) for the period 1952 – 1971, IV.

Again the use of seasonally adjusted data together with a seasonally unadjusted long-term interest rate may have distorted the point estimates to the extent that we cannot interpret them as estimates of the structural parameters in a demand for money function. In order to compare the predictive performance of model 1 with that of model 8 in (3.4), we compute the multi-step-ahead “predictions” of nominal money balances for the period 1972, I – 1976, III using the realized values of the exogenous variables. The results are given in Figure 11. The “predictions” from model 8 are more accurate than those from model 1 for which the forecasted values are systematically greater than the realized values. Theil’s inequality coefficient $\tau = [\Sigma u_r^2 / \Sigma r_r^2]^{1/2}$, where $u_r$ is the prediction error and $r_r$ is the realized value of the nominal money balances is equal to .20081 for model 1 and .09224 for model 8.

As a yardstick for the predictive performance of the models 1 and 8, we have fitted univariate ARIMA models to the nominal money balances $M_t$ (logarithmic transformation). The analysis of the autocorrelation and partial autocorrelation functions for $M_t$ suggested a second order autoregressive model with a low order, possibly a second order moving average error term.

The following model has been fitted to the data:

$$\Delta M_t = .932 \Delta M_{t-1} + .069 \Delta M_{t-2} + \hat{e}_t - .297 \hat{e}_{t-1} - .615 \hat{e}_{t-2}$$

(139.7) (6.346) (2.966) (6.528)

$R^2 = .324$, \hspace{1cm} RSS = .007, \hspace{1cm} DF = 75, \hspace{1cm} RSS/DF = .0001, \hspace{1cm} (3.9)$

$Q (12) = 13.5$, \hspace{1cm} DF = 8

$Q (24) = 19.6$, \hspace{1cm} DF = 20

$Q (36) = 26.0$, \hspace{1cm} DF = 32

Theil’s inequality coefficient = .11098.

Up to 19-step ahead forecasts have been calculated with this model and are plotted in Figure 11. Notice that model 8 reduces to a univariate ARIMA model if we approximate the error learning term and the exogenous variables by a moving average error. If the model given in (3.9) is correctly specified, its structure puts restrictions on the processes for the exogenous variables in model 8 (assumed to be correctly specified). The analysis of the autocorrelation function of the exogenous variables suggests random walk models for the income and the price variable and a first or second order autoregressive model for the interest rate variable. These univariate models and the model (3.9) for the endogenous variable are not entirely compatible with model 8, a point that will be investigated more extensively in the future along the lines proposed by Zellner/Palm [1974]. Finally, notice that a random walk model for income and the price variable rules out a rational expectations interpretation of the adaptives expectations schemes formulated by Fase/Kuné [see e.g. Sargent].

Although the model 8 has been extensively tested and its predictive power is fairly good, the point estimates of the parameters are not entirely satisfactory. In particular,
the sign of the estimated coefficient of $R_{t-2}$ implies that the steady state velocity of money with respect to transactions decreases when the interest rate increases. In order to see how the seasonal adjustment of the variables affects the estimation results, we have fitted the models 4, 6 and 8 to the seasonally unadjusted data. The results are given in Tables 6 and 7. Models 9 and 10 correspond to the model 8. The $D_t$'s in model
10 are seasonal dummies with \( D_i = 1 \) in quarter \( i \) and zero otherwise. In model 11, all the variables have been filtered using the annual difference operator \( 1 - L^4 \). The point estimates for model 11 are similar to those for model 8. However, the estimated coefficient of \( R_{t-2} \) is negative. This implies that the steady state velocity of money increases with the interest rate. Using a likelihood ratio test, we come to the conclusion that model 9 has to be rejected when compared to models 10 or 12. Model 11 is rejected when we test it against the model 14, but the model 14 is not rejected when compared with model 13 at the conventional 5% significance level. It should be clear that the method of seasonal adjustment of the data used in our analysis affects the outcome of the model comparison and the point estimates of the parameters. A more detailed analysis of the seasonally unadjusted data is presently under way.

According to the portfolio theory, the whole spectrum of interest rates is relevant for the amount of money demanded by agents in the economy. For the econometrician, it may be a difficult task to measure the effects of changes in the term structure of interest rates, because of the well-known problem of multicollinearity between the interest rates. However approximating the influence of the term structure of interest rates on the demand for money by including one interest rate in the demand equation may lead to a misspecification, a point, we shall briefly investigate now. In order to deal with the collinearity between interest rates with a different maturity period, we impose along the lines of Heller/Khan [1979] a priori restrictions on the term structure of the interest rates by assuming that at each period \( t \), the term structure can be described compactly by a quadratic function

\[
\ln R_{it} = \alpha_t + \beta_t \tau_i + \gamma_t \tau_i^2, \quad i = 1, 2, \ldots, n,
\]

where \( R_{it} \) is the \( i \)-th interest rate and \( \tau_i \) is its maturity period.

