Structural Invariance and Super Exogeneity in:
Macroeconometric Model Building:
MARIBEL's Consumption Function Revisited

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Dans cet article, nous illustrons l'importance et l'utilité des concepts d'invariance structurale et de super exogénéité dans la construction (empirique) de modèles macroéconométriques. A ces concepts, qui sont nécessaires pour qu'un modèle puisse être valablement utilisé dans un but de simulations de politiques économiques, correspondent des hypothèses testables qui nous permettent d'analyser les problèmes mis en évidence par la critique de Lucas. Nous illustrons ceci au travers d'une analyse empirique de la fonction de consommation du modèle MARIBEL version 1984. En utilisant à la fois une analyse de stabilité récursive (intra-échantillon) et des tests de super exogénéité nous trouvons que le modèle étudié ne satisfait pas aux conditions d'invariance structurelle et de super exogénéité.

1. Introduction

In this paper we illustrate the importance and the usefulness of the notions of structural invariance and super exogeneity in the elaboration of empirical models whose purpose is, among other things, policy simulation. For this purpose, we investigate the stability and structural

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* This paper is a revised version of a CREDEL Research Paper written while the first author was researcher at the department of economics. The authors want to thank J.K. Lindsey for a careful reading of this paper and David Hendry for making his unpublished paper ENGLE and HENDRY (1985) available to us. After our paper was completed, we have been informed that a revised version of ENGLE and HENDRY (1985) has been published under the same title : ENGLE and HENDRY (1990). The pages references mentionned in our paper do refer to the 1985 version. Any remaining errors or shortcoming are ours.
invariance properties of MARIBEL's consumption function in the version of the model which was extensively used in the mid-eighties. MARIBEL is a medium term annual macroeconometric model for the Belgium economy. Its main purpose is medium term projections (5 years) and policy simulation. As such, it is a perfect candidate potential invariance failure which is raised in the famous LUCAS (1976) critique. This is investigated for the private total consumption function. It must be clearly pointed out that the main purpose of this paper is to emphasize the usefulness of concepts such as invariance, weak and super exogeneity in the construction of a macroeconometric model whose purpose is basically policy simulation. We do not attempt to propose an alternative consumption model for the Belgium economy. The specific formulation of Maribel which is investigated here has been revised by the Bureau du Plan and is no longer used for practical simulation.

Our empirical analysis enable us to emphasize the importance and the usefulness of the L.S.E. econometric modelling approach for estimation, specification and evaluation of dynamic macroeconometric models (see inter alia HENDRY and RICHARD (1982; 1983), SPANOS (1988)). Section 2 briefly recalls the essence of the Lucas critique to econometric modelling. Section 3 presents the econometric tools that are used. MARIBEL's consumption function, in its 1984 version, is presented and extensively analysed in section 4.

2. Conditional Models and the Lucas Critique

As the concepts of structural invariance and super exogeneity were developed by ENGLE, HENDRY and RICHARD (1983) -denoted hereafter EHR- in order to deal with the problem raised by the Lucas critique, we shall first briefly recall the essence of the latter. Basically, LUCAS (1976)

1 MARIBEL stands for Model for Analysis and Rapid Investigation of the Belgian Economy.
2 As pointed out by a referee, we should also acknowledge that at the time of the construction of the Model, the methodological background to which this paper refers was emerging in the academic and research institutions but not yet used by institutions dealing with applied work such as the Planning Bureau.
considers a dynamic conditional macroeconometric model used and specified for macroeconomic policy simulation. The parameters of such an econometric relationship may reflect the economic agents' decisions rules. These are supposed to integrate knowledge about past policies as well as expectations about future possible policy shifts. Under these considerations, decision rules cannot be expected to stay invariant when subject to policy interventions. LUCAS (1976) claims that predicting the behaviour or evolution of the economy conditional on a certain path of the policy variables is meaningless unless the agents expectations are clearly modelled. A simple example is private agent behaviour (viewed as consumers) whose decisions about some consumption expenditures would not vary with government choice of varying tax rates. Shifts in the estimated parameters of such a consumption function are likely to occur and using a simple fixed parameter conditional macroeconometric model for policy simulation purposes is -according to Lucas- doubtful.

