Time Series and Econometric Models
with Unobservables *

1. Introduction

Time series methods have been designed for the analysis of the
dynamic features of one or several sequences of observations, in contrast
to econometric methods which aim at the unification of theoretical
results and factual evidence in economics. The scope of the Econometric
Society is described in section I of its Constitution as:

"Its [the Econometric Society] main object shall be to promote
studies that aim at the unification of the theoretical — quantitative
and the empirical — quantitative approach to economic problems . . . ."

A truly econometric approach can therefore not only consist in
pure theorizing or in pure data analysis, but it has to combine and
integrate both parts of the analysis into one methodology. Econometric
modeling requires to address the question of the connection between
the relationships established in theory and the equations fitted to the
data. As data collection and economic theory construction are usually
separate activities, it is probable that the theory variables are not di-
rectly observed. Therefore, a latent or unobservables framework is
frequently required for econometric modeling.

Well-known examples of unobserved theory variables are expecta-
tional variables, desired values in a partial adjustment model or in an
error correction model. Also, the theory often explains the behavior
of a microeconomic unit, whereas the data are temporal aggregates
over agents, commodities and services. Some variables might be
measured with a frequency that is assumed appropriate for econometric
modeling (e.g. quarterly series) whereas other variables are observed
less frequently (e.g. annually). The measurements may be subject to

* Revised and updated version of the paper presented at the Course Recent Advances
in Econometric Modelling: Causality, Time Series Methods and Computer Applications
held in Santa Margherita Ligure (Italy), 12-22 November, 1984.
seasonal variation which can only be partly explained by theory, or they may be contaminated with measurement errors or other forms of noise.

In a purely data-based approach, the aim is often to transform the observations into a second order stationary (time invariant structure) series and then represent their first and second moments by some ARMA-model, which is a reasonable approximation for most second order stationary processes. This type of approach is basically descriptive. One tries to get a fairly accurate, parsimoniously parametrized representation for the dynamics of the series under consideration. A data-based approach can also be a starting point for an econometric analysis, in which the aim is to formulate a model that is among other, theory-consistent and data-admissible. An econometric approach requires formulating a theory-model and a set of measurement or correspondence equations which relate the theoretical variables to the observations. The statistical analysis of an econometric model focusses on the joint process for the (latent) theoretical variables and the measurements.

In this paper, we present the general framework for the econometric analysis in the presence of unobservables, discuss the methodological problems that arise and show how these problems can be tackled in practice. To illustrate the approach some examples for the case of missing observations will be discussed. These examples are borrowed from joint work by Th. E. Nijman and the present author. Throughout the paper, we emphasize that the approach applies to many different types of unobservables.

The paper is organized as follows. In section 2, we introduce the general framework for econometric modeling and present several examples of models with unobservables and possible specifications for the measurement equations. In section 3, the estimation methods and numerical procedures will be presented. Section 4 contains an example of an empirical analysis. Finally, concluding remarks are given in section 5.

2. A Framework for Modeling Unobservables

One can distinguish between two basic approaches to modeling unobserved variables:
1. a data-based approach,

2. a model-based approach.

In a data-based analysis, the investigator's aim is to model the main features of the series at hand. For instance, when only annual values of a variable in a quarterly model are observed, one may assume that the series is smooth and construct 'quarterly observations' by means of interpolation.

The interpolation method proposed by Boot - Feibes - Lissman (1967) in which the missing values of a series \( y_t \) are approximated by minimizing the sum of squared \( d \)-th difference (usually \( d = 1 \) or \( 2 \)) of the series subject to the equality constraint between interpolated and observed values for all periods for which observations are available, is an example of a data-based approach.

Similarly, the fitting of say a multiplicative ARIMA-model along the lines of Box-Jenkins (1970) is data-based, when the series is an aggregate of say two variables, one of which is the sum of independent seasonal and nonseasonal ARIMA-components and the other is nonseasonal ARIMA. In this case, the structural unobserved component (UC) ARIMA-model (see e.g. (Nerlove et al., 1979)) is appropriate. The multiplicative seasonal ARIMA-scheme can at best be an approximation for the data-generating process (DGP). Using a structural econometric model to generate 'forecasts' of the missing observations to extract a signal from a series or to model latent variables illustrates model-based approaches. Econometric modeling is intrinsically model-based, although in practice quite frequently data-based procedures (e.g. seasonal adjustment of series before modeling them) are applied. We advocate a model-based analysis and try to show how one can proceed in practice.

