CHARACTERIZATION OF REVENUE EQUIVALENCE

BY BIRGIT HEYDENREICH,1 RUDOLF MÜLLER, MARC UETZ, AND RAKESH V. VOHRA2

The property of an allocation rule to be implementable in dominant strategies by a unique payment scheme is called revenue equivalence. We give a characterization of revenue equivalence based on a graph theoretic interpretation of the incentive compatibility constraints. The characterization holds for any (possibly infinite) outcome space and many of the known results are immediate consequences. Moreover, revenue equivalence can be identified in cases where existing theorems are silent.

KEYWORDS: Revenue equivalence, mechanism design, incentive compatibility.

1. INTRODUCTION

ONE OF THE MOST IMPORTANT RESULTS of auction theory is the revenue equivalence theorem. Subject to certain reasonable assumptions, it concludes that a variety of different auctions generate the same expected revenue for the seller. Klemperer (1999) wrote that “much of auction theory can be understood in terms of this theorem...”; hence the long line of papers that have attempted to relax the sufficient conditions under which revenue equivalence holds. The present paper provides necessary and sufficient conditions on the underlying primitives for revenue equivalence to hold.

We consider direct revelation mechanisms for agents with multidimensional types. Such mechanisms consist of an allocation rule and a payment scheme. The allocation rule selects an outcome depending on the agents’ reported types, whereas the payment scheme assigns a payment to every agent. We focus attention on allocation rules that are implementable in dominant strategies.3 Call such rules implementable. In this environment we characterize the uniqueness of the relevant payment scheme in terms of conditions that are easily verified in potential applications. The property of an allocation rule to be implementable in dominant strategies by a unique payment scheme is called revenue equivalence. Our characterization of revenue equivalence is based on a graph theoretic interpretation of the incentive compatibility constraints. This interpretation has been used before by Rochet (1987), Gui, Müller, and Vohra (2004), Saks and Yu (2005), and Müller, Perea, and Wolf (2007) to identify allocation rules that are implementable in dominant strategies or in Bayes–Nash. The characterization holds for any (possibly infinite) outcome space.

1The first author is supported by NWO Grant 2004/03545/MaGW, “Local Decisions in Decentralised Planning Environments.”
2We thank the editor and three anonymous referees for very useful comments and suggestions. We also thank participants of Dagstuhl Seminars 07271 and 07471 for their valuable feedback.
3With appropriate adjustments, our characterization of revenue equivalence holds for ex post as well as Bayes–Nash incentive compatibility.
The bulk of prior work on revenue equivalence (Green and Laffont (1977), Holmström (1979), Myerson (1981), Krishna and Maenner (2001), Milgrom and Segal (2002)) identifies sufficient conditions on the type space that ensure all allocation rules from a given class satisfy revenue equivalence. We know of only two papers that identify necessary as well as sufficient conditions—that is, characterizing conditions—for revenue equivalence to hold. If the outcome space is finite, Suijs (1996) characterized type spaces and valuation functions for which utilitarian maximizers satisfy revenue equivalence. Chung and Olczewski (2007) characterized type spaces and valuation functions for which every implementable allocation rule satisfies revenue equivalence, again under the assumption of a finite outcome space. Our characterization identifies, for general outcome spaces, a joint condition on the type space, valuation function, and implementable allocation rule that characterizes revenue equivalence. Given agents’ type spaces and valuation functions, several allocation rules may be implementable in dominant strategies, some of which satisfy revenue equivalence and some which do not. In such cases, all previous results have no bite. However, our characterization can be used to determine which of the allocation rules do satisfy revenue equivalence.

The remainder of the paper is organized as follows. In Section 2, we introduce notation and basic definitions. In Section 3, we derive our graph theoretic characterization of revenue equivalence. In Section 4, we briefly discuss how our characterization applies in various settings.

2. SETTING AND BASIC CONCEPTS

Denote by \{1, \ldots, n\} the set of agents and let \(A\) be the set of possible outcomes. Outcome space \(A\) is allowed to have infinitely many, even uncountably many, elements. Denote the type of agent \(i \in \{1, \ldots, n\}\) by \(t_i\). Let \(T_i\) be the type space of agent \(i\). Type spaces \(T_i\) can be arbitrary sets. Agent \(i\)’s preferences over outcomes are modeled by the valuation function \(v_i : A \times T_i \to \mathbb{R}\), where \(v_i(a, t_i)\) is the valuation of agent \(i\) for outcome \(a\) when he has type \(t_i\).

