Testing for Asset Market Linkages: A new Approach based on Time-Varying Copulas

by

Hans Manner\textsuperscript{1} and Bertrand Candelon\textsuperscript{2}

Abstract

This paper proposes a new approach based on time-varying copulas to test for the presence of increases in stock market interdependence (also labeled as shift-contagion) after a financial crisis. We show that it is very important to consider simultaneously separate breaks in the volatility and dependence, otherwise the contagion test turns out to be biased. A sequential algorithm is then elaborated to tackle this problem. Applied to the recent 1997 Asian crisis, it confirms that breaks in variances always precede those in the dependence. Moreover, a significant "J-shape" evolution of the dependence parameter is detected supporting the idea of shift-contagion.

Keywords: Financial crisis; Contagion; Copula; Structural Change.

JEL Classification: E32, F36.

\textsuperscript{1}Department of Quantitative Economics, Maastricht University, PoBOX 616, MD 6200, Maastricht, The Netherlands

\textsuperscript{2}Corresponding author. Department of Economics, Maastricht University, PoBOX 616, MD 6200, Maastricht, The Netherlands, Tel.: +31 43 3883442; fax: +31 43 3884864; E-mail address: b.candelon@algec.unimaas.nl (B. Candelon)

*The authors thank Christophe Hurlin, Franz Palm, Stefan Straetmans and Jean-Pierre Urbain for fruitful comments and remarks. This version of the paper also benefitted from reactions of attendance at at the internal seminars at the Universities of Orleans and Louvain-la-Neuve, and 2007 congress of the Method in International Finance Network for comments. The usual disclaimers apply.
1 Introduction

The recent financial crises have renewed the debate on the importance of vulnerability of international financial markets and the propagation of foreign shocks. These turmoils originate from a crash in a "ground-zero" country\(^3\) which spreads across the world, even though market analysts considered other countries "healthy" before the crisis. Financial market linkages are fostered by the trend towards an almost complete capital market liberalization (also called "globalization") and are consequently at the heart of the recent crises. Emerging countries wishing to finance domestic investments can find the capital they require on foreign capital markets. They are thus no longer bounded by their national saving and can then accelerate their growth. Nevertheless, it is achieved at the cost of a higher risk of financial instability: a negative shock in a "ground-zero" country will be quickly and strongly transmitted to its financial partners. The group of crisis-contingent theories explain the increase in market cross-correlation after a shock issued in a "ground-zero" country in several ways; multiple equilibria based on investor psychology, endogenous liquidity shocks causing a portfolio recomposition, and/or political disturbances affecting the exchange rate regime. The transmission of the crisis and the subsequent increase in cross correlation between markets is thus characterized by a "spill-over" or "shift-contagion" process.\(^4\) These crisis-contingent theories do not specify the channels of transmission, which are assumed to be unstable and crisis dependent. In contrast, the non-crisis-contingent theories consider that the propagation of shocks does not lead to a shift from a good to a bad equilibrium, but that the increase in cross-correlation is the continuation of linkages (trade and/or financial) existing before the crisis.

Empirical tests for "shift-contagion" avoid the identification of transmission channels and focus their attention on changing patterns of cross-market correlation. For example, the propagation of the Asian crisis from Thailand to Indonesia is revealed by a higher correlation

---

\(^3\)Thailand and the devaluation of the Thai Bath in July 1997 is considered as the event that initiated the Asian crisis.

\(^4\)Masson (1999) considers the particular case of "false" shift-contagion, where the increase in cross-correlation may be due to the simultaneous occurrence of macroeconomic shocks across countries. According to the "monsoonal effect" theory, this artefact for shift-contagion is likely to happen as macroeconomic shocks are correlated.
between these financial markets during the crisis period. This correlation breakdown has been considered by several empirical studies. For example, King and Wadhwani (1990) and Lee and Kim (1993) show that financial market cross-correlation in the largest financial markets exhibits a significant increase after the U.S. stock market crash. Similarly Calvo and Reinhart (1995) and Baig and Goldfajn (1998) offer similar results after the 1994 Mexican and the 1997 Asian crises.

The concept of "shift-contagion" appeared to be a robust standard stylized fact until the influential paper of Forbes and Rigobon (2002). Considering a simple linear framework they show that a any increase in spurious correlation is detected in the presence of a change in volatility. As during a crisis financial markets are subject to high volatility regimes, Forbes and Rigobon (2002) propose a stability test for correlation, which corrects for the influence of volatility changes. Applied to the 1994 Mexican and the 1997 Asian crises, the hypothesis of higher cross-market linkages is ruled out. Several recent studies have extended the framework proposed Forbes and Rigobon (2002) (see among other Candelon et al, 2005, Corsetti et al, 2005 and Dungey and Zumahbeika, 2001) ruling out the idea of absence of contagion.