Throughout this paper, we use the terms 'unobservables' and 'latent variables' as synonyms. We also limit ourselves to regular nonrandom sampling schemes. This means that it is a priori known whether a given variable is observed at a given time period. (For a discussion on econometric modelling see e.g. Zellner, 1979).

The following examples illustrate this. For instance, the yearly average of a macroeconomic variable can be available. The end of the year value of a stock variable is observed. Expectational variables, seasonal components, latent variables such as permanent income etc. are usually not directly observed. Random and irregular nonrandom sampling will not be further discussed here. To show the difference
between regular and irregular sampling, a few examples should be sufficient. In purely random sampling, the sampling process is independent of the variable in question. In many cases in econometrics, the probability of observing a variable depends on the realized value of that variable (e.g. sample selection). When sampling is regular, the vector of variables to be modeled can be a priori partitioned into observed values denoted by $y$ and unobserved values, $m$. In addition, we assume that $y$ and $m$ are generated by a joint process, which we denote by a family of conditional probability density functions

\begin{equation}
D(y, m | \theta, x),
\end{equation}

indexed by a vector of parameters $\theta \in \Theta$, where $\Theta$ is the parameter space and $x$ is a vector of observed weakly exogenous variables. We assume that the elements of $\theta$ are the parameters of interest. If some of the exogenous variables are not observed, a model-based approach requires that the model (2.1) is extended to include the process for the unobserved elements of $x$. In other words, the unobserved exogenous variables should be endogenized. Notice, however, that the weak exogeneity of the remaining elements of $x$ may not (and usually will not) hold any longer.

When the joint density (2.1) is given, the data generating process is obtained by marginalization of (2.1) with respect to $m$

\begin{equation}
D(y | \theta, x) = \int D(y, m | \theta, x) \, dm.
\end{equation}

If (2.1) represents a linear model, marginalization with respect to $m$ corresponds to solving out the unobservables $m$.

**Example 1**

Consider the following dynamic regression model

\begin{equation}
y_t = \varrho y_{t-1} + \beta x_t + \varepsilon_t, \quad |\varrho| < 1,
\end{equation}

with $\varepsilon_t$ being $\text{IN}(0, \sigma^2)$ and $x_t$ being strictly exogenous with respect to $\varepsilon_t$ and weakly exogenous with respect to $\varrho$ and $\beta$. The parameters of interest are $\theta = (\varrho, \beta, \sigma^2)$. Assume that $y_t$ is observed every second
period \(^1\), i.e. for \(t \in T_n\). Substituting for the unobserved value \(y_{t-1}\), we get the DGP for \(y_t\)

\[
y_t = \theta y_{t-2} + \beta x_t + \beta \theta y_{t-1} + \epsilon_t + \phi \epsilon_{t-1}, \quad t \in T_n.
\]

Conditionally on current and past values of \(x_t\) and on its own observed past, \(y_t\) is independently distributed for \(t \in T_n\) and it has variance \((1 + \phi^2) \sigma^2\).

Also, if an unobservable is an expectational variable, e.g. rational expectations, which is nonstochastic conditionally on the past, it can be substituted for to get a model for \(y\) conditional on \(x\).

**Example 2**

Consider a dynamic regression model

\[
y_t = \theta y_{t-1} + \beta x_t + \epsilon_t, \quad | \theta | < 1,
\]

where \(x_t\) and \(\epsilon_t\) satisfy the assumptions made in example 1 and \(y_t^e\) is the expected value of \(y_t\) given information on \(y\) and \(x\) up to period \(t-1\), denoted by \(I_{t-1}\)

\[
y_t^e = E (y_t | I_{t-1}).
\]

If \(x_t\) is generated by an AR(1)-model

\[
x_t = \gamma x_{t-1} + v_t, \quad v_t \sim IN (0, \sigma_v^2),
\]

after substitution of \(y_t^e\) in expression (2.5), the DGP is

\[
y_t = (1 - \phi)^{-1} \beta \theta y_{t-1} + \beta x_t + \epsilon_t.
\]