In fact, as SARGENT (1981) argues, part of the Lucas critique is based on the idea that parameters of observed decisions rules should not be viewed directly as structural. He also insists on the need of a stricter definition of what should be considered as structural parameters. Formulating models at the level of what he calls "deep parameters" ³ (SARGENT, 1981, p. 233) should provide empirical models whose parameters would exhibit invariance, even under changing constraints. As a response to this they propose a unified framework integrating expectation mechanisms for the elaboration of empirical models whose purposes are mainly policy prediction and simulation. Note that in their approach the specification (adjustment, expectations, dynamic,...) is derived from a well articulated dynamic economic theory. They thus implicitly assume that empirical and theoretical models do coincide.

Another approach to the empirical modelling of economic phenomena which takes the Lucas critique into account has emerged these last ten years : the so-called London School of Economics tradition, initiated by

³ Deep parameters in Sargent's conception are preferences, technologies,... (SARGENT, 1981).
the pioneer work of SARGAN (1964). In the modeling process, economic theory plays an important role in the determination of the underlying long run specification, but the dynamic adjustment schemes are derived from a careful analysis of the time series properties of the data series. The aim of the econometric modelling will then consist of trying to develop conditional submodels such that the transient features of the observed world (nonstationarity, instability,...) are affecting the marginal submodel only so that, if a valid cut is achieved (in the sense of FLORENS and MOUCHART, 1985; EHR, 1983), the analysis can be conducted on the conditional submodel only. In this framework, the empirical and theoretical models do clearly differ. Note that we do not consider the debate between feedforward (in the Lucas-Sargent tradition) and feedback (in the Sargan-Hendry tradition) representations of agents decisions rules. An interesting analysis is proposed in HENDRY (1988) to which we refer.

3. Structural Invariance and Super Exogeneity

In order to to present the basic tools used in this paper, we consider the case of two observable economic time series \( y_t \) and \( x_t \). We assume that the joint distribution of \((y_t \ x_t)'\) is conditional normal:

\[
(3.1) \quad \begin{pmatrix} y_t \\ x_t \end{pmatrix} | \Psi_t \sim N (\mu, \Sigma)
\]

where \( \mu = (\mu_1 \ \mu_2)'; \Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \)

and \( \Psi_t \) is the available information set, including past values and initial conditions. The underlying conditional model of \( y_t / x_t, \Psi_t \) is given by:

\[
(3.2) \quad y_t | x_t, \Psi_t \sim N (\mu_2 + \sigma_{12} (x_t - \mu_1), \sigma_{11,2})
\]

and \( \sigma_{11,2} = \sigma_{11} - \sigma_{12} \sigma_{22}^{-1} \sigma_{21} \)

If we consider this conditional model, it is known that sufficient conditions for valid inference on the parameters of interests are provided by the condition that \( x_t \) is weakly exogenous (in the sense of EHR, 1983) for the parameters of interest. Nevertheless, weak exogeneity alone does
not rule out the possibility that agents do modify their behaviour in response to an intervention affecting their constraints (see RICHARD, 1980; 1987). Consider now the joint distribution of $y_t$ and $x_t$. Some parameters of this joint distribution may be subject to important changes induced by taste evolutions, policy shifts, ... If we only consider certain classes of parameters, it is reasonable to assume that there may exist some parameters which remain constant. These will said to be *invariant* to policy changes.

*Definition 2.1. (see EHR, 1983, p. 284)*

A parameter is *invariant* for a class of interventions if it remains constant under these interventions. A model is invariant for such interventions if all its parameters are.

*Definition 2.2. (EHR, 1983, p. 284)*

A conditional model is *structurally invariant* if all its parameters are invariant for any changes in the distribution of the conditioning variables.

This last definition provides conditions for valid policy simulations, and reflects the assumption whose untested character was the essence of the Lucas critique. Structural invariance is naturally closely related to weak exogeneity. Conditional models with invalid weak exogeneity assumptions entails absence of a cut w.r.t. nuisance parameters and thus interventions affecting these will typically induce absence of structural invariance of the conditional model. By adding the condition of weak exogeneity for the parameters of interest of $x_t$ to definition 2.2., we get the concept of *super exogeneity* (EHR, 1983, p. 284) which is sufficient for inference and simulation in a conditional model subject to interventions. Finally conditional simulation requires strong exogeneity of $x_t$ for the parameters of interests (i.e. weak exogeneity and absence of feedback) ⁴ see EHR (1983), HENDRY and RICHARD, (1982).

