Increased Correlation in Bear Markets

Rachel Campbell, Kees Koedijk, and Paul Kofman

A number of studies have provided evidence of increased correlations in global financial market returns during bear markets. Other studies, however, have shown that some of this evidence may be biased. We derive an alternative to previous estimators for implied correlation that is based on measures of portfolio downside risk and that does not suffer from bias. The unbiased quantile correlation estimates are directly applicable to portfolio optimization and to risk management techniques in general. This simple and practical method captures the increasing correlation in extreme market conditions while providing a pragmatic approach to understanding correlation structure in multivariate return distributions. Based on data for international equity markets, we found evidence of significant increased correlation in international equity returns in bear markets. This finding proves the importance of providing a tail-adjusted mean–variance covariance matrix.

Correlation estimates form the core of analysis of the risk–return trade-off associated with investment portfolios. Low correlation is desirable from an investment perspective; diversification benefits materialize when a fall in one market is offset by a rise in another market. If the tendency is for all markets to fall simultaneously, however, the benefits of diversification will be overstated. Recent research into whether correlation has changed over time has generally used either multivariate GARCH (generalized autoregressive conditional heteroscedasticity) or regime-switching models, which can capture the time-varying nature of volatility and correlation. Longin and Solnik (1995) and Karolyi and Stulz (1996) found that time-varying correlation estimates have increased in recent sample periods. Although the finding that correlations vary over time does not necessarily imply that correlations depend on the size of market movements, correlations were found to have increased, particularly in periods of decreasing global market returns—that is, during bear markets.

Correlation estimates that are conditional on the size of market movements are of considerable relevance to investment analysts because it is in times of extreme market conditions that the benefits from diversification (and the effect of low correlations) are most urgently needed. A number of studies have dealt with the estimation of size-dependent correlation—Ramchand and Susmel (1998), Longin and Solnik (2001), Boyer, Gibson, and Loretan (1999), and Loretan and English (2000). In all these studies, a correlation structure was estimated conditional on market returns falling below (or above) a prespecified return level. The conditioning was applied to either a single component or to both components of the joint return distribution.

Such approaches to estimating size-dependent correlation invariably suffer from a theoretical estimation bias, which invalidates the estimates from a practical perspective. Proper adjustment for this bias is required, therefore, before applying the estimates to investment analysis. As it turns out, the bias depends not only on the choice of extreme returns (i.e., the nature of the conditioning on the size of the joint return) but also on the assumed underlying joint return distribution. Boyer et al. and Loretan and English, for example, conditioned the correlation estimates on one component of the bivariate normal return distribution. They then estimated the truncated conditional correlation for bivariate return observations in which one of the returns exceeded a certain threshold level. Unfortunately, one cannot straightforwardly compare these conditional truncated correlation estimates with unconditional correlation estimates to draw conclusions with regard to constant correlations in...
the tails of the joint return distributions. By truncating a joint return distribution with constant correlation, the conditional truncated correlation will be biased downward for increasing threshold levels (i.e., more extreme market conditions). The qualitative implication of this phenomenon is an increase in how often one would reject the assumption of constant correlation. Butler and Joaquin (2000) also used this approach and corrected their estimates based on estimated Sharpe ratios.

When researchers drop the normality assumption in favor of a fatter-tailed alternative bivariate distribution, the picture is quite different. In this approach, the truncation causes an upward bias for more extreme market movements. Hence, the frequency with which one would reject constant correlation decreases. This approach is discussed and illustrated for the $t$-distribution in detail in Campbell, Forbes, Koedijk, and Kofman (2001).

An alternative approach, which Longin and Solnik (2001) took, is to estimate conditional correlation that is conditional on the return series falling below or above a prespecified level of return. By considering only instances when both thresholds are exceeded, one obtains a “quadrant” of relevant return observations. When one is moving into the bivariate tail of the distribution, one includes only those joint return observations that fall into smaller and smaller quadrants of a scatterplot of the joint returns. The conditional correlation structure from such an approach converges to zero as one moves further into either tail of the distribution. Hence, a theoretical bias, similar to that of the previous approaches, occurs simply because of the way the conditioning extreme observations are defined.