**Example 3**

The structural unobserved component model used by Harvey - Todd (1983) reads as follows

\[
y_t = T_t + \epsilon_t, \quad \epsilon_t \sim IN (0, \sigma^2),
\]

\(^1\) We use the notation \(T_n\) and \(T_n^e\) to denote the sets \(\{m, 2m, \ldots, T\}\) and \(T_n \backslash T_n^e\) respectively, where \(m\) is some integer and \(T\) is the sample size assumed to be a multiple of \(m\).
with $T_t$, $s_t$ and $e_t$ being the trend-cycle, seasonal and purely random components respectively

$$T_t = T_{t-1} + b_{t-1} + u_t$$

(2.9)

$$b_t = b_{t-1} + v_t$$

$$s_t = -\sum_{j=1}^{s-1} s_{t-j} + \omega_t,$$

where $e_t$, $u_t$, $v_t$ and $\omega_t$ are independently normally distributed white noise with zero mean and variances $\sigma^2$, $\alpha^2$, $\sigma^2$, $\sigma^2$ respectively. Marginalization with respect to the unobserved components leads to the following DGP

(2.10) $\Delta \Delta s y_t = \Delta s u_t + (1 + L + \ldots + L^{s-1}) v_{t-1} + \Delta^2 \omega_t + \Delta \Delta_s e_t,$

where $A = 1 - L$ and $\Delta s = 1 - L^s$. In model (2.10), $z_t = \Delta \Delta_s y_t$ is expressed as an $(s + 1)$th order moving average process say $z_t = w(L) v_t^s$, where $w(L)$ is a lag polynomial of order $s + 1$ in $L$ and $v_t^s$ is a white noise.

The DGP in (2.2) can be reparametrized as

(2.11) $D(y | \theta^*, x),$ 

where $\theta^*$ is chosen such that it is identified. The vector of parameters $\theta^*$ is related to $\theta$ by a set of equations $\theta^* = f(\theta).$ The parameters of interest $\theta$ are identified if $f$ is injective.

Let us reconsider the three examples. One can choose $\theta^*$ as the unrestricted regression coefficients in (2.4). It is quite obvious that $\theta$ is identified in (2.4) provided $\beta \neq 0.$ In fact, then the parameters of (2.4) are subject to one nonlinear restriction. When $\beta = 0,$ $y_t$ is generated by a first order AR-model. From (2.4), it appears that $\alpha^2$ is identified. The sign of $\varrho$ is not determined. But the variance $\sigma^2$ is identified. The model is only locally identified. In the model (2.7) - (2.8), $\theta = (\varphi, \beta, \sigma^2)^{T}$ is identified provided $\gamma \neq 0$ and $\beta \neq 0.$ For example 3, $\theta = (\alpha^2, \alpha^2, \gamma, \sigma^2)^{T}$ is identified when $s \geq 2,$ i.e. when there is a seasonal component in $y_t.$ The model (2.9) is subject to $s - 2$
(s > 2) restrictions which can be expressed as linear relations between
the autocovariances of \( z_t \) up to lag \( s + 1 \) and the parameters \( \theta \).

Finally, for prediction and for signal extraction, the predictive
density function of \( m \) is needed at least partly

\[
D(m \mid y, \theta, x) = D(y, m \mid \theta, x) \big/ D(y \mid \theta, x).
\]

Again, in example 1, the expected value of \( y_t, t \in T_4' \), is

\[
y_t^* = \varrho y_{t-1} + \beta x_t,
\]

when we condition on the past observations for \( y \) and current and
past values of \( x_t \), and

\[
y_t^* = (1 + \varrho^2)^{-1} \left[ \varrho y_{t-1} + \varrho y_{t+1} + \beta x_t - \beta, q x_{t+1} \right],
\]

when we condition on all observations. Prediction and smoothing
the missing values of \( y_t \) by means of a minimum mean square error
predictor requires the identification of the parameters \( \varrho \) and \( \beta \). Similarly,
to compute the expectation in (2.6) or to extract the seasonals
from \( y_t \) in (2.9), the parameters in \( \theta \) have to be identified. More details
on the identification of parameters when observations are missing can
be found in Palm - Nijman (1984).