A mechanism \((f, \pi)\) consists of an allocation rule \(f : \prod_{i=1}^{n} T_i \to A\) and a payment scheme \(\pi : \prod_{i=1}^{n} T_i \to \mathbb{R}^n\). In a direct revelation mechanism, the allocation rule chooses for a vector \(t\) of aggregate type reports of all agents an outcome \(f(t)\), whereas the payment scheme assigns a payment \(\pi_i(t)\) to each agent \(i\). Let the vector \((t_i, t_{-i})\) denote the aggregate type report vector when \(i\) reports \(t_i\) and the other agents’ reports are represented by \(t_{-i}\). We assume quasi-linear utilities, that is, the utility of agent \(i\) when the aggregate report vector is \((t_i, t_{-i})\) is \(v_i(f(t_i, t_{-i}), t_i) - \pi_i(t_i, t_{-i})\).

**Definition 1**—Dominant Strategy Incentive Compatible: A direct revelation mechanism \((f, \pi)\) is called dominant strategy incentive compatible if for
every agent $i$, every type $t_i \in T_i$, all aggregate type vectors $t_{-i}$ that the other agents could report, and every type $s_i \in T_i$ that $i$ could report instead of $t_i$,

$$v_i(f(t_i, t_{-i}), t_i) - \pi_i(t_i, t_{-i}) \geq v_i(f(s_i, t_{-i}), t_i) - \pi_i(s_i, t_{-i}).$$

If for allocation rule $f$ there exists a payment scheme $\pi$ such that $(f, \pi)$ is a dominant strategy incentive compatible mechanism, then $f$ is called *implementable in dominant strategies*, in short *implementable*.

In this paper we assume that the allocation rule is implementable in dominant strategies and study the uniqueness of the corresponding payment scheme. We refer to the latter as revenue equivalence.\(^4\)

**Definition 2—Revenue Equivalence:** An allocation rule $f$, implementable in dominant strategies, satisfies the *revenue equivalence* property if for any two dominant strategy incentive compatible mechanisms $(f, \pi)$ and $(f, \pi')$ and any agent $i$ there exists a function $h_i$ that only depends on the reported types of the other agents $t_{-i}$ such that

$$\forall t_i \in T_i: \quad \pi_i(t_i, t_{-i}) = \pi'_i(t_i, t_{-i}) + h_i(t_{-i}).$$

### 3. Characterization of Revenue Equivalence

We give a necessary and sufficient condition for revenue equivalence with the aid of a graph theoretic interpretation used by Rochet (1987), Gui, Müller, and Vohra (2004), and Saks and Yu (2005) to characterize implementable allocation rules. We also adopt some of their notation.

Fix agent $i$ and the reports, $t_{-i}$, of the other agents. For simplicity of notation we write $T$ and $v$ instead of $T_i$ and $v_i$. Similarly, for any mechanism $(f, \pi)$, we regard $f$ and $\pi$ as functions of $i$’s type alone, that is, $f: T \to A$ and $\pi: T \to \mathbb{R}$. If $(f, \pi)$ is dominant strategy incentive compatible, it is easy to see that for any pair of types $s, t \in T$ such that $f(t) = f(s) = a$ for some $a \in A$, the payments must be equal, that is, $\pi(t) = \pi(s) =: \pi_a$. Hence, the payment of agent $i$ is completely defined if the numbers $\pi_a$ are defined for all outcomes $a \in A$ such that $f^{-1}(a)$ is nonempty. For ease of notation, we let $A$ denote the set of outcomes that can be achieved for some report of agent $i$. For an allocation rule $f$, let us define two different kinds of graphs. The *type graph* $T_f$ has node set $T$ and contains an arc from any node $s$ to any other node $t$ of length\(^5\)

$$\ell_{st} = v(f(t), t) - v(f(s), t).$$

\(^4\)We choose the term revenue equivalence in accordance with Krishna (2002). In our setting it is equivalent to payoff equivalence as used in Krishna and Maenner (2001). See Milgrom (2004, Section 4.3.1) for settings where it is not equivalent.
Here $\ell_{st}$ represents the gain in valuation for an agent truthfully reporting type $t$ instead of lying type $s$. This could be positive or negative. The allocation graph $G_f$ has node set $A$. Between any two nodes $a, b \in A$, there is a directed arc with length

$$\ell_{ab} = \inf_{t \in f^{-1}(b)} (v(b, t) - v(a, t)).$$

The arc lengths $\ell_{ab}$ in the allocation graph represent the least gain in valuation for an agent with any type $t \in f^{-1}(b)$ for reporting truthfully, instead of misreporting so as to get outcome $a$ (instead of $b$). The type graph and allocation graph are complete, directed, and possibly infinite graphs. We introduce our main results in terms of allocation graphs. Analogous results hold for type graphs as well.

