Iwan Bos, Wilko Letterie, Dries Vermeulen

Antitrust as Facilitating Factor for Collusion

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Abstract

We study collusion in an infinitely repeated prisoners’ dilemma when firms’ discount factor is private information. If tacit collusion is not feasible, firms that are capable of sustaining high prices may still be willing and able to collude explicitly. Firms eager to collude may signal their intentions when forming the agreement is costly, but not too costly. As antitrust makes explicit collusion costly in expected terms, it may in fact function as a signaling device. We show that there always exists a cost level for which explicit collusion is viable. Moreover, our analysis suggests that antitrust enforcement is unable to fully deter collusion.

Keywords: Antitrust Enforcement; Cartel Formation; Explicit Collusion; Tacit Collusion.

JEL Codes: L1; L4.
1 Introduction

An often made assumption in theories of industrial collusion is that firms’ intentions to collude are common knowledge. In the context of repeated games, these intentions are reflected by a discount factor that firms employ to determine the value of expected profits. Effective collusion commonly requires conspirators to put sufficient weight on future profits, which corresponds to firms having a sufficiently high discount factor. As collusive contracts (such as price-fixing) typically cannot be enforced in court, this holds independent of whether firms use express or implicit communication to form and sustain the agreement. In other words, both explicit and tacit collusive agreements must be self-enforcing.

Recently, Harrington and Zhao (2012) have shown that tacit collusion can be quite a challenge for firms that have incomplete knowledge about each other’s intentions to collude. Specifically, they study an infinitely repeated prisoners’ dilemma in which discount factors are private information. Firms have a low or a high discount factor and only the latter is capable of sustaining supra-competitive prices. In this setting, a firm that is eager to collude faces the problem that its rival may not be willing to increase prices. This firm can try to reveal its intentions by raising its price, but this is a risky strategy as it may not be matched by its competitor. To avoid this risk of losing business, it can alternatively wait in the hope that its rival will initiate collusion. Harrington and Zhao (2012) find that in this case it might take long before firms reach a collusive agreement and that they may not reach an agreement at all.

This finding raises the question of whether firms can do a better job by colluding explicitly. It is this issue that we address in this paper. We consider a similar setting as Harrington and Zhao (2012) and extend it by allowing firms to communicate directly. Specifically, each firm has to indicate at the beginning of the game whether it wants to engage in a cartel. In case of consensus, the cartel is formed. In principle, the option to communicate explicitly will not be of much help in this context when talking is cheap. However, firms can make communication costly by consciously acting in breach of antitrust laws. For instance, gathering to discuss selling prices makes communication costly in expected terms. In addition, there may be costs of setting up and maintaining the agreement (e.g., bargaining and monitoring).

The fact that forming a cartel is not cheap potentially provides an opportunity for firms to signal their intentions. In this paper, we establish the existence of a ‘chatty equilibrium’; a separating equilibrium in which firms with a high discount factor find it beneficial to com-
municate. This requires that forming the agreement is costly, but not too costly. It must be sufficiently costly to prevent firms not capable of collusion from pretending to be willing to cooperate. At the same time, it should not be too costly so that cartel formation is still profitable. We find that there exists a cost level that generates this separating effect. Additionally, we show that firms may always find it profitable to collude explicitly, even when the cartel gets caught. The reason being that cartel participants know each other’s type and therefore can continue colluding tacitly after the cartel has been discovered and prosecuted. Thus, by prohibiting firms to ‘talk the talk’, antitrust may in fact allow them to ‘walk the walk’. As such, it can be considered a facilitating factor for collusion.  

This paper proceeds as follows. In the next section, we present the basic structure and assumptions of the model. In Section 3, we explore the possibility for firms to collude both tacitly and explicitly. Specifically, we show that when tacit collusion is not feasible, firms may still be able to reach collusive market outcomes through express communication. Section 4 concludes with a brief summary and discussion of our main finding. All proofs are relegated to the Appendix.

2 Model

We study collusion in an infinitely repeated prisoners’ dilemma when players’ discount factor is private information. In this section, we present the basic structure of the model.