The identification problem in dynamic latent variables models
has received some attention in the literature.

For the identification of univariate AR models with missing ob-
servations we also refer to Telser (1967). Parameter identification
in dynamic models with measurement errors is considered by Haio (1977),
in dynamic factor analysis is discussed by Geweke - Singleton (1981)
and Wegge (1983). For rational expectations models we refer

3. Estimation of Models with Unobservables

In principle, as soon as the joint process (2.1) is specified, the
density for \( y \) in (2.2) can be obtained. The parameters \( \theta \) can be
estimated provided they are identified in (2.2). Existing estimation
methods for (non) linear models are appropriate. For instance, as the
data density function is assumed to be given, efficient Maximum Likelihood (ML) estimation is possible. Also, overidentifying restrictions on \( \theta \) can be tested to validate the model (2.1).

There are many ways for implementing ML. One way to obtain the ML estimate of \( \theta \) consists in estimating the unrestricted parameters \( \theta^* \) by ML (for the examples given above, this can be achieved by (non) linear least squares) and then apply the method of asymptotic least squares proposed by Gouriéroux et al., (1985) to the \( \text{'asymptotic model'} f(\theta) = \theta^* \). Alternatively, the restricted model (2.2) can be estimated directly by means of e.g. the Gauss-Newton algorithm using the chain rule to compute the partial derivatives of the likelihood function. More details can be found in Palm - Nijman (1984).

Some ML procedures do not require explicit marginalization with respect to the unobserved variables. For instance, the log-likelihood function in prediction error decomposition form and its derivatives can be evaluated by means of the Kalman filter in order to obtain ML estimates of the parameters and predictions or smoothed values of the unobservables. (See e.g. Harvey - McKenzie (1983) for missing observations). Also, the EM algorithm proposed by Dempster et al., (1977) can be applied to the joint process (2.1) to get ML estimates of the parameters \( \theta \) and predictions of the unobservables. As the EM algorithm is based on a sufficient statistic, it is less suited for models with MA parameters. Advantages of the EM algorithm are that the value of the likelihood function increases at each step of iteration and that it moves quickly to a region close to the maximum. Watson - Engle (1983) show how the EM algorithm can be implemented by means of the Kalman filter and how standard errors for the ML estimates can be computed.

In practice, a number of problems arise with efficient estimation of latent variables models. First, the dimension of the model or of the vector \( \theta \) in (2.2) may be too large to allow for joint estimation of the complete model subject to the (non) linear restrictions implied by \( \theta^* = f(\theta) \). Second, the model may not be completely specified and one may be interested in part of the model only, e.g. in one specific equation. Third, the model is tentatively specified and a specification analysis (see e.g. Mizon 1977 or Hendry et al., 1984) may be needed to validate the model. Then, inexpensive estimation procedures may be very useful. Fourth, only part of \( \theta \) is identified. If the parameters, which are of interest are identified, statistical inference remains possible. Fifth, it may be desirable to avoid MA-disturbances, which are often
the result of marginalization with respect to unobservables as is illustrated by example 2. [Another example is the aggregation of say two independent AR(p)-processes yielding an ARMA (p*, q*)-model with p* \leq 2 p and q* \leq p].

For these reasons, consistent estimation of parts of the model is often an alternative to fully efficient (ML) estimation. Most consistent estimators for latent variables models are based on proxy variables for the unobservables. To outline the steps of consistent proxy variables estimation, consider the linear model

\begin{equation}
\begin{aligned}
y &= X \beta^* + u, \quad u \sim I N(0, \Sigma). \\
T \times 1 & \quad T \times k \quad k \times 1 \quad T \times 1
\end{aligned}
\end{equation}

At present, we do not assume that $X$ is orthogonal to $u$. Some elements of $y$ and/or $X$ are not observed. We substitute proxies denoted by $y$, $X$ (The proxy is equal to the observed value, whenever the observation is available) into (3.1) to get

\begin{equation}
y = X \beta + v
\end{equation}

\begin{equation}
v = u + (X^* - X^*) \beta^*,
\end{equation}

where $X^* = (y X)$, $X^* = (y X)$, $\beta^* = (-1 \beta)'$.