A path from node $a$ to node $b$ in $G_f$, or the $(a,b)$ path for short, is defined as $P = (a = a_0, a_1, \ldots, a_k = b)$ such that $a_i \in A$ for $i = 0, \ldots, k$. Denote by length($P$) the length of this path. A cycle is a path with $a = b$. For any $a$, we regard the path from $a$ to $a$ without any arcs as an $(a,a)$ path and define its length to be 0. Define $\mathcal{P}(a,b)$ to be the set of all $(a,b)$ paths.

**Definition 3—Node Potential:** A node potential $p$ is a function $p : A \rightarrow \mathbb{R}$ such that for all $x, y \in A$, $p(y) \leq p(x) + \ell_{xy}$.

**Observation 1:** Let $f$ be an allocation rule. Payment schemes $\pi$ such that $(f, \pi)$ is a dominant strategy incentive compatible mechanism exactly correspond to node potentials in each of the allocation graphs $G_f$ that are obtained from a combination of an agent and a report vector of the other agents.

**Proof:** Assume $f$ is implementable. Fix agent $i$ and the reports $t_{-i}$ of the other agents. Consider the corresponding allocation graph $G_f$. For any pair of types $s, t \in T$ such that $f(t) = f(s) = a$ for some $a \in A$, the payments must be equal, that is, $\pi(t) = \pi(s) = \pi_a$. Therefore, $\pi$ assigns a real number to every node in the graph. Incentive compatibility implies for any two outcomes $a, b \in A$ and all $t \in f^{-1}(b)$ that $v(b, t) - \pi_b \geq v(a, t) - \pi_a$; hence, $\pi_b \leq \pi_a + \ell_{ab}$.

For the other direction, define the payment $\pi$ for agent $i$ as follows. For any report vector of the other agents $t_{-i}$, consider the corresponding allocation graph $G_f$ and fix a node potential $p$. At aggregate report vector $(t_i, t_{-i})$ with outcome $a = f(t_i, t_{-i})$, let the payment be $\pi_a := p(a)$. Incentive compatibility

---

5We assume that arc lengths are strictly larger than $-\infty$. For allocation rules implementable in dominant strategies, this is no restriction, as the incentive compatibility constraints imply finiteness of the arc lengths.

6Clearly, type and allocation graphs depend on the agent $i$ and reports $t_{-i}$ of the other agents. To keep the notation simple, we suppress the dependence on $i$ and $t_{-i}$, and will simply write $T_f$ and $G_f$. 
now follows from the fact that $p$ is a node potential in $G_f$, similarly to the above. $Q.E.D.$

Let

$$\text{dist}_{G_f}(a,b) = \inf_{P \in \mathcal{P}(a,b)} \text{length}(P).$$

In general, $\text{dist}_{G_f}(a,b)$ could be unbounded. However, if $G_f$ does not contain a negative cycle (the \textit{nonnegative cycle property}), then $\text{dist}_{G_f}(a,b)$ is finite, since the length of any $(a,b)$ path is lower bounded by $-\ell_{ba}$.

\textbf{LEMMA 1:} Fix an agent and some report vector of the other agents. The corresponding allocation graph $G_f$ has a node potential if and only if it satisfies the \textit{nonnegative cycle property}.

\textbf{PROOF:} Proofs can be found, for example, in Schrijver (2003) for finite $A$ and in Rochet (1987) for infinite $A$. For completeness, we give a simple proof. If $G_f$ has no negative cycle, then for any $a \in A$, $\text{dist}_{G_f}(a, \cdot)$ is well defined, that is, takes only finite values. The distances $\text{dist}_{G_f}(a, \cdot)$ define a node potential, because $\text{dist}_{G_f}(a, x) \leq \text{dist}_{G_f}(a, y) + \ell_{yx}$ for all $x, y \in A$. On the other hand, given a node potential $p$, add up the inequalities $p(y) - p(x) \leq \ell_{xy}$ for all arcs $(x, y)$ on a cycle to prove that the cycle has nonnegative length. $Q.E.D.$

\textbf{Observation 1} together with \textbf{Lemma 1} yields a characterization of allocation rules that are implementable in dominant strategies (see also, e.g., Rochet (1987)).