STAGE GAME  Let us start with a description of the stage game. There are two profit-maximizing firms, Firm 1 and Firm 2, that simultaneously choose prices. Each firm can either set a high price $p_H$ or a low price $p_L$. If both firms charge the high price, then each firm receives a ‘collusive’ profit $\pi^c$. If one firm charges the high price and one firm sets the low price, then the low-priced firm makes a profit of $\pi^d$. For simplicity, we make the innocuous assumption that the high-priced firm receives zero profit. When both firms choose the low price, each receives a ‘competitive’ profit $\pi^n$. In standard theories of industrial collusion it is commonly presumed that firms face a prisoners’ dilemma when choosing their prices. This corresponds to the following pay-off ranking: $\pi^d > \pi^c > \pi^n > 0$. As a result, it is a dominant strategy for both firms to charge the low price and therefore $(p_L, p_L)$ constitutes the unique

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1There are a few contributions that also highlight the facilitating potential of antitrust enforcement for cartel formation. McCutcheon (1997), for instance, argues that antitrust policy may enhance sustainability of collusion by preventing renegotiations in the event of defections, thereby allowing for severe and credible punishment strategies. Bos, Peeters and Pot (2013) show that when consumers are inert and tacit collusion is not feasible, firms may wish to collude explicitly when the probability of getting caught is sufficiently high.
static Nash equilibrium.

**REPEATED GAME** As is well-known, firms may obtain higher profits when they interact repeatedly. In the following, we will study the infinitely repeated version of the game specified above. To formalize, suppose that both firms simultaneously choose prices in each period $t = 1, 2, 3, \ldots$. Let $a_{it} \in \{p_L, p_H\}$ denote the action that firm $i$ chooses at time $t$. The pair of actions $a_t = (a_{1t}, a_{2t})$ induces a pair of pay-offs $\pi_{1t} = \pi_{1t}(a_t)$ and $\pi_{2t} = \pi_{2t}(a_t)$. Accordingly, firm $i$ receives a stream of pay-offs $\pi_{it}$, $\pi_{i2}, \pi_{i3}, \ldots$. To determine the value of future profit streams, each firm uses a discount factor $\delta_i$. Thus, the present value of receiving $\pi_{it}$ at time $t$ is $\delta_i^{t-1} \cdot \pi_{it}$. Cumulative profits are therefore given by

$$\Pi_i = \sum_{t=1}^{\infty} \delta_i^{t-1} \cdot \pi_{it}.$$ 

**INFORMATION** Each firm has either a high or a low discount factor: $\delta_i \in \{\delta_L, \delta_H\}$, with $0 < \delta_L < \delta_H < 1$. Let $\gamma \in (0, 1)$ be the probability that a firm is of the high type, $\delta_H$. With probability $1 - \gamma$, a firm is of the low type, $\delta_L$. To establish the firms’ discount factors, Nature performs an i.i.d. random draw from the probability distribution $(\gamma, 1 - \gamma)$ over types. After the realization of types, each firm is informed about its own type, but not about the type of its rival. Thus, discount factors are assumed private information.

**HISTORIES** Apart from the *ex ante* information that firms receive regarding their respective discount factors, firms are also updated about the pairs of actions that were taken in all previous periods at the start of each round. These updates are recorded in histories. A history is a sequence $h_t = (a_1, \ldots, a_{t-1})$ of pairs of actions $a_t = (a_{1t}, a_{2t})$. We denote the set of all histories by $\mathcal{H}$. Notice that there is only one possible history at time $t = 1$, which is the empty sequence $h_1 = ()$.

**PLANS OF ACTION** A *plan of action* is a function $P: \mathcal{H} \rightarrow \{p_L, p_H\}$ from the set of histories $\mathcal{H}$ to the set of actions $\{p_L, p_H\}$. The plan of action $P$ specifies that, for each history $h_t \in \mathcal{H}$, the action $P(h_t) \in \{p_L, p_H\}$ is chosen. In this paper, we consider the choice between a non-collusive and a collusive plan of action. The non-collusive plan of action $N: \mathcal{H} \rightarrow \{p_L, p_H\}$ is defined by $N(h_t) = p_L$ for all $h_t \in \mathcal{H}$. Alternatively, firms can attempt to collude by means of a grim-trigger strategy. Specifically, the collusive plan of action $T$ is defined by

$$T(h_t) = \begin{cases} 
  p_H & \text{if both firms have chosen } p_H \text{ in all previous rounds}, \\
  p_L & \text{otherwise}.
\end{cases}$$
tacit collusion and derive conditions under which tacit collusion is not feasible. Then, given