Ordinary least squares are consistent for $\beta$ when

\begin{equation}
\lim_{T \to \infty} T^{-1} X' v = 0.
\end{equation}

Notice that the orthogonality between $X$ and $u$ is not required. If $X$ is asymptotically orthogonal to $u$ and to its prediction error $(X^* - X^*)$, then (3.4) is satisfied. When the proxy $X$ is a conditional expectation with known parameters, the second condition is satisfied. When the parameters of the conditional expectation have to be estimated, it is not difficult to assure that the proxy $X$ is at least asymptotically orthogonal to its prediction error. A proxy $X$ which is an interpolation along the lines proposed by Boot et al., (1967) and briefly discussed in section 2 usually does not satisfy condition (3.4). In fact, the bias of OLS can be important as has been shown by Palm - Nijman (1984).
Proxy variables estimators \( (PVE) \) can be computed in a straightforward way. Consistent estimation of the standard errors \( (SEs) \), however, requires some care. Assume that condition \( (3.4) \) is satisfied. Then the asymptotic variance of \( \beta_{OLS} \) in \( (3.2) \) is given by

\[
V_{OLS} = \lim_{T \to \infty} T (X' X)^{-1} X' \Omega X (X' X)^{-1},
\]

where \( \Omega = E w w' \) (assumed to exist, \( E w = 0 \)).

To compute expression \( (3.5) \), an estimate of \( \Omega \) is needed. The following example illustrates the estimation of the \( SEs \) for a \( PVE \).

\[
\begin{align*}
\gamma_t &= \beta x_t + \epsilon_t, \quad \epsilon_t \sim I N (0, \sigma^2), \\
\delta_t &= \alpha z_t + \nu_t, \quad \nu_t \sim I N (0, \sigma_{\nu}^2),
\end{align*}
\]

where \( x_t \) is exogenous to \( \epsilon_t \) and \( z_t \) is exogenous to \( \epsilon_t \) and \( \nu_t \). The variables \( y_t \) and \( z_t \) are observed for \( t \in T_1 \), \( x_t \) is observed for \( t \in T_2 \).

A proxy for \( x_t \) is given by

\[
\begin{align*}
x_t &= x_t, \quad t \in T_2 \\
x_t &= \alpha z_t, \quad t \in T_\infty
\end{align*}
\]

where \( \alpha \) is the \( OLS \) estimate of \( \alpha \) in \( (3.6b) \), \( \alpha = (\sum_{T_2} z_t^{-1})^{-1} \sum_{T_2} z_t x_t \).

After substitution of the proxy \( (3.7) \) into \( (3.6a) \), the disturbance becomes heteroscedastic and autocorrelated

\[
\begin{align*}
\omega_t &= \epsilon_t, \quad t \in T_2 \\
&= \epsilon_t + \beta \nu_t + \beta z_t (\alpha - \alpha), \quad t \in T_\infty
\end{align*}
\]

Conditionally on \( z_t \), the covariance matrix of \( w \) is

\[
E w w' = \sigma^2 I_T + \sigma_{\nu}^2 \beta^2 \Lambda + \frac{\beta^2 \sigma_{\nu}^2 \beta^2}{\sum_{T_2} z_t^2} \pi z',
\]
where $A$ is a diagonal matrix with even and odd diagonal elements being equal to zero and one respectively and $z = [z_1, 0, z_3, \ldots]^\prime$.

Although the third r.h.s. term in (3.9) converges to zero in probability, it contributes to the large sample variance of $\hat{\beta}_{OLS}$, a point that is often missed in the literature. Its contribution to the asymptotic variance of $\hat{\beta}_{OLS}$ is given by

$$
(3.10) \quad \text{plim} \beta^2 \sigma^2 \left( \frac{x'x}{T} \right)^{-1} \left( \frac{x'z}{T} \right) \left( \frac{\Sigma z_i}{T} \right)^{-1} \left( \frac{x'x}{T} \right)^{-1},
$$

which is generally a constant different from zero. Notice also that consistent estimates of $\sigma^2$, $\sigma^2$, $\beta$ and of the moments of $z_i$ are needed to compute (3.9). More details on consistent estimation of the standard errors can be found in Nijman (1985) and Nijman - Palm (1986).