\textbf{Observation 2:} The allocation rule $f$ is implementable in dominant strategies if and only if all allocation graphs $G_f$ obtained from a combination of an agent and a report vector of the other agents satisfy the \textit{nonnegative cycle property}.

From \textbf{Lemma 1} and Observations 1 and 2 it follows that for any allocation rule $f$ implementable in dominant strategies, there exist node potentials in all allocation graphs $G_f$. The allocation rule $f$ satisfies revenue equivalence if and only if in each allocation graph $G_f$, the node potential is uniquely defined up to a constant. Our main result states that this is the case if and only if distances are antisymmetric in every $G_f$.

\textbf{THEOREM 1—Characterization of Revenue Equivalence:} Let $f$ be an allocation rule implementable in dominant strategies. $f$ satisfies revenue equivalence if and only if in all allocation graphs $G_f$ obtained from a combination of an agent and a report vector of the other agents, distances are antisymmetric, that is, $\text{dist}_{G_f}(a,b) = -\text{dist}_{G_f}(b,a)$ for all $a, b \in A$. 
PROOF: Suppose first that $f$ satisfies revenue equivalence. Fix a combination of an agent and a report vector of the other agents, and let $G_f$ be the corresponding allocation graph. Let $a, b \in A$. The functions $\text{dist}_{G_f}(a, \cdot)$ and $\text{dist}_{G_f}(b, \cdot)$ are node potentials in $G_f$. As any two node potentials differ only by a constant, we have that $\text{dist}_{G_f}(a, \cdot) - \text{dist}_{G_f}(b, \cdot)$ is a constant function. Especially, for $a$ and $b$ we get that $\text{dist}_{G_f}(a, a) - \text{dist}_{G_f}(b, a) = \text{dist}_{G_f}(a, b) - \text{dist}_{G_f}(b, b)$. Clearly, $\text{dist}_{G_f}(a, a) = \text{dist}_{G_f}(b, b) = 0$ and hence $\text{dist}_{G_f}(a, b) = -\text{dist}_{G_f}(b, a)$.

Now suppose that $\text{dist}_{G_f}(a, b) = -\text{dist}_{G_f}(b, a)$ for all $a, b \in A$. Let $a, b \in A$. Let $P_{ab}$ be an $(a, b)$ path with nodes $a = a_0, a_1, \ldots, a_k = b$. For any node potential $p$, add up the inequalities $p(a_i) - p(a_{i-1}) \leq \ell_{a_ia_{i+1}}$ for $i = 1, \ldots, k$. This yields $p(b) - p(a) \leq \text{length}(P_{ab})$. Therefore,

$$p(b) - p(a) \leq \inf_{P \in P(a,b)} \text{length}(P) = \text{dist}_{G_f}(a, b).$$

Similarly, $p(a) - p(b) \leq \text{dist}_{G_f}(b, a)$. Therefore, $-\text{dist}_{G_f}(b, a) \leq p(b) - p(a) \leq \text{dist}_{G_f}(a, b)$. Since $\text{dist}_{G_f}(a, b) = -\text{dist}_{G_f}(b, a)$, $p(b) - p(a) = \text{dist}_{G_f}(a, b)$ for any node potential $p$. Hence, any potential is completely defined, once $p(a)$ has been chosen for some outcome $a$. Thus, any two node potentials can only differ by a constant and $f$ satisfies revenue equivalence.

Q.E.D.