Let us now direct our attention to the possibilities for firms to collude. We start by analyzing

3 Collusion

The expected values $V_{i}(s_{1}, s_{2})$, $V_{2}(s_{1}, s_{2})$ and $V_{2h}(s_{1}, s_{2})$ are determined in a similar fashion.\(^3\)

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tacit collusion and derive conditions under which tacit collusion is not feasible. Then, given

\(^2\)Strictly speaking, we could do without the specification of actions for histories that a firm’s own actions prevent from happening. This, however, would significantly complicate our notation. We therefore follow standard practice in game theory by accepting the limited amount of redundancy associated with this way of modeling histories.

\(^3\)Observe that, for example, $V_{1h}(s_{1}, s_{2})$ does only depend on the part $s_{1h}$ of $s_{1}$ and not on $s_{1\ell}$. We have chosen this modest amount of redundancy in our notation to avoid having to define all four variants of expected pay-offs separately.
these conditions, we show that firms may still be willing and able to collude explicitly. In both cases, the solution concept is a Bayesian Nash Equilibrium.

**Bayesian Nash Equilibrium** A pair \((s_1, s_2)\) of strategies is a *Bayesian Nash Equilibrium* (BNE) when

\[
V_{1h}(s_1, s_2) \geq V_{1h}(\tilde{s}_1, s_2) \quad \text{and} \quad V_{1\ell}(s_1, s_2) \geq V_{1\ell}(\tilde{s}_1, s_2)
\]

for all strategies \(\tilde{s}_1\) of Firm 1, and

\[
V_{2h}(s_1, s_2) \geq V_{2h}(s_1, \tilde{s}_2) \quad \text{and} \quad V_{2\ell}(s_1, s_2) \geq V_{2\ell}(s_1, \tilde{s}_2)
\]

for all strategies \(\tilde{s}_2\) of Firm 2.

### 3.1 Tacit collusion

We consider a pair of strategies \((s_1, s_2)\) = \(((s_{1h}, s_{1\ell}), (s_{2h}, s_{2\ell}))\) with \(s_{i\ell}, s_{ih} \in \{N, T\}\). Thus, each firm chooses between the competitive plan of action \(N\) and the collusive plan of action \(T\). Let us start by analyzing the strategy choice of a firm with a low discount factor. The next result shows when \(s_{1\ell} = s_{2\ell} = N\) holds in equilibrium.

**Proposition 1.** For a firm with a low discount factor, \(N\) is a strictly dominant plan of action when

\[
\delta_{\ell} < \frac{\pi^d - \pi^c}{\pi^d - \pi^n}.
\]

Proposition 1 shows when firms with a low discount factor will not collude. Given this condition, the only remaining possible equilibria are \(((T, N)(T, N))\) and \(((N, N)(N, N))\). It is clear that \(((N, N)(N, N))\) is an equilibrium regardless of the values of \(\delta_h\) and \(\delta_{\ell}\). Moreover, if \(\delta_h < \frac{\pi^d - \pi^c}{\pi^d - \pi^n}\), then firms with a high discount factor will not collude either. Therefore, suppose that \(\delta_h \geq \frac{\pi^d - \pi^c}{\pi^d - \pi^n}\) so that firms with a high discount factor may have an interest to collude tacitly. In fact, given this condition, a firm with a high discount factor is willing to collude provided that its rival also has a high discount factor and both firms know each other’s type. Yet, in the current setting, firms do not know the type of their competitor. The next result shows when \(((T, N)(T, N))\) is still a BNE.