We estimate the parameters of equation (3.10) for each time period \( t \) and substitute them into the demand function for money. The parameter \( \alpha_t \) can be viewed as a shift parameter of the term structure, \( \beta_t \) and \( \gamma_t \) determine the slope and the curvature of the entire term structure at time \( t \). For a detailed discussion, we refer the reader to Heller/Khan [1979]. After substitution of the three term structure parameters for the interest rate, model 8 leads to the following results for the period 1952 – 1971, IV

\[
\Delta M_t = -0.083 + 0.446 \Delta M_{t-1} - 0.195 \Delta M_{t-2} - 0.025 (M - Y - P)_{t-1} \\
(1.10) \quad (3.75) \quad (1.71) \quad (1.26)
\]

\[
+ 0.634 \Delta Y_t - 0.063 \Delta \alpha_t - 0.529 \Delta \beta_t - 5.277 \Delta \gamma_t - 0.001 \alpha_{t-2} \\
(1.07) \quad (2.11) \quad (1.22) \quad (7.2) \quad (0.11)
\]

\[
- 0.078 \beta_{t-2} - 1.158 \gamma_{t-2} + 0.044 \Delta P_t + \hat{u}_t \\
(0.38) \quad (0.30) \quad (0.44)
\]

\[R^2 = 0.43, \text{D.W.} = 1.94, \ln L = 254.5, \text{DF} = 65.\]

The plot of the spectrum of the residuals is given in Figure 10.

Heller/Kahn show that the partial derivatives of \( M_t \) with respect to the parameters of the term structure of interest rates should be negative, except for values \( \beta_t \) and \( \gamma_t < 0 \) simultaneously. In equation (3.11), the point estimates of the interest rate variables have
### Tab. 6: Ordinary least squares estimates of equation (3.4) using seasonally unadjusted data, 1952–1971, IV

<table>
<thead>
<tr>
<th>Data transformation</th>
<th>constant</th>
<th>$\Delta M_{t-1}$</th>
<th>$\Delta M_{t-2}$</th>
<th>$(M_{t-1}-P_{t-1})$</th>
<th>$\Delta Y_t$</th>
<th>$\Delta R_t$</th>
<th>$R_{t-2}$</th>
<th>$\Delta P_t$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>D.W.</th>
<th>$R^2$</th>
<th>DF</th>
<th>lnL</th>
<th>nr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-$</td>
<td>.032 (.23)</td>
<td>.194 (1.56)</td>
<td>-.500 (4.34)</td>
<td>.009 (.30)</td>
<td>-.089 (2.31)</td>
<td>-.045 (.74)</td>
<td>.265</td>
<td>-.022 (.16)</td>
<td></td>
<td></td>
<td></td>
<td>2.08</td>
<td>.31</td>
<td>69</td>
<td>211.8</td>
<td>9</td>
</tr>
<tr>
<td>$-$</td>
<td>-.134 (1.12)</td>
<td>.18 (1.48)</td>
<td>-.081 (.65)</td>
<td>-.021 (.87)</td>
<td>.027 (.35)</td>
<td>-.079 (1.53)</td>
<td>.008</td>
<td>.045 (.38)</td>
<td>.028 (2.25)</td>
<td>.030</td>
<td>.026</td>
<td>2.02</td>
<td>.55</td>
<td>66</td>
<td>228.6</td>
<td>10</td>
</tr>
<tr>
<td>$1-L^4$</td>
<td>.001 (.44)</td>
<td>.317 (2.55)</td>
<td>-.142 (1.08)</td>
<td>-.017 (4.27)</td>
<td>.037 (.49)</td>
<td>-.071 (1.61)</td>
<td>-.012</td>
<td>-.233 (1.80)</td>
<td></td>
<td></td>
<td></td>
<td>2.02</td>
<td>.23</td>
<td>65</td>
<td>202.1</td>
<td>11</td>
</tr>
</tbody>
</table>