An analogous characterization holds for type graphs as well. One can check that all previous arguments still apply when using type graphs. On the other hand, note the following relation of node potentials in that all previous arguments still apply when using type graphs. On the other hand, given a node potential $p^G$ in $G_f$, we can define a node potential $p^T$ in $T_f$ by letting $p^T(t) := p^G(f(t))$ for any $t \in T$. In fact, let $\ell^G$ and $\ell^T$ denote the arc lengths in $G_f$ and $T_f$, respectively, and observe that $\ell^G_{ab} = \inf\{\ell^T_{st} : s \in f^{-1}(a), t \in f^{-1}(b)\}$. Then, for any $s, t \in T$, $p^T(t) = p^G(f(t)) \leq p^G(f(s)) + \ell^G_{f(s)f(t)} \leq p^T(s) + \ell^T_{st}$ and $p^T$ is a node potential. On the other hand, given a node potential $p^T$ in $T_f$, let $p^G(a) := p^T(s)$ for any $s \in f^{-1}(a)$. Note that $p^G$ is well defined as $f(s) = f(t) = a$ implies $\ell^T_{st} = 0$ and hence $p^T(s) = p^T(t)$. Furthermore, for any $a, b \in A$ and any $s \in f^{-1}(a), t \in f^{-1}(b)$, $p^G(a) = p^T(s) \leq p^T(t) + \ell^T_{ts} = p^G(b) + \ell^T_{ts}$. Hence, $p^G(a) \leq p^G(b) + \ell^T_{ba}$ and $p^G$ is a node potential in $G_f$. Consequently, there is a one-to-one relationship between node potentials in $G_f$ and node potentials in $T_f$. This insight together with a proof similar to the one of Theorem 1 yield the following corollary.

**COROLLARY 1**—Characterization of Revenue Equivalence on Type Graphs: Let $f$ be an allocation rule that is implementable in dominant strategies. Then $f$ satisfies revenue equivalence if and only if in all type graphs $T_f$ obtained from a combination of an agent and a report vector of the other agents, distances are antisymmetric, that is, $\text{dist}_{T_f}(s, t) = -\text{dist}_{T_f}(t, s)$ for all $s, t \in T$. 

4. DISCUSSION

In settings with multidimensional type spaces, finite \( A \), and valuation functions that are linear in types, implementability implies that the allocation rule \( f \), viewed from a single agent perspective as a vector field that maps multidimensional types on lotteries over outcomes, has a potential function \( F \). One can easily verify that this property has the following interpretation on type graphs: the length of a shortest path in \( T_f \) from some type \( s \) to some type \( t \) is upper bounded by a path integral of the vector field \( f \) or, equivalently, \( F(t) - F(s) \). From this it follows easily that \( \text{dist}_{T_f}(s, t) = -\text{dist}_{T_f}(t, s) \), that is, revenue equivalence holds. In particular, \( \text{dist}_{T_f}(s, t) = F(t) - F(s) \) for any potential function \( F \). This connection between implementability, potential functions, and revenue equivalence is also established in Jehiel, Moldovanu, and Stacchetti (1996, 1999), Jehiel and Moldovanu (2001), and Krishna and Maenner (2001).

It is interesting to compare our result with the characterization by Chung and Olszewski (2007). First, we introduce the notation used by Chung and Olszewski (2007) and restate their characterization theorem. Let \( A \) be countable. As before, regard everything from the perspective of a single agent. Let \( A_1, A_2 \) be disjoint subsets of \( A \) and let \( r : A_1 \cup A_2 \to \mathbb{R} \). For every \( \varepsilon > 0 \), let

\[
T_1(\varepsilon) = \bigcup_{a_1 \in A_1} \{ t \in T | \forall a_2 \in A_2 : v(a_1, t) - v(a_2, t) > r(a_1) - r(a_2) + \varepsilon \}
\]

and

\[
T_2(\varepsilon) = \bigcup_{a_2 \in A_2} \{ t \in T | \forall a_1 \in A_1 : v(a_1, t) - v(a_2, t) < r(a_1) - r(a_2) - \varepsilon \}.
\]

Finally, let \( T_i = \bigcup_{\varepsilon > 0} T_i(\varepsilon), i = 1, 2 \). Observe that \( T_1 \cap T_2 = \emptyset \). Call the type space \( T \) splittable if there are \( A_1, A_2 \), and \( r \) such that \( T = T_1 \cup T_2 \) and \( T_i \neq \emptyset \) for \( i = 1, 2 \).

**THEOREM 2**—Chung and Olszewski (2007): If \( A \) is finite, the following two statements are equivalent.

(i) All \( f \) that are implementable in dominant strategies satisfy revenue equivalence.

(ii) For all agents, \( T_i \) is not splittable.

If \( A \) is not finite, but countable, (ii) implies (i).

To see the connection between the allocation graph defined in Section 3 and the notion of splittable, we outline a proof that (ii) \( \Rightarrow \) (i). Suppose an allocation rule \( f \) implementable in dominant strategies that fails revenue equivalence. Since \( f \) is implementable, the allocation graphs satisfy the nonnegative cycle property. Since revenue equivalence is violated, Theorem 1 implies an agent...
i and reports of the other agents $t_{-i}$ such that in the corresponding allocation graph $G_f$, $\text{dist}_{G_f}(a^*, b^*) + \text{dist}_{G_f}(b^*, a^*) > 0$ for some $a^*, b^* \in A$. Assume the perspective of agent $i$. We show that this implies that $T_i$ is splittable.