Portfolio managers and risk managers require a meaningful yet practical interpretation of the conditional correlation structure of the joint return distributions of global financial assets. To apply conditional correlation estimates to investment analysis, the conditional correlation structure needs to be adjusted to account for the theoretical estimation bias and thus requires knowledge of the nature of the conditioning used. Ang and Chen (2000), for example, corrected for the induced bias by using both single and joint component conditioning techniques to analyze asymmetrical correlations in the U.S. equity markets.

Managers need a thorough understanding of the conditioning technique used to obtain size-dependent estimates before considering application of any size-conditional correlation measure. For a size-dependent correlation measure to be of practical use in portfolio analysis or risk management, it should relate to the correlation measure typically used by practitioners. Preferably, the conditional correlation measure would be independent of the nature of the conditioning used.

Therefore, instead of adjusting conditional correlation estimates for the theoretical bias by using the truncated or quadrant conditioning approaches, we propose an alternative approach to estimating the conditional correlation structure. We condition the correlation measure in a manner consistent with portfolio value at risk (VAR). The advantage is that our correlation estimates for portfolio returns are conditioned on portfolio returns falling below a prespecified worst-case portfolio quantile, instead of on the returns of either one or both of the assets falling below a given threshold level.

This approach has a number of advantages. First, it is in keeping with how correlation is applied in portfolio management and risk management. Our correlation measures can be applied directly to both Markowitz-style portfolio optimization and to VAR analysis. Second, the approach is easily generalized from the bivariate to a multivariate scenario. By using portfolio returns, we effectively collapse the multidimensional case into a one-dimensional case (i.e., a univariate portfolio return distribution). A third advantage is the characterization of the conditional correlation structure that is derived. For a broad class of elliptical distributions with constant correlations, the theoretical conditional correlation is equivalent to the theoretical unconditional correlation. A manager can directly attribute many deviations from this correlation to size-dependent correlations without having to worry about measurement bias. Hence, in a sense, the measure is “conditioning free.”

**Conditional Correlation Structure**

To determine how the correlation structure changes over the return distribution of two financial time series, one has to make a decision about which returns to condition. A correlation measure conditional on the size of the joint asset returns can be defined in various ways. We propose a measure derived from the literature on VAR. The size-conditional correlation measure follows directly from VAR measurement and provides a simple methodology for estimating the correlation structure implicit in joint portfolio returns without resort to extreme-value-theory estimation or fully parametric modeling of the joint return distribution. The use of our quantile correlation measure, with the conditioning on the downside support of the joint distribution of returns, is in line with current portfolio management techniques.
By using the VAR methodology, we are able to estimate the probability of a portfolio's return falling below a threshold (VAR) return with a pre-specified confidence level. With constant correlation over the joint return distribution assumed, the probability of the portfolio return falling below this VAR level is a weighted average of the probabilities of the individual assets' returns in the portfolio falling below this VAR level. As we increase confidence levels, we move further out into the tails of the joint portfolio distribution and the VAR increases. If at the same time, however, the correlation between the assets' returns increases, the portfolio's VAR will exceed the weighted average of the individual VARs. We invert this relationship, and by observing the difference between the portfolio VAR and the VAR levels for the individual assets in the portfolio, we can determine exactly how the correlation structure changes as we move further out into the tails of the joint return distribution.

VAR quantile estimation can be summarized in terms of a quantile return $q_c$, where $c$ denotes the confidence level that will not be exceeded with $(1 - c)$ percent probability. Assuming jointly normally distributed returns with a mean of 0 and a standard deviation of $\sigma$, the quantile return is simply a function of the standard deviation of the univariate normal distribution:

$$q_c = \Phi^{-1}((1 - c)\% \text{ of the standard normal distribution})$$  \hspace{1cm} (1)

where $\Phi$ is the standard normal cumulative distribution function. Writing Equation 1 as a portfolio quantile, squaring it, and substituting its components for the portfolio variance produces

$$q_{\text{portfolio}}^2 = \sum_{i=1}^{n} w_i^2 q_i^2 + 2 \sum_{i,j=1}^{n} w_i w_j q_i q_j$$  \hspace{1cm} (2)

where $w_i$ and $w_j$ are the weights of two assets, $q_i$ and $q_j$, and $q_{\text{portfolio}}$ is the portfolio return for confidence level $c$. Replacing the individual standard deviations by their VAR quantile companion estimates gives a conditional correlation measure:

$$\rho_Q = \frac{q_{\text{portfolio}}^2 - \sum_{i=1}^{n} w_i^2 q_i^2 - 2 \sum_{i,j=1}^{n} w_i w_j q_i q_j}{2 \sum_{i=1}^{n} w_i q_i q_i}$$  \hspace{1cm} (3)

We call the measure in Equation 3 the "quantile correlation measure." For a normal distribution, we can simplify Equation 3 so that the quantile correlation is constant for increasing quantiles:

$$\rho_Q = \rho$$  \hspace{1cm} (4)

Of course, in the case of normality, because correlation is bounded by $-1 < \rho < 1$, the quantile correlation is also bounded by $-1 < \rho < 1$.