### Tab. 7: Ordinary least squares estimates of equation (2.11) using seasonally unadjusted variables, 1952–1971, IV

| Data transformation | constant | $M_{t-1}$ | $M_{t-2}$ | $M_{t-3}$ | $Y_t$ | $Y_{t-1}$ | $R_t$ | $R_{t-1}$ | $R_{t-2}$ | $P_t$ | $P_{t-1}$ | $C_t$ | $C_{t-1}$ | $C_{t-2}$ | D.W. | $R^2$ | lnL | DF | nr. |
|---------------------|----------|-----------|-----------|-----------|-------|-----------|-------|-----------|----------|-------|-----------|------|-----------|--------|------|------|----|----|
| $-$                 | -.13 (.45) | 1.09 (8.3) | -.47 (2.52)| .32 (2.58)| -.09 (1.93)| -.10 (1.66)| .01   | -.32 (.50) | .14 (.93) | .05   | -.01 (1.97) | .01  | .005 (1.06) | .005 (1.06) | 2.0  | .99  | 223.4 | 63 | 12 |
| $1-L^4$             | .01 (2.16) | 1.16 (8.86)| -.40 (2.05)| .08 (.60) | .04 (.48) | -.08 (1.05) | -.06 (1.47)| .07   | -.03 (.57) | -.11 (.83) | .19  | .0002 (.04) | .007 (1.81) | .001 (2.7) | 2.0  | .84  | 210.8 | 59 | 13 |
| $1-L^4$             | .01 (2.17) | 1.17 (9.23)| -.38 (1.95)| .05 (.40) | .04 (.49) | -.12 (1.66) | -.05 (1.21)| .05   | -.19 (.43) | -.11 (.85) | .22  | .001 (2.7)  | .007 (1.81) | .001 (2.7) | 2.0  | .83  | 208.7 | 62 | 14 |
the expected sign. Those for the remaining variables are similar to the estimates for model 8 in (3.4). The steady state velocity of money implied by equation (3.11) increases with the interest rate variables.

In conclusion, model 8 seem to be appropriate to describe the short-run behaviour of the data and to predict future developments. The point estimates and the dynamic specification are sensitive to the method of seasonal adjustment and to the choice of interest rate variables. Therefore, it may be somewhat risky to take the point estimates of model 8 as estimates of the structural parameters in the demand function for money. Rather, our analysis suggests that model 8 is a transformation of a structural equation for the demand for money.

4. Some Final Conclusions

In the present paper we have analysed a model for the demand for money in the Netherlands. We have formally tested some of the assumptions and restrictions formulated by Fase/Kuné [1974]. In particular, the choice of the estimates \( \kappa = \lambda = .5 \) has to be rejected. Although the restricted model 1' is not rejected when compared with the unrestricted model 4 and that its residual correlation properties are acceptable (see Table 3), it had to be disregarded for at least two reasons: first, the point estimate of \( \theta \) is equal to .088 implying a very slow adjustment of nominal money balances to desired ones; second, its predictive performance is very bad.

It predicts a decrease of money balances during the period 1972–1976, III. The unrestricted version of the model formulated by Fase/Kuné is sufficiently general to be taken as a starting point for our specification analysis. The value of the likelihood ratio test statistic for comparing model (2.11) with a general unrestricted regression equation with the endogenous variable \( M_t \) lagged up to four quarters and the current and past values of the explanatory variables \( Y_t, P_t, R_t \) and \( C_t \), all lagged up to four periods, is equal to 11.3 (the nr. of degrees of freedom = 10), so that for the period 1952–1976 the model (2.11) is not rejected at the conventional significance level. Restrictions such as a unit elasticity of money with respect to income and prices in the steady state are not rejected by the data and are therefore imposed on the model (2.11). In this way we obtain a model, that is theoretically meaningful and that predicts rather well. For the years 1972–1974, its predictive properties are similar to those of the ARIMA model (3.9), while for 1975 and 1976 the predictive performance of the former model is superior.

The specification of the model and the point estimates are affected by the choice of a method for seasonal adjustment (the X-11 method or the annual difference operator) and by the choice of the interest rate variables. This aspect and the problem of simultaneity between variables such as \( M_t, P_t \) and \( R_t \) have to be investigated more extensively. The conclusion by Jager [1978] that the monetary approach to the balance of payments in the form of the global monetaristic version has to be rejected for Dutch data for the period 1967–1976 points to a possible problem of simultaneity. Some work in progress on testing for the direction of causality indicates that there is a feedback relationship between money and income in the Netherlands.

Finally, in line with the conclusion by Kuipers/Wilpstra [1978], that the demand for
money in a narrow sense is better explained than the demand for broadly defined money, investigated by Fase/Kuné and in this paper, the definitions of the variables and the quality of the quarterly data also require more attention.

Appendix: The Data

A more detailed description of the data is given by F & K. The data are quarterly seasonally adjusted (except for the interest rate) observations on:

- \( M_t \) = total domestic liquidities (M2) in hands of the public averaged over the quarter.
- \( Y_t \) = gross national expenses in quarter \( t \), measured in 1963 prices.
- \( R_t \) = average of 18 rates of return on long-term government bonds.
- \( P_t \) = price index of gross national expenses (basis = 100 in 1963).
- \( C_t \) = quarterly average of unemployed males in percentage of total male employees.

The interest rates used to compute the parameters of the term structure are averages of monthly treasury bill rates with maturity periods of three, twelve, twenty four, thirty six and sixty months and the rates of government loans with maturity periods between six and thirty years. For the period 1962 to 1971 we also included the three months Euro-guilder deposit rate. The data have been collected by De Nederlandsche Bank.

References


