MEAN REVERSION, CONDITIONAL HETEROSKEDASTICITY AND JUMPS IN THE EMS

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1 INTRODUCTION

In March 1979, the European Monetary System (EMS) was founded. A main element of the EMS is the Exchange Rate Mechanism (ERM) in which each currency has a central rate expressed in terms of the European Currency Unit (ECU). These central rates determine a grid of bilateral central rates, around which fluctuation margins are established. In order to keep the exchange rate within these margins, participating countries are obliged to intervene in the foreign exchange market when a bilateral exchange rate reaches the boundary of this band. For this purpose, special credit facilities have been established. Instead of intervening, it is also possible to realign the parities, provided all the members of the EMS agree.

In empirical work, the ERM has not yet received much attention. Most empirical studies on exchange rates are concerned with US dollar rates. These exchange rates exhibit leptokurtic behavior (fatter tails than usual), clusters of high and low volatility, but not significant serial correlation. These stylized facts can be reproduced by means of an ARCH (Engle [1982]) or GARCH (Bollerslev [1986]) specification, as done, for instance, in Diebold [1988], Hsieh [1989] or Baillie and Bollerslev [1989].

Especially for high frequency financial data however, the GARCH specification cum normal innovations cannot fully explain the leptokurtic behavior. That is why several authors used other distributions than the normal, such as the student-t, a discrete mixing of normals or a normal-Poisson distribution (see, for instance, Boothe and Glassman [1987], Tucker and Pond [1988], Jorion [1988], Hsieh [1989], or Baillie and Bollerslev [1989]).

Since the ERM currencies have to stay within a target zone, their statistical properties should be expected to differ from those of the free float rates. As long as no realignment occurs, the changes of the rates have to be small and mean-reverting in order to stay within the band. During realignments, however, large depreciations can
occur that result in much fatter tails for the ERM currencies than for the free float currencies.

In the literature, the effects of a target zone on exchange rates have been modeled mostly along the lines of Krugman [1991]. In his model, the expected future spot rates are affected by the target zone because central banks will intervene whenever the exchange rate reaches the boundary of the band. These types of models heavily rely on the assumption that the intervention policy is fully credible and known to the public. Also, the Krugman model does not account for parity realignments. Since, in reality, the nature of the policy interventions is not known, it is not surprising that the empirical support for these models is disappointing.

Another line of research on the ERM currencies has been followed by Nieuwland, Verschoor and Wolff [1991]. They analyze weekly D-mark rates of several ERM currencies by estimating a model with an ARCH(1) specification and a mixed normal-Poisson distribution, as in Jorion [1988]. The Poisson process generates jumps, which might reproduce the discontinuities arising from (anticipations of) parity adjustments.

In this paper, these models are generalized in three directions, using weekly observations on the D-mark rates of the Belgian franc, the Dutch guilder, the French franc, the Danish krone, the Irish pound, the Italian lira, and for comparison reasons, the British pound and the US dollar, for the period April 4, 1979 to March 27, 1991. First, a moving average (MA) specification is included to allow for mean reversion of the exchange rates. Second, the ARCH(1) specification is replaced by the more general GARCH(p,q) specification. Third, the normal-Poisson mixture is compared to mixtures of two, three and four normal distributions (compound normal distributions), respectively. Using an adjusted Pearson goodness of fit test, it is concluded that the easier to estimate compound normal distributions are at least as good as the normal-Poisson mixtures.

2 THE MODEL

Before specifying the model, the statistical properties of the log D-mark rates are investigated. The first row of table 1 shows that the unit root hypothesis is rejected for the exchange rate within the band (s-p). However, if realignments are not accounted for, the unit root hypothesis can no longer be rejected. That is why the model is estimated in first differences. The parity reversion is partly accomplished by a negative moving average parameter.