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⁴ As far, we have not discussed the relation between parameter constancy and parameter invariance. This last concept is more demanding and requires constancy of the parameters even under hypothetical (and various type of) interventions. It is thus by nature conjectural (HENDRY and RICHARD, 1982).
From the above discussion, it is clear that empirical models whose purpose is policy simulation should, besides some traditional criteria, satisfy the above mentioned one. As a by-product, the usefulness of the LSE approach and its related criteria emerge as a useful tool for empirical specification. In this framework, an empirical model aimed to represent the underlying unknown data generating process must satisfy the conditions of being able to be considered as a tentatively adequate conditional data characterization (TACD ⁵), i.e. it must be congruent with the evidence provided by the data. This entails, roughly speaking, 6 criteria (see HENDRY, 1987): (1) data admissibility; (2) valid conditioning; (3) parameter constancy; (4) data coherency; (5) encompassing; (6) theory consistency. Conditions (2) and (3) ensure the empirical model not to be subject to the Lucas Critique. Note that (2) depends on the purpose of the analysis ⁶. Since all these conditions can be viewed as design criteria, the empirical modelling can be interpreted as an exercise in design (HENDRY, 1987) where all these different points imply testable null hypotheses.

4. Econometric Methodology and Testing Procedures

Structural invariance is by nature conjectural and there are several ways to check this hypothesis. Explicit test for superexogeneity may be conducted along the lines of ENGLE and HENDRY (1985). This will be seen in section 4.2. The other possibility is to consider within sample parameter change as a sufficient condition to reject that conjecture (see HENDRY and RICHARD, 1982 and DUFOUR, 1986 for an application on investment function).

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⁵ For a detailed presentation, definition, concepts and criteria, we refer to GILBERT (1986); HENDRY (1987) or SPANOS (1988).

⁶ For inference, weak exogeneity is required; if the purpose is conditional predictions (or dynamic simulation), then we need to add the condition of Granger non-causality from \( y_t \) to \( x_t \), i.e. strong exogeneity of \( x_t \) is required.
4.1. Within sample stability analysis

We first consider a recursive stability analysis following the pioneer paper of BROWN, DURBIN and EVANS (1975) - hereafter BDE- and its extensions (DUFOUR, 1982). Consider the following simple regression model:

\begin{equation}
(4.1) \quad y_t = x'_t \beta_t + u_t \quad t = 1, \ldots, T
\end{equation}

where \( y_t \) is the dependent variable, \( x_t \) is a \((k \times 1)\) vector of regressors, \( \beta_t \) is a \((k \times 1)\) vector of coefficients and \( u_t \) is the disturbance term supposed to be N.I.I.D.\((0, \sigma^2)\). The null to be tested is the constancy of the \( \beta_t \)'s over different observations: \( H_0: \beta_t = \beta_0 \) for \( t = k+1, \ldots, T \). A natural way to proceed is to estimate by OLS (4.1) recursively, using the first \( k \) observations to get an initial \( \hat{\beta} \) and then adding one observation at a time in order to get a sequence of estimates.

\begin{equation}
(4.2) \quad \hat{\beta}_p = (X'_p X_p)^{-1}X_p Y_p \quad \text{for } p : k, \ldots, T
\end{equation}

where \( X_p = (x_1, x_2, \ldots, x_p)' \).

This sequence, in graphical form, constitutes a first useful source for the detection of possible departure from the constancy hypothesis. Note that they are (DUFOUR, 1982, p.24) highly autocorrelated as they follow a heteroskedastic random walk. Apparent trend in these sequences must thus be carefully interpreted. If we consider the associated standardized one-step ahead prediction errors we get the wellknown (one-step ahead) recursive residuals:

\begin{equation}
(4.3) \quad \tilde{u}_t = v_t / \left[ 1 + x'_t (X'_{t-1} X_{t-1})^{-1} x_t \right]^2
\end{equation}

where \( v_t = y_t - x'_t \hat{\beta}_{t-1} \).

Under \( H_0 \), the sequence of \( \tilde{u}_t \)'s is normal white noise. A shift is \( \beta_t \), at

\[7\text{ The initial formulation required fixed regresors and thus ruled out dynamic models. However, recent work by KRÄMER et al. (1987) shows that the straightforward cusum test retains its asymptotic significance level in dynamic models.} \]
say $t^*$, will usually imply an over or under prediction of $y_t$ for $t > t^*$. Defining $k$-step ahead residuals ($k > 1$) can provide better defined patterns and are thus also useful (Dufour, 1982).

Further investigation can be carried by distinguishing two types of instability: systematic or erratic. The former can be captured by for example the CUSUM test (see BDE, 1975; Dufour, 1982) or the perturbation test (Ploberger et al., 1989). The latter may require the use of heteroskedasticity tests, CUSUMSQ test or sequences of Chow (1960) tests.