The linear framework used in Forbes and Rigobon (2002) is also subject to strong criticism. Hartman et al (2004), Bae et al (2003) stress that contagion is not characterized by an increase in correlation over the whole sample, but only during a period of extreme events, i.e. a financial crises. It would support the idea that contagion is a transitory process and that dependence between markets deviates only temporarily from its long-run path. They consequently propose to test for an increase in tail dependence (also called "co-exceedence") around the financial crisis dates.\footnote{Formal tests for the stability tail-dependence are proposed in Straetmans (1997).}

Other studies (Ramchand and Susmel, 1998 and Ang and Beckaert, 2002 to name but a few), prefer to consider another non-linear framework, namely the Markov-switching approach. They test for differences of the sample correlations among different volatility regimes identified as crisis and non-crisis periods. Maximum likelihood techniques are used to estimate the coefficients of a SWARCH and the probability matrix of staying or leaving a particular volatility regime.

More recently, Rodriguez (2007) investigates contagion using the concept of copulas. A Copula is the part of a joint distribution that completely describes its dependence struc-
Copulas allow the modeling of the dependence between variables in a flexible way and independently of the marginal distribution. Using copulas it is possible to model different dependencies for losses and gains (asymmetric dependence) and dependence of extreme events (tail dependence). They thus appear to allow for a more general characterization of contagion than linear correlation. Using asymmetric copulas and assuming that the same regimes govern the volatility and the (tail-)dependence structure, Rodriguez (2007) finds support for changing tail-dependence during periods of turmoil.

This paper proposes to use copulas in order to investigate asset market shift-contagion. It is worth noticing that contagion is defined here as a significant increase in overall dependence, namely correlation, as in Forbes and Rigobon (2002). This point of view differs from Rodriguez (2007), who focuses on tail-dependence. Moreover, we do not to impose the dependence to have a break coinciding with the changes in volatility regime. It may be that both dates are identical, supporting the idea that change in dependence is synchronized with change in volatility regime. However, because of propagation time or information transfer, they are in our opinion unlikely to perfectly coincide. Typically, when tensions occur on the ”ground zero” market we expect the linkage with the other markets to remain constant or even to slightly decrease until the transmission of the crisis is not complete. Then, interdependence would rise to become higher than before the turmoil. Anyway, the possibility to investigate such a ”J-shape” is possible in the framework proposed in this paper.

For this reason, this paper contributes to the literature by setting up a sequential algorithm. Based on a time varying copula, it allows for an efficient joint estimation of distinct breakpoints in dependence and volatility. Elaborating on Dias and Embrechts (2004), which proposes a formal test for the presence of a structural break in the dependence at an unknown period of time. The sequential algorithm also includes a formal estimation of the date of break in the variance parameters. Then, a Monte-Carlo study shows that the sequential algorithm exhibits correct size and power properties whatever the type of copula. In the empirical application, which deals with the Asian 1997 crisis, it turns out that contagion, defined as an increase in correlation, is a dominating feature among the Asian economies.

6The sequential algorithm does not have a formal estimation of the break in tail-dependence, which is assumed to be synchronous with the break in dependence.
Furthermore, the assumption that dependence and volatility exhibit a simultaneous change in regime is rejected. The date of the change in regime is different, supporting the idea that transmission process may take some time after to the occurrence of a financial crisis.

The rest of the paper is organized as follows. Methodological tools are introduced in section 2. Section 3 is devoted to an extensive Monte-Carlo analysis. Section 4 presents the results of our empirical study in the case of the Asian crisis and section 5 offers some conclusions.