**Proposition 2.** The pair of strategies \(((T, N)(T, N))\) is a BNE when

\[
\delta_h \geq \frac{\gamma(\pi^d - \pi^c) + (1 - \gamma)\pi^n}{\gamma(\pi^d - \pi^n) + (1 - \gamma)\pi^n}.
\]
Observe that for $\gamma \rightarrow 1$ the above condition reduces to $\delta_h \geq \frac{\pi_d - \pi^c}{\pi_d - \pi^n}$. Thus, a firm that is of the high type may still be willing to collude when it is sufficiently certain that its rival is also of the high type. By contrast, when $\gamma \rightarrow 0$, then the above condition reduces to $\delta_h \geq 1$, which does not hold. Clearly, when a high type is sufficiently certain that its rival is of the low type, then it will prefer the non-collusive plan of action $N$. Finally, notice that collusion is more of a challenge in comparison to a situation where discount factors are common knowledge. In particular, two firms that are of the high type may no longer be willing to collude despite the fact that both might have the capability to sustain high prices.

### 3.2 Explicit Collusion

On the basis of the above analysis, we now make the following assumption.

**Assumption 1:**

$$0 < \delta_\ell < \frac{\pi_d - \pi^c}{\pi_d - \pi^n} \leq \delta_h < \frac{\gamma(\pi_d - \pi^c) + (1 - \gamma)\pi^n}{\gamma(\pi_d - \pi^n) + (1 - \gamma)\pi^n}.$$  

This assumption implies that both firms, independent of their type, prefer $N$ to $T$ and therefore excludes the possibility of tacit collusion.

In this section, we add an extra feature to the model by allowing firms to communicate directly and form a cartel. Specifically, after the information phase, both firms have to indicate whether they have an intention to collude. This is modeled as a choice from the set $\{A, R\}$, where $A$ signifies the willingness to cartelize and $R$ signifies refusal to collude. For a cartel to form, both firms have to choose $A$. We assume that explicit collusion is costly and when both firms choose $A$, their expected profit is reduced by a lump sum amount $X > 0$. The amount $X$ has a broad interpretation. It may be thought of as an investment of private resources required to form and maintain the cartel agreement. As explicit collusion typically constitutes a violation of antitrust laws, it also captures the anticipated costs of antitrust enforcement.

With the above extension, a strategy for a low-type firm is now a triplet $c_\ell = (K_{\ell h}, c^e_h, c^n_h)$, where $K_{\ell h} \in \{A, R\}$ is the choice whether to signal intentions to collude, $c^e_h : \mathcal{H} \rightarrow \{p_L, p_H\}$ is the plan of action when both firms have indicated their willingness to form a cartel and $c^n_h : \mathcal{H} \rightarrow \{p_L, p_H\}$ is the plan of action when at least one firm refuses to collude. The strategy for a firm with a high discount factor is determined in a similar way.

In the following, let the rival of firm $i$ be denoted by $j$. For a strategy pair $c = (c_1, c_2)$, with $c_i = (K_{ih}, c^e_{ih}, c^n_{ih}, K_{\ell h}, c^e_{\ell h}, c^n_{\ell h})$, the pay-off $V_i(c_1, c_2)$ is defined as follows.
A strategy pair

CHATTY EQUILIBRIUM

The expected pay-off

\[ V \]

The next result shows when the pair of strategies \( (c, \tilde{c}) \)

for all strategies \( \tilde{c} \) for all strategies \( c \)

of Firm 1, and

\[ V_{2h}(c_1, c_2) \geq V_{2h}(\tilde{c}_1, \tilde{c}_2) \]

for all strategies \( \tilde{c} \) of Firm 2.

The next result shows when the pair of strategies \( (c_1, c_2) \), with \( c_1 = c_2 = (A, T, N, R, N, N) \), is a chatty equilibrium. In this case, firms with a high discount factor signal their intention to collude and follow the plan of action \( T \) accordingly provided that collusion is agreed upon. The non-collusive plan of action \( N \) is chosen in all other cases.\(^4\)

**Proposition 3.** The pair of strategies \( (c_1, c_2) \), with \( c_1 = c_2 = (A, T, N, R, N, N) \), is a chatty equilibrium precisely when

\[
\frac{\pi^c - \pi^n}{1 - \delta_h} \geq X \geq \pi^d - \pi^n.
\]

Proposition 3 provides conditions under which firms find it beneficial to collude explicitly. Firms that are willing and able to collude may successfully engage in a cartel when this is costly, but not too costly. It must be sufficiently costly to prevent firms that are not willing

\[^4\text{Notice that, in principle, the fifth strategy element is irrelevant due to the choice of } R \text{ by low-type firms. As a result, any pair of strategies } (c_1, c_2), \text{ with } c_1 = c_2 = (A, T, N, R, *, N), \text{ will yield the same outcome.}\]
to cooperate from fooling those that are. At the same time, it should not be too costly so that collusion is still profitable. The next result indicates that the cost level that is required to generate this separating effect exists.