The CUSUM test is based on cumulative sums

\[
U_p = \frac{1}{\hat{\sigma}} \sum_{t=k+1}^{p} \hat{\epsilon}_t \quad p = k+1, \ldots, T
\]

where $\hat{\sigma}$ is the estimated standard error of the one-step recursive residuals. The null of constancy is rejected when, for a certain $\alpha$, the $U_p$'s crosses one of the critical values lines constructed as in BDE (1975). Recently, Ploberger et al. (1989) have proposed a simple procedure for the detection of structural instability in a situation where no prior information is available on the location of the possible shift point. It is directly based on the successive OLS parameter estimates. The null of parameter constancy is rejected whenever the $\hat{\beta}'s$ exhibit too important a fluctuation, i.e. when there is excessive variation in:

\[
|| \hat{\beta}_t - \beta_T ||_\infty = \max_{i=1,\ldots,k} || \hat{\beta}_{it} - \hat{\beta}_{iT} ||
\]

where $|| . ||_\infty$ is the maximum norm. The test statistic they propose to use is:

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8 BDE proposed to estimate $\hat{\sigma}$ by the estimated standard error of the OLS residuals. A comment by Harvey (1975) pointed out that this is likely to reduce the power of the procedure. He therefore advise to use recursive residuals for the calculations. For Monte Carlo evidence favoring this point, see Alt and Krämer (1986).

(4.6) \[ S_T = \max_{t=1}^{T^*} T \]

\[ = \max_{t=k,...,T} \frac{1}{T} \sum_{t=k}^{T} \left( X_{p}^1 X_{T}^1 \right)^{1/2} - \left( \hat{\beta}_t - \hat{\beta}_T \right) \| \infty \]

and \( \hat{\sigma} = \left[ \frac{1}{T-k} \sum_{t=k}^{T} \left( y_t - X_t \hat{\beta}_T \right)^2 \right]^{1/2} \). The standardisation by \( (X_p^1 X_T^1)^{1/2} \) ensures the existence of some non degenerate limiting distribution for \( S_T \) (see Ploberger et al., 1989, p 312). Using this, we can graph the trajectory of \( S_t \) (with \( t \) varying from \( k+1 \) to \( T \)). Under the null, \( S_t \) should remain below a critical significance line whose value depends on the number of regressors and on the chosen level (see Ploberger et al., 1989, p 315).

Finally, various sequences of successive Chow (1960) tests for parameter constancy provide additional tools for within sample analysis. Note that the sequence of CUSUMSQ statistics is numerically equivalent to a sequence of Chow test with decreasing horizon.

### 4.2. Testing for super exogeneity 10

Super exogeneity is defined as weak exogeneity and structural invariance. Inference on it can thus be conducted by means of weak exogeneity, invariance tests or both jointly.

Consider again the joint distribution of \( (y_t x_t)' \) as being conditional normal but where we now allow \( \mu \) and \( \Sigma \) to be both functions of time.

\[ (4.7) \quad \begin{pmatrix} y_t \\ x_t \end{pmatrix} | \Psi_t \sim N \left( \mu_t, \Sigma_t \right) \]

where \( \mu_t = (\mu_{1t}, \mu_{2t})' \) and \( \Sigma_t = \begin{pmatrix} \sigma_{11t} & \sigma_{12t} \\ \sigma_{21t} & \sigma_{22t} \end{pmatrix} \) and \( \Psi_t \) is the entire information set.

The underlying conditional model of \( y_t \) given \( x_t \), \( \Psi_t \) is given by:

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10 This subsection is directly based on an unpublished paper by Rob Engle and David Hendry (1985).
(4.8) \[ y_t \mid x_t, \Psi_t \sim N \left( \mu_{2t} + \frac{\sigma_{12t}}{\sigma_{22t}} (x_t - \mu_{1t}), \sigma_{11.2t} \right) \]

with \[ \sigma_{11.2t} = \sigma_{11t} - \sigma_{12t} \sigma_{22t}^{-1} \sigma_{21t} \cdot \sigma_{21t}. \]

We further assume that we are interested in a behavioral relation relating \( \mu_{1t} \) to \( \mu_{2t} \) and a set \( z_t \in \Psi_t \) of variables. Our parameters of interest are \( (\beta, \gamma) \) in the following relationship:

(4.9) \[ \mu_{1t} = \beta \mu_{2t} + z_t' \gamma \]

By substituting (4.8) in (4.9), the conditional model becomes:

(4.10) \[ y_t \mid x_t, \Psi_t \sim N \left( \beta x_t + z_t' \gamma + \left( \frac{\sigma_{12t}}{\sigma_{22t}} - \beta \right)(x_t - \mu_{2t}), \sigma_{11.2t} \right) \]

In this model, the condition for super exogeneity of \( x_t \) for \( (\beta, \gamma) \) is simply given by \( \beta = (\sigma_{12t}/\sigma_{22t}) \). If this is not the case, \( \mu_{2t} \) and \( \sigma_{22t} \) appear in the conditional model and changes in these quantities will directly affect the conditional model, i.e. both weak exogeneity and invariance fail. If we now suppose constant \( \Sigma_t \), testing the null hypothesis of super exogeneity of \( x_t \) for \( (\beta, \gamma) \) in

(4.11) \[ y_t = x_t' \beta + z_t' \gamma + \varepsilon_t \]

simplifies to the wellknown case of testing for weak exogeneity following for example the work of ENGLÉ (1984) or HOLLY (1985). We may model \( \mu_{1t} \) by means of a set of instruments \( Z_t \) (with \( z_t \in Z_t \)) through

(4.12) \[ x_t = Z_t' \Pi_X + \eta_t \]

The LM test for weak exogeneity can then be simply conducted by testing for significance of \( \hat{\mu}_{1t} = x_t - \hat{x}_t \) in (4.11) \(^{11}\).

Consider now an alternative model for \( \mu_{1t} \) that allows \( \beta \) to be function

\(^{11}\) It is obvious that one can alternatively test for \( \hat{x} \) in (4.11).
of changes in the distribution of \( x_t \) (4. 9) may be rewritten as

\[
\mu_{1t} = \beta_0 \mu_{2t} + z_t \gamma + \beta_1 (\mu_{2t})^2 + \beta_2 \sigma_{22t} + \beta_3 \mu_{2t} \sigma_{22t}
\]

where one has simply used a Taylor expansion for expressing \( \beta \) as a function of the moments of \( x_t \) (see ENGLE and HENDRY, 1985). With (4.13), (4.10) becomes:

\[
y_t | x_t, \Psi_t \sim N (x_t \beta_0 + z_t \gamma + (\frac{\sigma_{12t}}{\sigma_{22t}} - \beta_0)(x_t - \mu_{2t}) + \beta_1 (\mu_{2t})^2 + \beta_2 \sigma_{22t} + \beta_3 \sigma_{22t} \mu_{2t}, \sigma_{11,2t})
\]

This modelling of \( \mu_{1t} \) allows for direct test of the Lucas critique. By following a similar argument as above, testing for the superexogeneity of \( x_t \) for \((\beta, \gamma)\) can be conducted by performing a joint test of the significance of \( \hat{x} \) and \( \hat{x}^2 \) in (4.11) where \( \hat{x} \) is obtained by estimating (4.12).

5. Maribel's Private Consumption Function

The specification of MARIBEL's private consumption function is derived within a Keynesian framework. Using among others the work of EVANS (1969) \(^{12}\) they finally introduce an "error correction" type adjustment scheme in such a way that their "preferred" specification is:

\[
C_t = \lambda_1 \hat{\beta} (YD/PC)_t + \alpha \lambda_2 + \lambda_2 \beta (YD/PC)_{t-1} + (1-I_2) C_{t-1} + \gamma \Delta (UL/NA)_t + e_t
\]

where \( \Delta \) denotes the first difference operator, \( C \) is the unadjusted private consumption at current prices; \( YD \) is the unadjusted disposable income; \( PC \) is the unadjusted private consumption price index, \( UL \) the number of unemployed and \( NA \) the total active population. All variables are

\(^{12}\) For further details see BUREAU DU PLAN (1984, p. 151-160).
expressed in levels. The underlying (long run) equilibrium model is assumed to be:

\[ C_t^* = \alpha + \beta (YD/PC)_t^* \]

where * denotes the long run equilibrium level. The sample period covers 1953-1981, the data are annual figures and the estimation was carried by simple OLS. Absolute t-values are reported in brackets. The following results are reported (BUREAU DU PLAN, 1984, p. 176):

\[ C_t = 26.971 + 0.515 \Delta (YD/PC)_t + 0.305 (YD/PC)_{t-1} + 0.631 C_{t-1} \]

\[
(1.68) \quad (4.55) \quad (2.89) \quad (4.738) \\
- 830.89 \Delta (UL/NA)_t \\
(-2.32)
\]

\[ R^2 = 0.999 \quad S.E. = 11.0094 \quad D.W. = 1.80 \]

6. Revisiting Maribel's Consumption Function

We first reestimate (5.1) by OLS. The sample covers the period 1953-1986; the first observation is used for the initialization of the lagged variables, observations 1982-1986 are kept for predictive failure tests. Data sources are reported in an appendix. In brackets, we report the heteroskedastic consistent standard errors (see WHITE, 1981). Other notations are: \( R^2 \) for the squared multiple correlation coefficient, RSS for the sum of squared residuals, \( \sigma \) for the standard deviation of the regression error, and D.W. for the traditional Durbin-Watson statistic. The following test statistics are also reported.