2 Methodology

2.1 Copulas

Copulas are multivariate distribution functions, which have uniform marginal distributions. They capture dependence between the random variables of interest independently of their marginal distributions and hence are scale invariant. Copulas find their applications mainly in finance when calculating the Value-at-Risk of a portfolio, pricing exotic options and credit derivatives, or for simply estimating the joint distribution of asset returns. In this study we only focus on bivariate copulas. Definitions and most results of bivariate copulas carry over to the multivariate setting. In practice, however, the extensions are trivial only for very specific cases and we thus limit the analysis to the bivariate case. The most important result on copulas, Sklar’s theorem, can be found with a proof in Nelson (2006) and states the following. Let $F$ be the marginal distribution function of $X$, $G$ be the marginal distribution function of $Y$, and let $H$ be the joint distribution function of $(X, Y)$. Then there exists a copula $C$ such that

$$H(x, y) = C(F(x), G(y)), \forall (x, y) \in \overline{\mathbb{R}} \times \overline{\mathbb{R}},$$

where $\overline{\mathbb{R}}$ denotes the extended real line. If $F$ and $G$ are continuous then $C$ is unique. Conversely if we have distribution functions $F$ and $G$ and a copula $C$, then $H$ is a bivariate

\footnote{For a good exposition of financial applications of copulas see Cherubini et al. (2004). Applications in settings apart from measuring financial risk are rather rare and examples can be found in Granger et al. (2006) for modeling the income consumption relationship or Bonhomme and Robin (2004) who model earnings trajectories with the help of copulas.}
distribution function. Recalling the probability integral transform for continuous distributions, which states that the random variable $U = F(X)$ has a $U(0, 1)$ distribution regardless of the original distribution $F$, it becomes clear that a copula is no more than a multivariate distribution function with uniform marginals.\(^8\) It captures all the dependence between random variables of interest, as all the dynamics of the marginal distributions are captured by $F$ and $G$ for $X$ and $Y$, respectively. In the case of bivariate normal distribution, $F$ and $G$ are just normally distributed and the copula is completely described by the correlation between the margins. Other copulas allow for more complex and possibly non-linear dependence structures. For formal introductions to copulas and related functions, as well as a large number of examples of copulas we refer to the books by Joe (1997) and Nelson (2006). Patton (2006a) extended the theory by allowing the copula to be time varying and to depend on an exogenous conditioning set $\mathcal{F}_{t-1}$. In this way both the functional form of the copula and the copula parameter may vary over time. It is crucial, however, that the conditioning set is the same for marginal distributions as for the copula, since otherwise the extension of Sklar’s theorem to conditional distributions is not valid.

### 2.2 The Model

This paper opts for a model of contagion and interdependence between two asset markets, which consists in keeping the conditional mean process a simple linear process and in considering time-varying copula based distributions for the error terms, evolving with volatility and correlation regimes.

To this aim, the class of semiparametric copula-based multivariate dynamic (SCOMDY) models by Chen and Fan (2006) is considered. They propose a parametric estimation the conditional mean and variance of multivariate time series (using VAR or AR and GARCH models). In contrast, the multivariate distribution of the standardized innovations is estimated via a semiparametric copula model. Surprisingly, the estimation of the first step model does not influence the estimation of the copula parameter. The model for the conditional mean is given by the following stationary VAR model:

\(^8\)Note that it is crucial that the marginal distributions are well specified so that the variables $F(x)$ and $G(y)$ are i.i.d. $U(0, 1)$ distributed.
\[ R_t = \Gamma(L) \cdot R_{t-1} + \epsilon_t, \] (2)

where \( R_t \) are the stacked returns in markets \( r_1 \) and \( r_2 \), \( R_t = [r'_1 \ r'_2]' \) and \( \Gamma(L) \) is a lag polynomial with roots lying outside the unit circle. \( \epsilon_t = (\epsilon_{1t}, \epsilon_{2t}) \) are the VAR errors which have the following conditional distribution:

\[
\begin{pmatrix}
\epsilon_{1t} \\
\epsilon_{2t}
\end{pmatrix} | \mathcal{F}_{t-1} \sim C(F(\epsilon_1; \eta_t), G(\epsilon_2; \eta_t); \theta_t),
\]
(3)

where \( \mathcal{F}_{t-1} \) is the \( \sigma \)-space generated by the past returns. The variances of the marginal series \( \sigma^2 \) are included in the parameter vector \( \eta_t = [\sigma^2_t, \xi] \), where \( \xi \) captures the other moments of the marginal distribution, which we will treat as time constant nuisance parameters. The copula is chosen to provide the best fit to the data, which is the one giving the smallest value for the Akaike information criterion (AIC).

The marginal distributions \( F \) and \( G \) may be specified parametrically or non-parametrically. We allow the conditional volatility to follow a GARCH process, but we leave the distribution of the standardized innovations unspecified and model it by the empirical distribution function

\[
\hat{F}(x) = \frac{1}{T} \sum_{t=1}^{T} 1\{X_t \leq x\}.
\]

Furthermore, we allow for a single breakpoint in the unconditional variance. Similarly, the copula parameter \( \theta_t \) is characterized by a single (unknown) breakpoint. Alternatively, it can dynamically evolve over time as proposed in Patton (2006a), which will be described below. Both the breakpoints in volatility and in correlation are determined endogenously. Thus, changes in volatility and correlation induced by the regime shift are captured in the conditional distribution of the VAR residuals.