### Table 1: Summary Statistics of Log D-mark rates

<table>
<thead>
<tr>
<th>Statistics</th>
<th>BFr</th>
<th>DFI</th>
<th>FFr</th>
<th>DKr</th>
<th>IPd</th>
<th>LI</th>
<th>BPd</th>
<th>US$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF s-p</td>
<td>-5.84</td>
<td>-6.39</td>
<td>-4.32</td>
<td>-5.04</td>
<td>-4.94</td>
<td>-3.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ADF s</td>
<td>-2.09</td>
<td>-2.79</td>
<td>-1.68</td>
<td>-3.60</td>
<td>-1.29</td>
<td>-1.97</td>
<td>-2.74</td>
<td>-0.86</td>
</tr>
<tr>
<td>Mean (x10^4)</td>
<td>.042</td>
<td>.007</td>
<td>.062</td>
<td>.052</td>
<td>.058</td>
<td>.080</td>
<td>.040</td>
<td>.014</td>
</tr>
<tr>
<td>Std (x10^4)</td>
<td>.508</td>
<td>.242</td>
<td>.478</td>
<td>.483</td>
<td>.490</td>
<td>.577</td>
<td>1.138</td>
<td>1.567</td>
</tr>
<tr>
<td>Skewness</td>
<td>5.074</td>
<td>2.55</td>
<td>6.180</td>
<td>1.411</td>
<td>4.278</td>
<td>2.071</td>
<td>.382</td>
<td>.430</td>
</tr>
<tr>
<td>p(1)</td>
<td>-1.55</td>
<td>-1.95</td>
<td>-0.23</td>
<td>-2.00</td>
<td>-1.02</td>
<td>-1.14</td>
<td>-1.30</td>
<td>0.068</td>
</tr>
<tr>
<td>Q(25)</td>
<td>74.6</td>
<td>80.3</td>
<td>29.3</td>
<td>69.5</td>
<td>44.8</td>
<td>65.7</td>
<td>48.1</td>
<td>24.4</td>
</tr>
<tr>
<td>Q(25)</td>
<td>10.4</td>
<td>143.5</td>
<td>1.53</td>
<td>73.2</td>
<td>4.42</td>
<td>68.7</td>
<td>56.8</td>
<td>44.9</td>
</tr>
</tbody>
</table>

The data consist of 626 weekly Wednesday spot rates, expressed in domestic currency per D-mark, from April 4, 1979 to March 27, 1991. s is the log D-mark rate and p is the log central parity.

ADF is the augmented Dickey-Fuller test statistic with a constant and one lag of the differenced series. The 5% critical value is -2.87.

The skewness is significantly positive for all ERM currencies. This might be a result of the asymmetry in the movements of the parity adjustments. Compared to the US dollar, the excess kurtosis is, due to large outliers, extremely high for all ERM currencies, especially for the Belgian franc, the French franc and the Irish pound. This is not what one would expect to see when looking at the Box-Pierce statistic of the squared data (Q(25)). Notice that, strictly speaking, this test requires normality. It turns out that the currencies with the highest excess kurtosis do not exhibit any significant correlation in their second moments. This finding indicates that a GARCH specification might not reduce the kurtosis very much.

Finally, the first order serial correlation coefficient p(1) and the Box-Pierce statistic of the raw data (Q(25)) are highly significant for all currencies, except for the French franc and the US dollar, indicating serial correlation in the series. The negative autocorrelation for the ERM currencies, which is probably due to the stabilizing effects of the intervention policy, partly produces mean reversion.

The results for the ERM currencies differ from those for the US dollar. To begin with, first differences of ERM rates exhibit significant negative autocorrelation. Second, the excess kurtosis for ERM currencies is much larger than for free float currencies. Lastly, unlike US dollar rates, ERM exchange rates in terms of the D-mark are not symmetric. As a consequence, symmetric distributions such as the normal or student-t are unlikely to give adequate results. The model here combines normal distributions and a stochastic jump process. The jump intensity (λ) is modeled by
of a Bernoulli distribution and, for comparison, by a Poisson distribution. The jump size is assumed to be normally distributed with expectation \( \mu \) and variance \( \sigma^2 \). These distributions can explain the skewness and the leptokurtic behavior of a series (Vlaar and Palm [1992]). Economic explanations for the presence of jumps are changes and realignments in the expected deviation probability after news announcements, for example. Since deviation sizes can be very large, and since the political willingness to sustain the parities is not known to the public, the market is very sensitive to news in times of economic weakness, resulting in volatile behavior within the band.

In order to specify the log-likelihood functions, first define the error term

\[
e_i = \delta a_i + \mu - \lambda \nu - \xi_i e_{i-1}.
\]

Notice that it has expectation 0 (Vlaar and Palm [1992] appendix 2).

The normal-Poisson log-likelihood \( \ln(L_n) \) can be written as

\[
\ln(L_n) = -\frac{T}{2} \ln(2\pi) + \ln(\lambda_{n-1}^{\lambda_{n-1}}) - \sum_{j=1}^{\lambda_{n-1}} \frac{1}{j!} \exp\left(-\frac{(e_j - (\lambda - \mu))^2}{2(\sigma^2 + \delta_j^2)}\right).
\]

The normal-Bernoulli log-likelihood function has the following form

\[
\ln(L_n) = -\frac{T}{2} \ln(2\pi) + \sum_{i=1}^{\lambda_{n-1}} \ln(b_i) \exp\left(-\frac{(e_i - (\lambda - \mu))^2}{2\sigma_i^2}\right) + \frac{\lambda}{\sqrt{h_i^2 + \sigma_i^2}} \exp\left(-\frac{(e_i - (\lambda - \mu))^2}{2(h_i^2 + \sigma_i^2)}\right).
\]