Different consistent $PVE$'s for latent variables models and methods to consistently estimate their $SE$s or to get bounds for the $SE$s are presented in Nijman - Palm (1984). The $PVE$'s can be applied to models with different types of unobservables. For any proxy, which satisfies (3.4), OLS will be consistent. Otherwise, one may use an instrumental variables methods with instruments that are (asymptotically) orthogonal to $\omega$ in (3.2). In Nijman - Palm (1984), it is shown that the efficiency of $PVE$'s can be basically increased in four ways:

1) when the proxies are given, by carefully selecting the set of instrumental variables, and

2) by taking into account the serial correlation of the disturbances $\omega$ (e.g. generalized least squares instead of OLS).

3) by conditioning on all appropriate variables to construct the proxy $X$,

4) by efficiently estimating the parameters of the conditional expectation of $X$ when the information set has been chosen.

It appears that for models with missing observations or with rational expectations, $PVE$'s that are almost as efficient asymptotically as the ML estimator can be obtained along these lines.

The results in Nijman - Palm (1984) suggest that many $PVE$'s are computationally more attractive than ML estimation and that they have fairly good asymptotic properties. As stated above, the
proxies will usually be conditional expectations of the corresponding variable given some information set. To construct a conditional expectation, a proxy variables generating model (PGM) will often have to be formulated. In the 3 examples discussed in section 2, the PGM is an intrinsic part of the model to be analyzed, except that for the rational expectations, the process for \( x_t \) in (2.7) has to be specified. Endogenization is generally required when the unobservable is an exogenous variable. Once the PGM is given and its parameters are estimated, the proxy variables can be computed as one- or two-sided conditional expectations. For simple models, an analytical expression of the conditional expectation is readily available (see e.g. (2.13) and (2.14)). For more complicated stationary models the Wiener-Kolmogorov filtering theory applies when one conditions on the infinite history of the process. The Kalman filter technique and smoothing algorithms such as the fixed point and the fixed interval smoothers (see Anderson - Moore (1979)) are very useful to compute proxies (conditional expectations) for stationary and non-stationary processes.

4. An Example

Nijman - Palm (1986) consider the following error-correction model (see Davidson et al. (1978), Granger - Weiss (1983) for a discussion of these models) for the quarterly aggregate demand for labor in the Netherlands, 1967-1981.

\[(1 - \gamma) \Delta Ld = \beta [\Delta d - I d^{*}]_{t-1} + \delta_1 \Delta d^{*} + \varepsilon_t,\]

where \( Ld \) is the demand for labor and \( d^{*} \) is the desired demand for labor, in the private sector. \( \Delta \) denotes the difference operator and \( L \) is the lag operator. The disturbance \( \varepsilon_t \) is assumed to be \( IN(0, \sigma^2) \) and independent of \( Ld^{*} \), for all \( t \) and \( t' \). Further, \( d^{*} \) is assumed to be quarterly measured without error. The variable \( Ld \) is not directly observed. Using a disequilibrium model developed by Kooiman - Kloek (1979) and aggregating over a normally distributed continuum of demand and supply-constrained micro labor markets, employment in the private sector, \( Ld \), is approximately a weighted average of labor demand and supply in this sector. This relationship is used to determine labor demand in the private sector

\[Ld = a_{1t}(N_t - Ld^{*}) + a_{2t}(Ld^{*} - Ld), \quad N_t - Ld^{*} = L_t,\]
where \( N_t \) is total employment, \( L^p_t \) is employment in the public sector, \( L^s_t \) is total labor supply, \( \alpha_{it} \) and \( \alpha_{st} \) are weights assumed to be given (predetermined).

The variables \( \hat{N}_t = \frac{1}{4} (N_t + N_{t-1} + N_{t-2} + N_{t-3}) \), \( L^p_t \) and \( L^s_t \) are annually observed. For each of them, a PGM is specified and used to generate smoothed values (i.e. twosided conditional expectations) for the quarters. Subsequently these values are substituted into (4.2) to get a proxy for \( L^d_t \). The proxy for \( L^d_t \) is then used to estimate (4.1). The first model for \( \hat{N}_t, t \in T_4 \), is a univariate AR(1)-model obtained from an analysis of the autocorrelation functions for \( \hat{N}_t \)

\[
A_0 \hat{N}_t = 0.05 + 0.50 A_4 \hat{N}_{t-4} + \varepsilon_t, \quad \sigma^2 = 1.4 \times 10^{-3} .
\]

with \( A_4 = 1 - L^4 \). Standard errors are given between parentheses.