Using copulas has three main advantages compared to using a known multivariate distribution such as the multivariate normal or student t-distribution. First, the individual series are likely to be not normally distributed (i.e. leptokurtic and skewed). The marginals underlying standard multivariate distributions do not allow for these features. Leaving the marginal distributions unspecified, eliminates the risk of misspecification, which may influence the estimation of the dependence parameter.\(^9\) Second, the dependence between two stock mar-

\(^9\)The advantage of not having to specify the marginal distributions comes at the cost of less efficient estimation of the copula parameter, see Genest et al. (1995).
kets may show tail dependence (dependence of extreme losses), which may be modeled by several types of copulas.\textsuperscript{10} Third, using a copula representation as in (2) allows sequential estimation of marginal distribution as well as the copula itself, which is the basis of our test for a breakpoint in the dependence parameter conditional on a break in volatility. It also leads to a significant decrease in the computing time.

A test for contagion can then be performed as follows: First, one looks for a breakpoint in the dependence parameter. If correlation does not increase, there is clearly no evidence for shift-contagion. When a breakpoint in the dependence parameter is detected, it may support the hypothesis of contagion, but it may also simply be due to an increase in volatility (see Forbes and Rigobon, 2002). To discriminate between these possibilities, several methods are available. First, it is possible to compare the confidence intervals of the breakpoint in variance in the “ground-zero” country and those of the breakpoint in correlation. Second, the model (2) is extended to allow for the dependence parameter to vary over time, conditional on volatility. It then becomes possible to build a likelihood ratio test to determine whether the level or the regime of the conditional variance can explain the variation of the dependence parameter.

The tests for contagion based on correlation proposed by Forbes and Rigobon (2002) consist of estimating the VAR model (2)\textsuperscript{11} and restricting the distribution of the error terms to a bivariate normal distribution, which is estimated over two predetermined periods (one preceding and the other one during the turmoil). A significant increase in the correlation of the error term during the turmoil (i.e. $\rho_1 < \rho_2$) indicates the presence of contagion. Several studies have elaborated on this seminal paper keeping a similar framework.\textsuperscript{12} This approach suffers from several drawbacks. First, it may be subject to sample selection bias as the choice of tranquil and crisis periods is done beforehand. Second, the test assumed that the residuals are normally distributed. It is obvious that such a condition is violated

\textsuperscript{10}However, tail dependence analysis will not constitute the focus of this paper.

\textsuperscript{11}Forbes and Rigobon (2002) also add interest rate as an exogenous control variable.

\textsuperscript{12}(2) can be restated without adding more information as a common factor representation in order to separate the common factor from the idiosyncratic country-specific component. Corsetti et al. (2005) extract the common factor using principal components whereas Candelon et al. (2005) perform a common feature approach. The test for contagion boils down to a stability analysis of the common component. If its weight is larger after the crisis it can be concluded that shift-contagion occurred.
in our case. Comparing the models before and during the crisis leads to the introduction of non-linearities, and transforms (2) into a regime-dependent model. Forbes and Rigobon (2002) associate these regimes to volatility stance, and propose a correction of the original test. It may nevertheless miss other explanatory factors and introduce again an endogeneity bias as the volatility regimes are determined beforehand.

2.3 Testing for structural breaks in copula models (Dias and Embrechts, 2004)

A formal test for the presence of a breakpoint in the dependence parameter of a copula is developed by Dias and Embrechts (2004). They assume a sample \((x_1, y_1) \ldots (x_T, y_T)\) where \(t = 1, \ldots, T\) generated by the bivariate distribution functions \(H(x, y; \theta_1, \eta_1) \ldots H(x, y; \theta_T, \eta_T)\). The \(\theta_i\)'s are the parameters of the underlying copula, whereas the \(\eta_i\)'s are the parameters of the marginal distributions and are treated as nuisance parameters. Formally, the null hypothesis of no structural break in the copula becomes

\[
H_0 : \theta_1 = \theta_2 = \ldots = \theta_T \quad \text{and} \quad \eta_1 = \eta_2 = \ldots = \eta_T
\]

whereas the alternative hypothesis of the presence of a single structural break is formulated as:

\[
H_1 : \theta_1 = \ldots = \theta_k \neq \theta_{k+1} = \ldots = \theta_T \equiv \theta_k^* \quad \text{and} \quad \eta_1 = \eta_2 = \ldots = \eta_T.
\]