\( Z_1(\ldots) \) : Chow test for predictive failure (PEASARAN et al, 1985)
\( Z_2(\ldots) \) : Hendry's predictive failure test (HENDRY, 1979)
\( Z_3(\ldots) \) : Box-Pierce statistic (HARVEY, 1981)
\( Z_4(q,\ldots) \) : LM test for qth order autocorrelation (HARVEY, 1981)
$Z_5(\cdot \cdot) :$ Test for qth order ARCH effect (Engle, 1982)
$Z_6 (\cdot) :$ Test for normality (Beran and Jarque, 1987)
$Z_7(\cdot \cdot) :$ Test for heteroskedastic errors (White, 1981)
$Z_8(\cdot \cdot) :$ 2nd order RESET test (Spanos, 1986)

The $Z_i (\cdot)$ are $\chi^2$ statistics and $Z_i (\cdot \cdot)$ are F statistics with the corresponding degrees of freedom.

We obtained the following results

$$C_t = 86.763 + 0.4753 \Delta (YD/PC)_t + 0.5083 (YD/PC)_{t-1} + 0.3641 C_{t-1}$$

$$- 987.4407 \Delta (UL/NA)_t$$

$$R^2 = 0.998 \quad \sigma = 18.1043 \quad RSS = 7538.5574 \quad DW = 2.113$$

$$Z_1 (5.23) = 2.64^* \quad Z_2 (5) = 6.69 \quad Z_4 (1.22) = 0.21 \quad Z_4(2.21) = 0.71$$

$$Z_4 (3.20) = 3.36^* \quad Z_5 (3.19) = 1.48 \quad Z_5 (2) = 1.67 \quad Z_7 (8.14) = 0.21$$

$$Z_8 (2.21) = 2.954$$

Reported values of the above test statistics do not indicate evidence of important misspecification although a significant 3rd order serial correlation ($Z_4 (3.20) = 3.36$ for a 5% critical value of 3.10) is detected. $Z_1$ and $Z_2$ show some predictive failure.

The next step is to reestimate recursively (5.1) by means of the Recursive Least Squares method. Figures 1-4 reports the evolution of the coefficients estimated recursively by enlarging the sample from k+1 to T by step one.

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13 All the computations were carried with P.C. GIVE econometric package.
FIGURE 1: Recursive Estimation: $\Delta (YD/PC)$ Coefficient

FIGURE 2: Recursive Estimation: $C(t-1)$ Coefficient
FIGURE 3: Recursive Estimation: \((YD/PC)_{t-1}\) Coefficient

FIGURE 4: Recursive Estimation: \(\Delta(UL/NA)\) Coefficient
Figures 6-9 are various figures of CUSUM quantities. Figure 6 is based on one-step ahead recursive residuals, Figure 7 1-step backward recursive residuals while Figures 8-9 are respectively 3-step ahead and backward recursive residuals. The sequences of Chow tests are reported in Figures 10-12. Finally, the evolution of the perturbation test quantity $S_t^\star$ is reported in Figure 13.

**FIGURE 5 : Recursive Estimation : Constant Term**

![Recursive Estimation: Constant Term](image1)

