Synergies are a reason to prefer first-price auctions!

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Received 21 November 2006; received in revised form 2 February 2007; accepted 14 February 2007
Available online 22 June 2007

Abstract

This paper studies the performance of auction formats when synergies are present between sequentially auctioned objects. Although the second-price auction format performs better in terms of efficiency and revenue, bankruptcy problems are less severe in the first-price auction format.

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Keywords: Auctions; Bankruptcy; Synergies; Procurement

JEL classification: D44; H57

1. Introduction

In private value settings second-price sealed-bid auctions are generally preferred to first-price sealed-bid auctions due to the desirable properties they have. First of all, the dominant bidding strategy is truth-revealing and therefore does not require any information on the situation or intention of competitors. Second, in equilibrium the auction is guaranteed to be efficient. Besides, the revenue-equivalence theorem (Myerson, 1981) shows that the expected revenues from both auction formats are identical under certain assumptions.

Still, second-price auctions are hardly observed in practice and several theoretical explanations have been given for this. Rothkopf et al. (1990) argue that bidders might be reluctant to reveal their true valuation, since this can have negative implications for future auctions. Both in one-shot and

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0165-1765/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.econlet.2007.02.022
repeated settings, collusion among bidders is less stable in first-price auctions than in second-price auctions (see for instance Robinson, 1985; Fehl and Güth, 1987; Skrzypacz and Hopenhayn, 2004). Finally, when bidders are risk-averse or when their valuations are asymmetrically distributed, the revenue-equivalence theorem does not hold anymore. Holt (1980) shows that when bidders are risk-averse the first-price auction generates higher revenues since bidders then shade their bids less. When the valuation distribution of one type of bidders stochastically dominates that of the other type of bidders, Maskin and Riley (2000) show that a first-price auction generates more revenue than a second-price auction. However in such a situation the first-price auction is not guaranteed to be efficient anymore.

This paper also discusses asymmetries between the valuation distributions of two bidders but here the asymmetry is not exogenously given. We analyze a sequential auction of two stochastically equivalent objects in which positive synergies are present and compare the first- and second-price format. We find that both efficiency and revenue are higher for the second-price auction format. However, bidders are more likely to receive losses under this auction format. These losses can ultimately lead to bankruptcy—in particular for auctions where potential synergies are high—and therefore induce disastrous welfare consequences for society.

2. The model

Two objects, indexed by $k=1, 2$, are auctioned sequentially among two risk-neutral bidders, indexed by $i=1, 2$, who participate in both auctions. Valuations are private information and independently drawn across bidders and objects from the interval $[0, 1]$ according to the uniform distribution. The valuation for the second object is not known during the auction of the first object. After each auction bidders are informed whether or not they won the object. We incorporate the presence of positive synergies by multiplying the second auction valuation with a factor $s>1$ if the first auction is won.\footnote{Although most applications are in procurement settings, we follow the convention and analyze ‘highest bid wins’ auctions for expositional ease and without loss of generality.} Although the valuation for the second object is not known during the first auction, bidders know that winning the first object increases this valuation from $v_2^i$ to $sv_2^i$ and thus increases their expected payoff of the second auction.

The present setting represents a recurring auction like the annual auctioning of contracts for public services. The exact details of future contracts are not specified yet and therefore contracts are considered as a priori identical. The benefits from synergies are only attributed to the second object as the result of the order in which projects are executed. For instance, expertise is created during the first project and this gives benefits for the second, or specialized equipment is needed which then does not need to be acquired for a possible second project. Our setting can also find its application in other situations such as spectrum auctions where bidders benefit from a win in one area in the creation of a domain consisting of multiple contiguous areas.

A similar auction setting has been studied in Jeitschko and Wolfstetter (2002) and Tang Sørensen (2006). Tang Sørensen (2006) considers a second-price sealed-bid auction with synergies modeled by a constant that is added to the value of the second object. A drawback of this approach is that the marginal
synergy is infinite if the valuation is close to zero (see Menezes and Monteiro, 2004). The present way of modeling synergies ensures there is an observable relationship between the benefits from synergies and the intrinsic value of the second object. This approach is also adopted in Jeitschko and Wolfstetter (2002) where the case \( s = 2 \) is studied for the second-price sealed-bid auction format. In both articles the focus is only on revenues and price trends whereas the present study focuses on the wider (social) consequences the presence of positive synergies has.

**Proposition 1.** For the first-price sealed-bid auction format, the bidding strategies given by

\[
b_1^i(v_1^i) = \frac{1}{2}v_1^i + \Delta^1 \quad \text{with} \quad \Delta^1 = \frac{s^2}{s^2-1} \left\{ \frac{1}{2}s^{-1} \frac{1}{s} + \frac{1}{2}s^2 + \frac{1}{2} \arcsinh(\sqrt{s^2-1}) + \arcsin\left(\frac{1}{s} \frac{\sqrt{s^2-1}}{s^2-1}\right) \right\}
\]

and

\[
b_2^i(v_2^i) = \begin{cases} v_2^i & \text{if auction 1 is lost}; \\ \frac{1}{s} v_2^i & \text{if auction 1 is won} \end{cases}
\]

constitute a unique symmetric subgame-perfect Bayesian Nash equilibrium.