**Corollary 4.** There exists an $X$ for which $(c_1, c_2)$, with $c_1 = c_2 = (A, T, N, R, N, N)$, is a chatty equilibrium.

Thus, when tacit collusion is not feasible, firms can collude explicitly provided that the price of forming the agreement is high, but not too high. From a policy perspective, it seems that there is not much antitrust authorities can do to prevent cartel formation from being sufficiently costly. That is to say, the lower bound on $X$ is endogenous and can be arranged to hold by, for example, an up-front participation fee. The same does not hold for the upper bound on $X$. Here, antitrust agencies may attempt to make explicit collusion unprofitable by enhancing enforcement efforts. Yet, no matter how high the (expected) costs of forming a cartel, it might remain a profitable alternative.

**Corollary 5.** Fix $X \geq \pi^d - \pi^n$. There exists a $\gamma^* \in (0, 1)$ such that for all $\gamma < \gamma^*$ there is a $\delta_h$ for which $(c_1, c_2)$, with $c_1 = c_2 = (A, T, N, R, N, N)$, is a chatty equilibrium.

This result reveals that antitrust enforcement may be unable to create a full deterrent effect when firms are uncertain about each other’s intentions to collude. The logic is clear. As $\delta_h \to 1$, the expected gains from colluding become arbitrarily high, thereby justifying any investment. Note that this holds even when the cartel gets caught and the expected antitrust penalty is exceptionally high. The reason is that, after the cartel is discovered and prosecuted, conspirators can continue colluding tacitly as they know each other’s type. Therefore, antitrust activity can make explicit collusion less attractive, but it cannot avoid being used as a signaling device.

It should be noted that this result is in part an artifact of the model. After all, it may not seem very likely that potential conspirators expect the gains from collusion to be very, perhaps even arbitrarily, large. Yet, the underlying intuition remains. In this respect, it is noteworthy that there are several empirical studies that lend some support to this finding. Sproul (1993), for example, discovered that for a sample of industries in which firms were convicted for price-fixing, prices rose in the years following the indictment. Block, Nold and Sidak (1981), Newmark (1988) and Harrington (2004) also provide evidence of post-cartel prices that did not (immediately) return to their non-collusive levels. Finally, Feinberg (1980) and Choi and
Philippatos (1983) report a relatively small negative impact of antitrust enforcement on the price level of convicted colluders.

In principle, there may be several explanations for these observations. For instance, there could have been a substantial rise of input prices during the cartel phase. Also, the convicted cartels might have been efficient, cost-reducing coalitions. Our analysis suggests an alternative explanation, namely that tacit collusion has replaced explicit collusion.\(^5\) In that sense, antitrust may be considered a facilitating factor for collusion, independent of whether the cartel gets caught.

4 Concluding Remarks

In recent years, there is a growing interest in the distinction between the economic and legal approach to industrial collusion.\(^6\) Whereas economists generally do not distinguish between tacit and explicit collusion, lawyers require that firms have reached an agreement on one or more key strategic variables (e.g., prices or outputs).\(^7\) Therefore, in order to better guide antitrust enforcement, it is of importance to improve our understanding of when and why conspirators communicate directly rather than through the market. This paper contributes to this agenda by providing a rationale for explicit collusion.

In line with the findings of Harrington and Zhao (2012), uncertainty regarding intentions to collude may prevent firms from reaching collusive market outcomes in a tacit manner. In that case, we have shown that explicit collusion may be a profitable alternative. Firms eager to collude can credibly signal their intentions when forming the collusive agreement is sufficiently costly. The underlying logic is somewhat similar to the ‘blood-in’ strategy as sometimes employed by criminal gangs. This strategy urges a person who wants to join a gang to kill somebody in order to prove that he is a trustworthy partner in crime. In a related fashion, albeit less dramatic, firms may reveal their true intentions by gathering together and explicitly discuss their selling prices. As antitrust activity makes such discussions costly in expected terms, it might effectively function as a signaling device.