**FIGURE 6 : One-Step Ahead Cusum Test**

![One-Step Ahead Cusum Test](image2)
FIGURE 7: One-Step Backward Cusum Test

FIGURE 8: Three-Step Backward Cusum Test
FIGURE 9: Three-Step Ahead Cusum Test

FIGURE 10: One-Step Chow Test
FIGURE 11: Increasing Horizon Chow Test

FIGURE 12: Decreasing Horizon Chow Test
A simple analysis of these graphical representations allows us already to express some doubts about the structural invariance character of Maribel's consumption function. Although the 1- and 3- step ahead cusum quantities do remain below their critical lines, it is worth noting that the fluctuation test is above its critical value for almost the entire sample period. More details can be obtained from the figures reporting the different evolutions of the recursively estimated coefficients (Figures 1-5). The more drastic situation seems to occur with the coefficient associated with C_{t-1} which changes sign around 1978, going from a significant negative value up to a point estimate of .63 for the period 1981. Similarly the coefficient associated with Δ(UL/NA)_{t} changes sign around 1968 while further instability is seen for the other coefficients where for example that associated with (YD/PC)_{t-1} shows a downward trend over the period 1964-1981. This is also true (although less clearly) for Δ(YD/PC)_{t}. Finally, the various sequences of Chow tests point out an important instability where the major breaks seem to be located around 1965 and 1978. One might thus suspect that in this model parametrisation both UL/NA and YD/PC are not super exogenous conditioning variables for the parameters of interest.
The final step of this empirical analysis is to perform explicit tests for weak and super exogeneity. As usual, the weak (and/or super) exogeneity of YD/PC will be questioned by means of the LM test presented above. It first require the estimation and specification of an auxiliary instrumental regression. As pointed out for example by KIVIET (1986) variable addition tests do strongly depend on the specification of the auxiliary equation, i.e. on the choice of the instruments. Modelling disposable income is not obvious, but in this case all that matters is to obtain a reasonably well behaved model for YD/PC with (at least) white noise disturbances. Various sets of instruments have been investigated, including both lagged values of the variables entering (5.1) and also some additional variables such as cumulated saving (denoted by L)\(^{14}\), public consumption, total consumption (C), some inflation measure (P), current national saving (S), Gross National Product (GNP)... We also tried to add some shift dummies interacting with several instruments in order to take into account different regimes but these appeared as insignificant in the auxiliary regressions. In order to show the sensitivity of the results to the specification of the auxiliary regression, we applied the same testing procedure with three alternative specifications of the auxiliary reaction function for disposable income. Note that equation (6.4) has the set of instruments restricted to lagged values of the variables entering (6.1). The selected specifications are reported below.

\[
\Delta (YD/PC)_t = -87.097 + 0 + 46675 \text{ GNP}_t - 0.58915 \text{ GNP}_{t-1} \\
+ 0.20257 \text{ S}_t - 0.11703 \text{ S}_{t-1} + 470.781 \Delta_1 \text{ P}_t + 0.24344 \text{ C}_{t-1} \\
(46.755) \quad (0.06968) \quad (0.13438) \quad (0.06907) \quad (0.07572) \quad (316.27) \quad (0.15131)
\]

\[
R^2 = 0.849 \quad \sigma = 20.23 \quad \text{RSS} = 7364.98 \quad \text{DW} = 2.159
\]

\[
Z_1 (5,18) = 3.24 \quad Z_2 (5) = 10.89 \quad Z_3 (1,17) = 0.14 \quad Z_4 (2.16) = 1.061
\]

\[
Z_5 (1,16) = 0.03 \quad Z_6 (2) = 0.924 \quad Z_7 (14,3) = 0.63 \quad Z_8 (2,16) = 0.292
\]

\[\text{As used and constructed in STEEL (1987), LAHAYE (1989).}\]
(6.3) \[ \Delta(YD/PC)_t = 4529.9589 - 0.38155 (YD/PC)_{t-1} + 0.32394 \text{GNPt} \]
\[ \begin{array}{ccc}
\text{(7953)} & \text{(0.0840)} & \text{(0.0697)} \\
- 0.32394 \Delta_1 \text{GNPt}_{t-1} + 1056.3045 \Delta_1 \text{Pt} + 0.30972 \Delta_1 \text{Ct-1} \\
\text{(0.08523)} & \text{(367.60)} & \text{(0.1586)} \\
- 0.088 \text{Lt} + 0.0813 \text{Lt-1} \\
\text{(0.0431)} & \text{(0.0356)} 
\end{array} \]

\[ R^2 = 0.879 \quad \sigma = 18.278 \quad \text{RSS} = 6013.589 \quad \text{DW} = 2.46 \]
\[ Z_1 (5.18) = 2.49 \quad Z_2 (5) = 6.14 \quad Z_3 (1.17) = 2.06 \quad Z_4 (2.16) = 1.71 \]
\[ Z_5 (2.15) = 0.39 \quad Z_6 (2) = 1.113 \quad Z_7 (14.3) = 0.51 \quad Z_8 (2.16) = 0.404 \]