In the case of a known break-point \(k\), the test statistics can be derived from a generalized likelihood ratio test. Let \(L_k(\theta, \eta)\), \(L_k^*(\theta, \eta)\) and \(L_T(\theta, \eta)\) be the log-likelihood functions of our copula given in (3) using the first \(k\) observations, the observations from \(k + 1\) to \(T\) and all observations, respectively. Then the likelihood ratio statistic can be written as

\[
LR_k = 2[L_k(\hat{\theta}_k, \hat{\eta}_T) + L_k^*(\hat{\theta}_k^*, \hat{\eta}_T) - L_T(\hat{\theta}_T, \hat{\eta}_T)],
\]

where a hat denotes the maximizer of the corresponding likelihood function. Note that \(\hat{\theta}_k\) and \(\hat{\theta}_k^*\) denote the estimates of \(\theta\) before and after the break, whereas \(\hat{\theta}_T\) and \(\hat{\eta}_T\) are the estimates of \(\theta\) and \(\eta\) using the full sample. In the case of an unknown break date \(k\), a recursive procedure similar to the one proposed by Andrews (1993) can be used. The test
statistic is the supremum of the sequence of statistic for known \( k \):

\[
Z_T = \max_{1 \leq k < T} LR_k.
\]  

(4)

Dias and Embrechts (2004) recommend obtaining critical values using the approximation provided by Gombay and Horváth (1996), which we present here. Under \( H_0 \) it holds that for \( T \to \infty \)

\[
\left| Z_T^{1/2} - \sup_{1/T \leq t \leq 1/T} \left( \frac{B^{(d)}_n(T)}{t(1-t)} \right)^{1/2} \right| = o_P(\exp(-\log(T)^{1-\varepsilon}))
\]  

(5)

for all \( 0 < \varepsilon < 1 \), where \{\( B^{(d)}_T \): \( 0 \leq t \leq 1 \}\} is a sequence of stochastic processes such that \{\( B^{(d)}_T \): \( 0 \leq t \leq 1 \}\} \( \overset{d}{=} \) \{\( B^{(d)} : 0 \leq t \leq 1 \}\} for each \( T \) and \( B^{(d)}_T = \sum_{1 \leq i \leq d} B_i^2(t) \), where \{\( B_s \): \( 0 \leq t \leq 1 \}\}, \( s = 1, \ldots, d \) are independent Brownian bridges. There is no simple closed form expression for the distribution in (5). The following approximation can be used in practice. For \( 0 < h < l < 1 \)

\[
P\left( \sup_{h \leq t \leq 1-l} \left\{ \frac{B^{(d)}_T(t)}{t(1-t)} \right\}^{1/2} \geq x \right) = \frac{x^d \exp(-x^2/2)}{2^{d/2}\Gamma(d/2)} \times \left( \log \frac{(1-h)(1-l)}{hl} - \frac{d}{x^2} \log \frac{(1-h)(1-l)}{hl} + \frac{4}{x^2} + O\left( \frac{1}{x^4} \right) \right),
\]  

(6)

as \( x \to \infty \). Note that this limiting distribution is strictly identical to the one proposed in Andrews (1993), which applies in a more general context. The only difference is that Gombay and Horváth (1996) let the trimming parameters \( l \) and \( h \) depend on the sample size through \( l(t) = h(t) = \log(t)^{3/2}/t \), whereas Andrews (1993) considers a constant trimming value. We thus opt for the use of the critical values tabulated by Andrews (1993) and to trim the first and last 15% of the observations.

### 2.4 Testing for a structural break in unconditional volatility

We test for and locate a breakpoint in volatility using a quasi likelihood ratio test. To this end we model the return data with a normal distribution that has a structural break in variance at an unknown point in time \( p \). This does not mean that we assume the return data to be normally distributed, but the maximum likelihood estimate of the variance parameter
\( \sigma^2 \) converges in probability to its pseudo true value, which is the unconditional variance of the (sub-)sample.