### International Equity Markets

To see how the correlation structure changes over the joint distribution of international equity returns, we compared, under the assumption of bivariate normal return distributions, the empirical correlation structure to the (constant) theoretical quantile correlation structure. We could then comprehensively answer the question of whether large movements (of either sign) in equity markets are more highly correlated than small movements. We used daily data from DataStream for the United States, the United Kingdom, France, and Germany. These markets were studied by Longin and Solnik (2001), but we used a higher frequency of sampling. This data set extends from May 1990 through December 1999 (i.e., 2,500 observations).

The average return on the S&P 500, FTSE 100, CAC 40, and DAX 100 indexes was about 15 percent in the sample period, close to twice the return on the 10-year U.S. Government Bond Index. The equity index returns were two to three times as volatile, however, as the U.S. Government Bond returns. Summary statistics for the data are in Table 1.

### Table 1. Summary Index Statistics, May 1990–December 1999

<table>
<thead>
<tr>
<th>Statistic</th>
<th>S&amp;P 500</th>
<th>FTSE 100</th>
<th>CAC 40</th>
<th>DAX 100</th>
<th>10-Year U.S. Government Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average annual return</td>
<td>17.9%</td>
<td>15.5%</td>
<td>14.3%</td>
<td>12.2%</td>
<td>7.5%</td>
</tr>
<tr>
<td>Annual standard deviation</td>
<td>14.0</td>
<td>12.7</td>
<td>19.0</td>
<td>18.0</td>
<td>6.3</td>
</tr>
<tr>
<td>Maximum return</td>
<td>51.1</td>
<td>5.8</td>
<td>7.0</td>
<td>6.6</td>
<td>1.7</td>
</tr>
<tr>
<td>Minimum return</td>
<td>-6.9</td>
<td>-3.5</td>
<td>-7.3</td>
<td>-9.6</td>
<td>-2.8</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.346</td>
<td>0.078</td>
<td>-0.16</td>
<td>-0.655</td>
<td>-0.401</td>
</tr>
</tbody>
</table>

Note: Number of observations = 2,500.
All of the return series exhibit highly significant excess kurtosis, and all except the FTSE 100 exhibit significant negative skewness. The implication is that all the return distributions have greater probability mass in the tails of the distributions than the normal distribution would predict. Therefore, the probability of large movements in the stock and bond markets is greater than the assumption of normally distributed returns would predict. Deviations from normality may have implications for the correlation structure of the bivariate distribution, which we estimate later.

In Table 2, we provide the unconditional correlation estimates for the returns of the various international indexes. The unconditional correlation between the S&P 500 and the European stock markets averaged 0.338 in the period studied; the unconditional correlation between the European markets was much higher, averaging 0.620. This greater comovement of European stock markets resulted in almost 40 percent (0.620^2) of stock price movements being common to the European markets, whereas only about 10 percent (0.338^2) of stock price movements were common to both the U.S. and European markets. The unconditional correlation between stock market and bond market returns was low, as expected; the domestic correlation between the S&P 500 and the U.S. Government Bond indexes was only 0.272. The correlation between the European markets and U.S. government bonds averaged a mere 0.084.