Our analysis further suggests that it may be more than difficult to deter collusion. In particular, acting in breach of antitrust laws is potentially very profitable as it allows firms to sustain

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\(^5\)This explanation is also pointed out by Harrington (2004) and Whinston (2006, p. 32).

\(^6\)See, for example, Motta (2004), Whinston (2006), Martin (2006) and Davies and Olczak (2008).

\(^7\)It is noteworthy that pinning down the exact meaning of ‘agreement’ is a challenging exercise. See Kaplow (2011) for an extensive and detailed discussion of this matter.
high prices even after the cartel is caught. As such, explicit collusion, whether successful or not, might set the stage for effective tacit collusion. We thus cannot exclude the possibility that antitrust enforcement leads to higher rather than lower prices, thereby doing more harm than good. This finding seems important enough to warrant a more detailed study of the precise price effects. Clearly, this requires a richer framework than the one we have used in this paper. We leave this issue for future research.
Appendix: Proofs

**Proof of Proposition 1** Suppose that a firm with a low discount factor is confronted with a competitor that employs plan of action $N$. The plan of action $N$ gives a strictly higher pay-off when
\[
\sum_{t=1}^{\infty} \delta_t^{t-1} \cdot \pi^n > \sum_{t=2}^{\infty} \delta_t^{t-1} \cdot \pi^n,
\]
which holds.

Next, suppose that a firm with a low discount factor is confronted with a competitor that employs plan of action $T$. The plan of action $N$ gives a strictly higher pay-off when
\[
\pi^d + \sum_{t=2}^{\infty} \delta_t^{t-1} \cdot \pi^n > \sum_{t=1}^{\infty} \delta_t^{t-1} \cdot \pi^c.
\]
Rearranging gives,
\[
\delta_\ell < \frac{\pi^d - \pi^c}{\pi^d - \pi^n}.
\]

**Proof of Proposition 2** Suppose that a firm with a high discount factor faces a competitor that employs strategy $(T, N)$. The plan of action $T$ gives a weakly higher pay-off than $N$ when
\[
\gamma \cdot \left[ \frac{\pi^c}{1 - \delta_h} \right] + (1 - \gamma) \cdot \left[ \frac{\delta_h \cdot \pi^n}{1 - \delta_h} \right] \geq \gamma \cdot \left[ \frac{\pi^d + \delta_h \cdot \pi^n}{1 - \delta_h} \right] + (1 - \gamma) \cdot \left[ \frac{\pi^n}{1 - \delta_h} \right].
\]
Rearranging gives
\[
\delta_h \geq \frac{\gamma(\pi^d - \pi^c) + (1 - \gamma)\pi^n}{\gamma(\pi^d - \pi^n) + (1 - \gamma)\pi^n}.
\]

**Proof of Proposition 3** Consider firm $i$ and suppose that the strategy of its rival is $c_j = (A, T, N, R, N, N)$. We check under which conditions $c_i = (A, T, N, R, N, N)$ is a best response.

To begin, suppose that firm $i$ has a low discount factor. Given the above pair of strategies, it will reject collusion and choose $N$. As collusion is rejected, its rival will choose $N$ regardless of its type. Its pay-off is therefore given by
\[
\frac{\pi^n}{1 - \delta_\ell}.
\]
Now suppose that firm $i$ chooses $A$ instead. In that case, it faces a competitor that plays $T$ when of the high type and $N$ when of the low type. Since for a low type it is always a best response to play $N$, its pay-off is given by

$$\gamma \cdot \left[ \pi^d + \frac{\delta \cdot \pi^n}{1 - \delta_h} - X \right] + (1 - \gamma) \cdot \left[ \frac{\pi^n}{1 - \delta_h} \right].$$

Thus, the equilibrium condition for this case is

$$\frac{\pi^n}{1 - \delta_h} \geq \gamma \cdot \left[ \pi^d + \frac{\delta \cdot \pi^n}{1 - \delta_h} - X \right] + (1 - \gamma) \cdot \left[ \frac{\pi^n}{1 - \delta_h} \right].$$