The GARCH(\(p,q\)) specification is the same for the two models

\[
h_i^2 = \alpha_0 + \beta h_{i-1} + \gamma^2 e_{i-1} + \delta_{i-1} f_{i-1}.
\]

A difficulty with a Poisson function, which has been used in most empirical studies, is that it contains an infinite sum. This sum has to be truncated for the process to become estimable. Even if it is truncated after only five terms, maximum likelihood estimation is quite complicated. The solution turns out to be sensitive to the choice of the starting values. Ball and Torous [1985] suggested to use the Bernoulli distribution to get starting values for the Poisson distribution. This procedure is adopted here.

### 3 EMPIRICAL RESULTS

Table 2 contains the empirical results of MA(\(q\))-GARCH(\(p,q\)) models for each currency. Three different distributions are used to estimate these models: a normal distribution, a normal-Bernoulli mixture (a discrete mix of two normal distributions) and a normal-Poisson mixture. The lengths of the lag structures for the MA and GARCH specifications have been determined using a likelihood-ratio test with a 5% marginal significance level. The critical values of this statistic are adjusted because nonnegativity constraints (Kodeled and Palm [1986]) of the GARCH parameters lead to a test of a one-sided hypothesis. All models are estimated with the maximum likelihood routine from the software package Gauss.

In the models without stochastic jumps, negative autocorrelation is not always present. For the French franc and the Italian lira, the MA parameters are not significant. Furthermore the estimated GARCH parameters for the Belgian franc, the Dutch guilder, the French franc and the Irish pound seem to be too high since they sum to more than one. This can be explained from misspecification. In these models, the increased volatility resulting from a jump might be captured through high values of \( \alpha_i \) or \( \beta_i \).

When jumps are taken into account, negative autocorrelation becomes significant for all ERM currencies. The jumps reduce the influence of the outliers on the MA-GARCH specification. The MA(1) specifications turn out to be adequate in terms of likelihood ratio except for the free float currencies. The GARCH(1,1) specification is appropriate for all series, except for the French franc and the Danish kroner, for which an ARCH(1) specification is sufficient.

The Bernoulli and Poisson jump specifications give almost similar results for all ERM currencies. The estimated jump intensities are slightly lower for the Poisson model, giving rise to a lower jump size. For the Bernoulli model, the jump intensity ranges from 2.3% (Belgium franc) to 9.4% (Italian lira). Since the sample consists of 626 observations, this means the estimated expected number of jumps lies between 14 and 59. Given the fact that there have only been 12 parity adjustments, part of the jumps must have taken place within the band, as a result of speculative attacks, for instance.

For all currencies, the expected jump size \( \nu \) is positive, which is in accordance with the positive skewness (Vlaar and Palm [1992] appendix 2). It is significantly different from zero (at the 5% level) for the French franc, the Danish Kroner, the Italian lira and the US dollar.
Table 2: Maximum Likelihood results of D(t) (M5) and D(t) (M5) jump models.

<table>
<thead>
<tr>
<th>Year</th>
<th>Model</th>
<th>Parameter 1</th>
<th>Parameter 2</th>
<th>Parameter 3</th>
<th>Parameter 4</th>
<th>Parameter 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>BLM5</td>
<td>0.064</td>
<td>0.064</td>
<td>0.064</td>
<td>0.064</td>
<td>0.064</td>
</tr>
<tr>
<td>1981</td>
<td>BLM5</td>
<td>0.064</td>
<td>0.064</td>
<td>0.064</td>
<td>0.064</td>
<td>0.064</td>
</tr>
<tr>
<td>1982</td>
<td>BLM5</td>
<td>0.064</td>
<td>0.064</td>
<td>0.064</td>
<td>0.064</td>
<td>0.064</td>
</tr>
</tbody>
</table>

Note: The results are based on a series of tests and simulations to determine the most appropriate model for each parameter.
Notice, also, that the differences in terms of fit between the models with Bernoulli and Poisson jump structures are negligible for most currencies.

The improvement of the fit for ERM currencies can also be seen from the first column of table 2. The value of the log-likelihood function increases by 53 (Dutch guilder) to 281 (French franc) points when stochastic jumps are included. Again, the differences between the Bernoulli and Poisson processes are negligible.

From these results, it may be concluded that both the normal-Poisson mixture and the normal-Bernoulli mixture perform rather well as representations of the weekly ERM exchange rates data, except for the French franc and the Irish pound. One possible reason for the failure of the normal-Bernoulli distribution to pass the goodness of fit test for these currencies might be that the number of distributions included in the mixture is too low. The normal-Bernoulli mixture is essentially a mixture of two normal distributions (a compound normal distribution). A possible reason for adding one extra normal in the mixture could be for instance, that the jump sizes at realignment dates are much larger than the jumps inside the band.