Based on the estimates of the unconditional correlations, we can parameterize the bivariate normal distribution. We are now in a position to estimate the conditional quantile correlation structure based on the historical data and compare it with the theoretical quantile correlation structure. Figure 1 is a plot of the estimated empirical quantile correlation structure for the S&P 500 and the FTSE against the theoretical correlation structure, under the assumption of bivariate normality for the left tail of the distribution. We also plotted the 95 percent confidence interval for the estimates, and indeed, the data violate the assumption of normality and constant correlation only in the tails of the

### Table 2. Unconditional Correlations of Index Returns, May 1990–December 1999

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
<th>FTSE 100</th>
<th>CAC 40</th>
<th>DAX 100</th>
<th>10-Year U.S. Government Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FTSE 100</td>
<td>0.349</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAC 40</td>
<td>0.371</td>
<td>0.644</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DAX 100</td>
<td>0.296</td>
<td>0.575</td>
<td>0.622</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>10-Year U.S.</td>
<td>0.272</td>
<td>0.095</td>
<td>0.123</td>
<td>0.034</td>
<td>1.000</td>
</tr>
<tr>
<td>Government Bond</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Figure 1. Left-Tail Cumulative Correlation Using VAR: FTSE 100 and S&P 500

#### Notes:
Based on data for May 1990 through December 1999. The empirical quantile correlation estimates for cumulative percentiles of the joint return distribution are compared with the bivariate normal distribution with correlation coefficient 0.349.
Increased Correlation in Bear Markets

Up to the 95 percent level, we cannot reject the null of constant correlation. Thereafter (i.e., further out in the tails of the distribution), the correlation appears to increase significantly above the unconditional correlation. If the assumption of bivariate normality is reasonable, then allowances should be made for greater correlation of large movements in bull and bear markets. We also simulated quantile correlations with bivariate Student t-distributions. The same qualitative results held for the quantile correlation estimator. The theoretical quantile correlation is still constant and equal to the unconditional correlation. The only difference is that the standard error bands are considerably wider; hence, we would not reject constant correlation as frequently.

Implications for Portfolio Diversification

Having observed an increase in conditional quantile correlation during bull and bear markets, we now consider its relevance to the portfolio allocation decision. In modern portfolio theory, mean-variance investors maximize expected return for a given level of risk, as determined by the variance of the unconditional return distribution. No allowance is made for investors who weigh losses more heavily than gains or weigh large losses more heavily than small losses. Risk is a purely symmetrical measure, and correlation is assumed to be constant (i.e., size independent). So, even if investors did attribute greater risk to large losses, the assumption of joint normality would still result in the same mean-variance-efficient portfolio. However, as we have seen, deviations from normality exist, with increased probability mass in the tails of the return distribution, which results in an increase in the correlations of large negative movements in equity markets. This phenomenon has serious implications for portfolio management.

The benefits to international diversification depend crucially on assets being less than perfectly correlated. These benefits would be severely eroded by increasing correlation between asset returns in the tails of their joint distributions. In fact, when most needed, during bear markets, the protection offered by diversification would rapidly erode. Investors worried about greater downside risk from increased conditional correlation require a reformulation of the mean-variance portfolio allocation model. Instead of maximizing expected returns given the unconditional variance-covariance matrix, such investors need to maximize expected returns given a tail-adjusted variance-covariance matrix. Once we know the appropriate level of downside risk of concern to the investor, we can substitute the unconditional correlations by their conditional quantile correlation equivalents for that level of downside risk. Table 3 contains the average percentage increase in correlation (conditional quantile minus unconditional) required for a range of downside-risk levels.

The results illustrate that for investors or risk managers concerned with downside risk at the 90-95 percent level, little adjustment in correlation input is required for estimating the risk-return

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Table 3. Quantile Correlation Adjustments, May 1990–December 1999