**Proof.** See Plum (1992) for the second round bid function and Leufkens and Peeters (2006) for the derivation of the first round bid function $\square$

The factor \( \Delta^1 \) in the proposition is the option value of winning the first auction. This option value is the difference in expected instantaneous payoffs for the second auction, before knowing the second auction valuation, between winning and losing. In line with intuition, the option value is positive and increasing in the synergy factor.

**Proposition 2.** For the second-price sealed-bid auction format, the bidding strategies given by

\[
b_1^i(v_1^i) = v_1^i + \Delta^2 \quad \text{with} \quad \Delta^2 = \frac{s^2}{s^2-1} - \frac{1}{s}
\]

and

\[
b_2^i(v_2^i) = \begin{cases} v_2^i & \text{if auction 1 is lost}; \\ \frac{1}{sv_2^i} & \text{if auction 1 is won} \end{cases}
\]

constitute a unique symmetric linear subgame-perfect Bayesian Nash equilibrium in weakly dominant strategies.

**Proof.** See Leufkens et al. (2006). $\square$

Again the factor \( \Delta^2 \) represents the option value effect of winning the first auction and equals the difference in expected instantaneous payoffs for the second auction between winning and losing the first auction before the second auction valuation is known. Also here the option value is positive and increasing in the synergy factor.
3. Comparison between the first- and second-price format

The option value may cause bidders to overbid their valuation in the first auction. In the first-price format bidders overbid if the option value exceeds half the valuation for the first object. In the second-price format bidders always overbid. As a consequence of the overbidding, the winner of the first auction can make an instantaneous loss in the first auction.

Losses that are made in the first auction can, but are not guaranteed to, be recovered in the second auction. In case the first auction winner is not able to recover its loss in the second auction, bankruptcy results in our setting. Especially for large procurement contracts, the term bankruptcy is warranted. Fig. 1 displays the likelihood that bankruptcy occurs in both the first-price and the second-price sealed-bid sequential auction with positive synergies for different values of the synergy factor. We see that for most values of $s$ bankruptcy is more likely to occur in the second-price format than in the first-price format.

The presence of synergies causes the second auction to be asymmetric. Asymmetric first-price sealed-bid auctions are known to possess inefficiencies, even when bidders are risk neutral. The second-price counterpart never possesses such inefficiencies. As a consequence, in addition to possible ex post inefficiencies due to absence of hindsight in future valuations, even ex ante inefficiencies can be observed for the first-price format. To be more specific, the inefficiency that can occur is that the loser of the first auction can win the second auction although that the synergy-adjusted value of the first auction winner is higher. Fig. 2 displays the likelihood by which inefficiencies occur in the two auction formats. We see that the probability on an inefficient outcome is bounded from above by 8.6% for the first-price auction format.

From the two figures we can conclude that for only a small region of the synergy factor the second-price auction performs better on both efficiency and bankruptcy. For all remaining values of the synergy factor the first-price auction performs better regarding bankruptcy but is outperformed by the second-price auction regarding efficiency.

The smaller probability on bankruptcy in a first-price auction is a consequence of the inefficiency in the second auction. As already mentioned, the only kind of ex ante inefficiency that can occur is that the loser of the first auction wins even though the (synergy adjusted) valuation of the first auction’s winner is higher. This inefficiency has a negative impact on the option value effect of winning the first auction. In a second-price auction this inefficiency does not exist and consequently the option value in the second-price

Fig. 1. The probability of bankruptcy as a function of $s$ for the first-price (●) and the second-price (○) format.
format is larger than for the first-price format (see Güth et al., 2005). Hence in the first-price auction the bidding competition will be less fierce in the first auction and consequently decreases the probability on bankruptcy.

As predicted by Maskin and Riley (2000), the expected revenue from the second auction is higher for the first-price format. The expected revenue from the first auction is higher for the second-price format owing to the larger option value. The total revenue from both auctions is larger for the second-price format but can be shown to be never more than 4% above that from the first-price format.

4. Summary

In sequential auctions with synergies, the second-price sealed-bid auction format guarantees efficiency in contrast to the first-price sealed-bid auction format. Also, the total revenue resulting from the second-price auction is higher. Still, the first-price format can be preferred since the probability that the winner of the first object goes bankrupt is smaller for most synergy levels. Our findings support the common use of the first-price auction format when synergies are present, most notably for procurement settings.

Acknowledgement

Financial support was received from the Dutch Science Foundation (NWO).

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