In order to investigate this possibility, we also estimated the MA(1)-GARCH(1,1) model for mixtures of three and four normal distributions (Vlaar and Palm [1992]). These models are both judged on the Schwarz [1978] criterion and on the goodness of fit test. For four out of eight currencies, the Poisson mixture has the lowest SC-value, but for these currencies the mixture of two normals is acceptable as well, in terms of fit. For three currencies, the mixture of three normals performs best, according the SC criterion. For the US dollar, the normal distribution shows the lowest SC criterion.

Two conclusions might be drawn from this finding. First, more than for free float currencies, jumps are necessary for modeling ERM currencies. Second, the jumps generated by a compound normal distribution are at least as appropriate, in terms of fit, as the jumps generated by the more difficult to estimate Poisson-normal mixture.

4 CONCLUSIONS

In this paper, the time series properties of weekly exchange rates participating in the exchange rate mechanism of the EMS are examined. Since these currencies have to stay within an agreed target zone, they all exhibit parity reversion. Moreover, since there have been several parity adjustments in the last twelve years, resulting in large changes in the spot rate compared to the usual variations within the band, the excess kurtosis for ERM currencies is much larger than that for free float currencies such as the US dollar or the British pound. Finally, since all realignments within the ERM have, in fact, been appreciations of the D-mark, all ERM rates in terms of the D-mark are positively skewed.

These features have been modeled using an MA(1)-GARCH(1,1)-jump model. The parity reversion is captured by negative moving average parameters, which turn out to be highly significant for all models. The parity adjustments are taken into account by means of a stochastic jump process. The Bernoulli and Poisson specification is compared; for most currencies, the two give similar results. A GARCH specification is included to take account of the changing volatility over time. If these stochastic jumps are not included, the MA-GARCH specification changes dramatically. For four out of six currencies, the GARCH specification becomes explosive and for two the MA parameter is no longer significant. These changes are the result of the influence of outliers on the model.

These MA(1)-GARCH(1,1)-jump models are checked by means of an adjusted Pearson chi-squared goodness of fit test. For four out of six ERM exchange rates, both the Bernoulli and the Poisson specifications pass the test. For the other two, a mixture of three normal distributions, which can be interpreted as a normal distribution plus two independent jump specifications, performs best.

One of the main aims of future research will be to allow for a time-varying jump process. Since the economic performance of the ERM countries has become more similar and, as a consequence, frequency and size of realignments have been declining, we should expect the influence of the jumps in our models to have been declined as well. This will be modeled by making the jump intensity (or size) a function of economic indicators, such as trade deficits, inflation differentials or interest differentials.

REFERENCES


A COMMENT ON:
MEAN REVERSION, CONDITIONAL HETEROSKEDASTICITY AND JUMPS IN THE EMS

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As stated in the conclusion of the article, the authors have examined the time series properties of weekly exchange rates of currencies participating in the exchange rate mechanism of the EMS. The authors demonstrate an extensive and detailed knowledge of the current state of the art regarding exchange rate modeling, in general, and, more specifically, the statistical properties of intra-EMS exchange rate movements.

They easily combine some of the main articles on this area and discuss them shortly, which results in a concise overview of the recent advances of empirical studies on exchange rates. More impressive is that they also succeed, and very convincingly so, in providing evident improvements in the currently available time series models of intra-EMS exchange rates. Distributional properties are modeled by combining stylized facts of intra-EMS exchange rate behavior with a stochastic jump-process. They integrate GARCH, mean reversion and a variety of jump-processes. By means of a series of tests, they clearly show which specification renders the best fit. Through combining the various elements and estimating the full model specification, they succeed in producing a precise description of the distributional properties. It is made manifest that inclusion of a jump-process contributes significantly to the explainatory power of the model.

Modeling the exchange rate behavior in such a detailed way inherently brings with it two drawbacks. The trade-offs are the large number of parameters needed and the hard-to-explain economic reasoning behind the model. For those people who know their way around these processes, the paper provides an interesting extension of the existing literature. For those who are not familiar with this area of research, an MA(1)-

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1 A negative autocorrelation of the logarithmic first differences of the exchange rate does not necessarily imply mean reversion, but merely that consecutive changes in the exchange rate tend to be of opposite sign. Henceforth, these changes are not necessarily reverting towards the mean, but to the proceeding level of the exchange rate. For a detailed discussion of mean reversion in the EMS, see Ruse and Svensson (1991) or Svensson (1991).