Rearranging gives

$$X \geq \pi^d - \pi^n.$$

Next, suppose that firm $i$ has a high discount factor. According to $(c_1, c_2)$, firm $i$ colludes with firm $j$ when firm $j$ is also of the high type, but not when firm $j$ is of the low type. Hence, its expected pay-off in this case is given by

$$\gamma \cdot \left[ \frac{\pi^c}{1 - \delta_h} - X \right] + (1 - \gamma) \cdot \left[ \frac{\pi^n}{1 - \delta_h} \right].$$

(1) Suppose that firm $i$ chooses $R$. In this case, it faces a competitor that plays $N$, irrespective of firm $j$’s type. Consequently, it is a best response for firm $i$ to also play $N$, which gives a pay-off

$$\frac{\pi^n}{1 - \delta_h}.$$ 

Thus, the equilibrium condition is

$$\gamma \cdot \left[ \frac{\pi^c}{1 - \delta_h} - X \right] + (1 - \gamma) \cdot \left[ \frac{\pi^n}{1 - \delta_h} \right] \geq \frac{\pi^n}{1 - \delta_h},$$

which is equivalent to

$$\frac{\pi^c - \pi^n}{1 - \delta_h} \geq X.$$ 

(2) Now suppose that firm $i$ chooses $A$. In this case, it faces a competitor that plays $T$ when of the high type and $N$ when of the low type. If firm $i$ now chooses $N$ instead, its pay-off is

$$\gamma \cdot \left[ \pi^d + \frac{\delta \cdot \pi^n}{1 - \delta_h} - X \right] + (1 - \gamma) \cdot \left[ \frac{\pi^n}{1 - \delta_h} \right].$$

Therefore, the equilibrium condition in this case is

$$\gamma \cdot \left[ \frac{\pi^c}{1 - \delta_h} - X \right] + (1 - \gamma) \cdot \left[ \frac{\pi^n}{1 - \delta_h} \right] \geq \gamma \cdot \left[ \pi^d + \frac{\delta \cdot \pi^n}{1 - \delta_h} - X \right] + (1 - \gamma) \cdot \left[ \frac{\pi^n}{1 - \delta_h} \right].$$

Rearranging gives

$$\delta_h \geq \frac{\pi^d - \pi^c}{\pi^d - \pi^n},$$

which holds by Assumption 1. □
Proof of Corollary 4  By Proposition 3, it must hold that $\frac{\pi_c - \pi_n}{1 - \delta_h} \geq X \geq \pi_d - \pi_n$. There exists an $X$ for which this condition is satisfied when $\frac{\pi_c - \pi_n}{1 - \delta_h} \geq \pi_d - \pi_n$. Rearranging gives $\delta_h \geq \frac{\pi_d - \pi_c}{\pi_d - \pi_n}$, which holds by Assumption 1. □

Proof of Corollary 5  By Proposition 3, it must hold that $\frac{\pi_c - \pi_n}{1 - \delta_h} \geq X \geq \pi_d - \pi_n$. By assumption, $X \geq \pi_d - \pi_n$. The condition $\frac{\pi_c - \pi_n}{1 - \delta_h} \geq X$ is satisfied when $\delta_h \geq 1 - \frac{\pi_c - \pi_n}{X}$. In addition, we know by Assumption 1 that

$$\frac{\pi_d - \pi_c}{\pi_d - \pi_n} \leq \delta_h < \frac{\gamma (\pi_d - \pi_c) + (1 - \gamma) \pi_n}{\gamma (\pi_d - \pi_n) + (1 - \gamma) \pi_n}.$$  

As $X \geq \pi_d - \pi_n$, if $\delta_h \geq 1 - \frac{\pi_c - \pi_n}{X}$, then $\delta_h \geq \frac{\pi_d - \pi_c}{\pi_d - \pi_n}$. Thus, there exists a $\delta_h$ for which $(c_1, c_2)$, with $c_1 = c_2 = (A, T, N, R, N, N)$, is a chatty equilibrium when

$$\frac{\gamma (\pi_d - \pi_c) + (1 - \gamma) \pi_n}{\gamma (\pi_d - \pi_n) + (1 - \gamma) \pi_n} > 1 - \frac{\pi_c - \pi_n}{X},$$

which holds for $\gamma$ sufficiently small. □
References