Define $d(a) = \text{dist}_{G_f}(a^*, a) + \text{dist}_{G_f}(a, a^*)$ for all $a \in A$. Since the function $d$ takes only countably many values, there exists $z \in \mathbb{R}$ such that the following sets form a nontrivial partition of $A$: $A_1 = \{a \in A \mid d(a) > z\}, A_2 = \{a \in A \mid d(a) < z\}$. Observe that for every $a_1 \in A_1$, there exists $\epsilon(a_1) > 0$ such that $d(a_1) > z + \epsilon(a_1)$. Similarly, for every $a_2 \in A_2$, there exists $\epsilon(a_2) > 0$ such that $d(a_2) < z - \epsilon(a_2)$. It is now straightforward to verify that the sets $A_i$ “split” the type space.

Notice that in Theorem 1 no assumption on the cardinality of $A$ is made, whereas in Theorem 2, $A$ is assumed finite or countable. On the other hand, Theorem 1 imposes a condition on the allocation rule, whereas Theorem 2 characterizes $T$ and $v$ such that all allocation rules that are implementable in dominant strategies satisfy revenue equivalence. The principle difference between these settings is illustrated by the following example.

A principal has one unit of a perfectly divisible good to be distributed among $n$ agents. The type of agent $i$ is his demand $t_i \in (0, 1]$. Given the reports $t \in (0, 1]^n$ of all agents, an allocation rule $f : (0, 1]^n \to [0, 1]^n$ assigns a fraction of the good to every agent such that $\sum_{j=1}^n f_i(t) \leq 1$. If an agent’s demand is met, he incurs a disutility of 0; otherwise, his disutility is linear in the amount of unmet demand. More precisely, agent $i$’s valuation if he is assigned quantity $q_i$ is

$$v_i(q_i, t_i) = \begin{cases} 
0, & \text{if } q_i \geq t_i, \\
q_i - t_i, & \text{if } q_i < t_i. 
\end{cases}$$

In this context, payments are reimbursements from the principal for unmet demand. This valuation function appears in Holmström (1979) as an example to demonstrate that his smooth path-connectedness assumption cannot be weakened. Likewise, the example can be used to show that the convexity assumption of the valuation function in Krishna and Maenner (2001) cannot be relaxed.

Call an allocation rule $f$ dictatorial if there is an agent $i$ who always gets precisely his demanded quantity $f_i(t_i, t_{-i}) = t_i$ for all $t_{-i}$. This rule is implementable and, as shown in Holmström (1979), fails revenue equivalence. However there are implementable rules in this setting that satisfy revenue equivalence:

THEOREM 3: For the demand rationing problem, the proportional allocation rule $f$ with $f_i(t) = t_i / \sum_{j=1}^n t_j$ for $i = 1, \ldots, n$ is implementable and satisfies revenue equivalence.

The proof uses the type graph for any agent $i$ and any fixed report $t_{-i}$ of other agents, and verifies implementability by using Lemma 1. A fixed point
argument is used to show that the distance function is antisymmetric. Thus revenue equivalence holds due to Theorem 1.  

As this setting of $T$ and $v$ allows for allocation rules that satisfy revenue equivalence as well as for rules that do not, any theorem that describes sufficient conditions for all implementable $f$ to satisfy revenue equivalence is necessarily silent.

REFERENCES


A complete proof can be found in Heydenreich, Müller, Uetz, and Vohra (2008).
Dept. of Quantitative Economics, Maastricht University, P.O. Box 616, 7500 AE Enschede, The Netherlands; b.heydenreich@ke.unimaas.nl,
Dept. of Quantitative Economics, Maastricht University, P.O. Box 616, 7500 AE Enschede, The Netherlands; r.muller@ke.unimaas.nl,
Dept. of Applied Mathematics, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands; muetz@utwente.nl,

and

Dept. of Managerial Economics and Decision Sciences, Kellogg Graduate School of Management, Northwestern University, Evanston, IL 60208, U.S.A.; r-vohra@kellogg.northwestern.edu.

Manuscript received May, 2007; final revision received July, 2008.