Next, we have to find a model for the quarterly values of \( N_t \), which is consistent with the result in (4.3). The choice is not unique. The following model has been chosen for \( N_t, t \in T_4 \)

\[
(1 - 0.80 L) (A N_t - 0.000) = \varepsilon_t, \quad \sigma^2 = 6.0 \times 10^{-5}
\]

and estimated by ML using annual observations on \( \hat{N}_t \) only. Notice that the model (4.4) implies an ARIMA (1, 1, 2)-model in \( L^4 \) for \( \hat{N}_t \) with small MA-parameters so that (4.4) is approximately in agreement with the result in (4.3).

In a similar way, PGM’s for \( L^p_t \) and \( L^s_t \) have been formulated and implemented to generate proxies for these variables.

Finally, after substitution of the proxy for \( L^d_t \), equation (4.1) has been estimated by OLS. The results are

\[
(1 - 0.789 L) A L^d_t = 0.92 A L^d_{t-1} + 0.36 [L^d_t - L^d_{t-1}] + \varepsilon_t .
\]

The figures between parentheses correspond to the SE’s using the formula \( s^2 (X'X)^{-1} \), which are inconsistent, and using the White - Domowitz (1984) estimates for the standard errors. The White - Domowitz (1984) estimates have been computed by taking into account up to a four period dependence in the sum of the first two terms
in (3.9), using a consistent estimate of the contribution of the third term of (3.9) (see (3.10)) and applying the Cauchy - Schwartz inequality to approximate the covariance between the sum of the first two terms and the third term of (3.9). In general, the covariance is not zero. The estimates appear to be insensitive to increases of the dependence in the first two terms of (3.9). Moreover the first four decimal points of lower and upper bounds for the standard errors obtained from the Cauchy - Schwartz inequality are found to be identical. Therefore, the figures in the last row of (4.5) are taken as the asymptotic SEs of the OLS estimator. Notice that these SEs are larger than the inconsistently estimated SEs. OLS might not be entirely appropriate here as one might a priori expect that the orthogonality condition (3.4) is not satisfied. However various instrumental variables estimates of equation (4.1) were found to be almost identical with the OLS estimates so that we conclude that the dependence between $X$ and $w$ is weak if not zero.

This example shows how one can analyze latent variables models by means of PVE's and that the results are both meaningful and fairly accurate. Detailed results are given in Nijman - Palm (1986) and Nijman (1985).

5. Concluding Remarks

We have discussed some issues of modeling in the presence of unobservables. As econometric modeling focuses on the use of economic theory, which is often formulated in terms of variables that are not directly observed, models with unobservables often arise in applied work (see also Palm (1986)). Moreover most of the data are very noisy, a feature that ought to be taken into account by any model-builder. Kalman (1982) has forcefully argued that modeling should take into account this very feature of the data.

Attention has been paid in the literature to the implications of the presence of latent variables for the econometric analysis. In our opinion, given the state of the art, more care could in many instances be taken of the implications of a latent variables structure when modeling economic time series and when interpreting empirical findings in the light of economic theory. Quite often ad hoc assumptions are introduced to handle latent variables instead of specifying the complete model
for the observations and the unobserved variables and adopting a modeling procedure that is consistent with the assumptions underlying the complete model.

FRANZ C. PALM

University of Limburg, Maastricht, The Netherlands

REFERENCES


**SUMMARY**

In this article we present a general model-based approach for the econometric analysis in the presence of unobservables. The approach requires formulating a theory-model and a set of measurement equations which relate the theoretical variables to the observations. The statistical analysis then focuses on the marginal process of the measurements which arises once the unobserved (theory) variables have been integrated out. Examples of models with unobservables and possible specifications for the measurement equations are considered.

The identification and estimation of the parameters of the theory-model from the marginal process for the measurements is discussed. An empirical example illustrates the analysis.