(6.4) \[ \Delta(YD/PC)_t = 42.17948 - 0.14075 \Delta(YD/PC)_{t-1} \]
\[ \begin{array}{ccc}
\text{(28.261)} & \text{(0.3656)} \\
+ 0.45074 (YD/PC)_{t-2} + 0.38669 \text{Ct-1} - 0.62816 \text{Ct-2} + 0.23917 \text{Ct-3} \\
\text{(0.3778)} & \text{(0.2728)} & \text{(0.5262)} & \text{(0.4869)} 
\end{array} \]

\[ R^2 = 0.423 \quad \sigma = 44.246 \quad \text{RSS} = 39154.019 \quad \text{DW} = 2.109 \]
\[ Z_1 (5.20) = 2.07 \quad Z_2 (5) = 3.41 \quad Z_4 (1.19) = 1.41 \quad Z_4 (2.18) = 1.30 \]
\[ Z_5 (1.18) = 4.37 \quad Z_6 (2) = 0.353 \quad Z_7 (14.3) = 1.19 \quad Z_8 (2.16) = 0.684 \]

The overall specification checks are not too bad and the forecast performances are rather satisfying although an important fall in the variation of disposable income occurred in 1982. Once the auxiliary regression is estimated, the weak exogeneity test can then easily be performed by adding to (6.1) the estimated residuals (denoted respectively \( \hat{\nu_i} \); i=1,2,3) from (6.2)-(6.4) and tests for their significance. Alternatively we could conduct a simple LM test for omitted, \( \hat{\nu}_i \) (i=1,2,3) in (6.1) \(^{15}\). For the super exogeneity hypothesis, a similar LM procedure is applied but where the joint significance of both the fitted \( \Delta(YD/PC) \) and its squared value is

\(^{15}\) The results are not reported since the outcome of this LM test (in F-form) is simply the square of the t-test.
tested in (6.1). The results are reported below in table 1.

<table>
<thead>
<tr>
<th>Aux. Equation</th>
<th>t-test on $\hat{V}_i$</th>
<th>Joint LM test on fitted $\Delta(YD/PC)$ and its square</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6.2)</td>
<td>2.34870</td>
<td>5.928</td>
</tr>
<tr>
<td>(6.3)</td>
<td>0.26905</td>
<td>0.966</td>
</tr>
<tr>
<td>(6.4)</td>
<td>0.87131</td>
<td>0.410</td>
</tr>
</tbody>
</table>

Note that the LM-test for omitted $\Delta(YD/PC)$ and its square is distributed as a F-statistic with (2,19) degrees of freedom. The 5% level critical value is approximately 3.52. It appears thus that depending on which set of instruments the analysis is based, one may or not reject the hypothesis of weak and super exogeneity. In the case of the set used in (6.2) we can reject both weak and strong exogeneity for our parameter of interests; while with the instruments set underlying (6.3) and (6.4) we may not reject both hypotheses of weak and super exogeneity of disposable income. This is particularly puzzling since the within sample stability analysis results which showed clear evidence of instability, rendering structural invariance more than a doubtful character of the selected consumption model specification.

7. Conclusions

In this study we have investigated several invariance properties of Maribel's consumption function as it was used in 1981. The first thing we have to point out is that when a model is design for policy simulation purposes, the traditional statistics usually reported as $R^2$ and DW are insufficient to ensure the model to be useful. A simple recursive analysis shows clear within sample instability, which is sufficient to reject the "conjecture" of structural invariance. We did also conduct explicit tests for super exogeneity, following the approach of ENGLE and HENDRY (1985). However, as these procedures are extremely sensitive to possible
mispresentation of the auxiliary regression, they should be interpreted with care. In any case, in its 1984 version, which was extensively used in the mid-eighties- the total consumption equation of MARIBEL appears to lack structural invariance rendering its projection and simulation outcome doubtful.

We must also mention that we have followed a very traditional analysis for investigating the properties of Maribel's consumption model. In particular, the recent theoretical and applied econometric literature has provided strong evidence that many macroeconomic series are well represented by integrated process for which many traditional statistical tools are no more valid (for a simple and clear overview, see CAMPBELL and PERRON, 1991; DOLADO et al, 1991). In this case, the study of the link between different series requires the use of cointegrated models and a careful analysis of weak exogeneity requires additional investigation as pointed out in URBAIN (1991) - see also ERICSSON (1992) for an overview of the concepts of cointegration, exogeneity and invariance.

Reference


URBAIN, J.P. (1988) "Further results on the instability of the demand for money during the German Hyperinflation", Credel Research Paper 8801., University of Liège


Data sources:

All data are taken from the publications of the I.N.S. (Institut nationa de Statistiques); except NA (total population) which is taken from the IMF statistics. Full details can be found in LAHAYE (1988).