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
<th>FTSE 100</th>
<th>CAC 40</th>
<th>DAX 100</th>
<th>10-Year U.S. Government Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. 90–95 percent confidence level</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>—</td>
<td>—</td>
<td>8.0</td>
<td>11.2</td>
<td>19.4</td>
</tr>
<tr>
<td>FTSE 100</td>
<td>—</td>
<td>5.2%</td>
<td>—</td>
<td>—</td>
<td>-3.4%</td>
</tr>
<tr>
<td>CAC 40</td>
<td>8.0</td>
<td>—</td>
<td>—</td>
<td>12.5%</td>
<td>-38.6</td>
</tr>
<tr>
<td>DAX 100</td>
<td>-9.8</td>
<td>-14.5</td>
<td>-3.4%</td>
<td>—</td>
<td>-96.0%</td>
</tr>
<tr>
<td>10-Year U.S. Government Bond</td>
<td>-41.7</td>
<td>-6.8</td>
<td>-8.6</td>
<td>-13.4%</td>
<td>-</td>
</tr>
<tr>
<td><strong>B. 95–99 percent confidence level</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>—</td>
<td>—</td>
<td>31.0%</td>
<td>11.1%</td>
<td>19.4</td>
</tr>
<tr>
<td>FTSE 100</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>-10.0</td>
<td>-38.6</td>
</tr>
<tr>
<td>CAC 40</td>
<td>11.2</td>
<td>—</td>
<td>12.5%</td>
<td>—</td>
<td>-96.0%</td>
</tr>
<tr>
<td>DAX 100</td>
<td>19.4</td>
<td>-10.0</td>
<td>—</td>
<td>—</td>
<td>-</td>
</tr>
<tr>
<td>10-Year U.S. Government Bond</td>
<td>13.0</td>
<td>18.4</td>
<td>-38.6</td>
<td>-96.0%</td>
<td>-</td>
</tr>
</tbody>
</table>
trade-off in global equity markets. The conditional quantile correlation estimates provide evidence of slightly lower correlations between global markets when conditioning on the 90–95 percent downside quantile of the joint return distribution; the decrease is insignificant, however, for all series. Only for the combination of S&P 500 returns and the CAC 40 returns do we find any evidence of increased correlation in this range. In particular, the quantile correlation between the U.S. equity and U.S. bond market shows evidence of greater diversification benefits than when the analysis is based on the unconditional correlation estimate.

Once we focus on more extreme downside risk, however, we find significant evidence of large increases in the conditional quantile correlation within global equity returns and between bond market returns and global equity returns. The S&P 500 and the FTSE show the greatest increase, 31 percent, in conditional quantile correlation. Thus, for investors and risk managers who require a greater degree of confidence in their portfolio or risk management recommendations, the benefits arising from international diversification are significantly curtailed.

In Figure 2, we have plotted two efficient frontiers to gauge the trade-off between risk and return as a portfolio moves from a 100 percent investment in the FTSE to a 100 percent investment in the S&P 500. The trade-off is significantly greater when using the higher conditional correlation of 0.457 for the 95–99 percent quantiles than when using the unconditional correlation of 0.349. The risk incurred to achieve a given amount of return is thus greater.

The implication is that downside risk increases during bear markets. Therefore, an investor will be better off using conditional mean–variance optimization in such circumstances. For example, if we assume a risk-free rate of 7 percent, the optimal allocation on the efficient frontier when we use the unconditional correlation measure is a 55 percent holding in the S&P 500 and a 45 percent holding in the FTSE. To maintain the same level of risk as when using the unconditional correlation measure would, then, require a 10.5 percent holding in the risk-free rate, a 50 percent holding in the S&P 500, and a 39.5 percent holding in the FTSE.

Conclusions
Volatility is time varying and should be modeled accordingly. In a multivariate context, therefore, correlation should also be modeled as a time-dependent variable. Market lore and intuition tell us that the observed time variation in correlation may also be a proxy for size dependency in correlation. In particular, large negative returns from international equity markets tend to coincide much more frequently than would reasonably be expected from the unconditional return correlation. Early attempts to capture the size dependency in correlations have so far been hampered by biased estimates and size conditioning that defy practical use in portfolio allocation. We have proposed using a correlation estimator that does not suffer from these shortcomings.

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**Figure 2. Efficient Frontiers Based on Unconditional and Conditional Correlations: S&P 500 and FTSE 100**

![Efficient Frontiers Diagram](image)

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**Note:** Based on annual data for May 1990 through December 1999.
Increased Correlation in Bear Markets

Our quantile correlation estimator conditions on the quantiles of the multivariate distribution of portfolio returns. Because these quantiles are well established in the portfolio VAR literature, they allow the conditional correlation estimator to be analyzed in a portfolio context. Thus, the quantile correlation measure can be directly applied to portfolio allocation, which makes it appealing from a practitioner's point of view. An additional advantage is its lack of bias, which allows a direct comparison of the conditional quantile correlation with the unconditional correlation.

Using data on international stock market index returns, we found evidence of increasing correlation in the tails, indicating contagion between financial markets for more extreme market movements. This phenomenon requires that investors use an amended variance-covariance matrix for mean-variance portfolio analysis and risk management when concerned about downside risk. During times of extreme bear markets, the effect from diversification is crucial; at such times, therefore, using the correlation estimates that incorporate the additional downside risk is crucial. The implications for portfolio allocation and risk management are serious because the benefits of diversification are partly eroded when they are needed most.

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Notes


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