Let \( L_p(\sigma), L_p^*(\sigma) \) and \( L_T(\sigma) \) be the log-likelihood function of the Gaussian distribution using the first \( p \) observations, the observations from \( p + 1 \) to \( T \) and the whole sample, respectively. Again \( \hat{\sigma}_p \) and \( \hat{\sigma}_p^* \) stand for the estimates of \( \sigma \) before and after the candidate breakpoint and \( \hat{\sigma}_T \) is the estimate of \( \sigma \) using the whole sample. Similar to the approach for testing for a breakpoint in correlation for

\[
LR_p = 2[ L_p(\hat{\sigma}_p) + L_p^*(\hat{\sigma}_p^*) - L_T(\hat{\sigma}_T) ],
\]

the test statistic of interest is

\[
Z_T = \max_{1 \leq p < T} LR_p. \tag{7}
\]

In practice, series should be demeaned over both subsamples, in order to remove the possible problems induced by different means. The asymptotic theory driven the behavior of the statistic \( Z_T \) for i.i.d. data is similar to the test above and the same critical values can be used.

Besides, it is quite likely that conditional volatility exhibits clusters and then follows a GARCH process. In such a case, previous asymptotic theory is not valid anymore: the time variation of conditional volatilities affects the estimates of the unconditional volatilities in finite (sub-)samples and therefore also the distribution of the test statistic under the null hypothesis. Thus, in such a case, critical values will be simulated using the estimated GARCH model.

Once the break in variance \( \hat{\sigma} \) is estimated, it is necessary to build its confidence interval at a certain level. To this aim, we develop a bootstrap procedure. We draw 2 bootstrap samples from our data set before and after \( \hat{\sigma} \). The test for a structural break in the variance is then performed on the complete bootstrap sample, leading to the detection of a variance breakpoint \( \hat{\sigma}^b \), which may be different from \( \hat{\sigma} \). This procedure is repeated a sufficiently large number of times \( B \) and the empirical 95\% confidence interval is calculated from the distribution of \( \hat{\sigma}^b \) where \( b = 1, \ldots, B \).
2.5 Testing for structural breaks in dependence conditional on a break in volatility: a sequential algorithm

The problem of testing for a structural break in the dependence parameter becomes more complicated when there is a breakpoint in the variances of the individual series. We take this into account by allowing for a breakpoint in the volatility parameter $\sigma_t^2$ at an unknown point in time $p$ both under the null of no change in dependence and under the alternative. The other moments of the marginal distribution, captured by $\xi$, are still treated as nuisance parameters and are not allowed to change over time. Consequently, we test for a break in dependence conditional on the fact that a break in volatility has occurred. Formally, the null hypothesis of our conditional test is

$$H_0 : \theta_1 = \theta_2 = ... = \theta_T, \quad \xi_1 = \xi_2 = ... = \xi_T \equiv \xi, \quad \sigma_1 = ... = \sigma_p \neq \sigma_{p+1} = ... = \sigma_T \equiv \sigma^*_p,$$

versus

$$H_1 : \theta_1 = ... = \theta_k \neq \theta_{k+1} = ... = \theta_T \equiv \theta^*_k, \quad \xi_1 = \xi_2 = ... = \xi_T \equiv \xi, \quad \sigma_1 = ... = \sigma_p \neq \sigma_{p+1} = ... = \sigma_T \equiv \sigma^*_p.$$ 

Under the alternative, a single break is present in both the dependence (at time $k$) and the variance (at time $p$). A time-varying copula model allows for a simultaneous estimation of the break in the variance and in the dependence parameter. Intuitively, the LR statistics is calculated for all possible points $k$ and $p$. We then retain the supremum and compared the LR value to the simulated critical value at a fixed nominal size. More precisely, let us assume without loss of generality that $p < k$ and let $L_{k\setminus p}(\theta, \sigma, \xi)$ be the likelihood function of the joint distribution in our model (3) using the observations between $p$ and $k$. In this situation the LR statistic becomes

$$LR_{k,p} = 2[L_p(\hat{\theta}_k, [\hat{\sigma}_p, \hat{\xi}]) + L_{k\setminus p}(\hat{\theta}_k, [\hat{\sigma}^*_p, \hat{\xi}]) + L_k(\hat{\theta}^*_k, [\hat{\sigma}^*_p, \hat{\xi}]) - L_T(\hat{\theta}_T, [\hat{\sigma}_T, \hat{\xi}])]$$

The test statistic then has the following form

$$S_T = \max_{1 \leq k < T} \min_{1 \leq p < T} LR_{k,p}.$$
This supremum statistics becomes more complicated when the variances of the two series exhibit a breakpoint at distinct points \( p_1 \) and \( p_2 \). In any case, the asymptotic distribution of the LR statistic \( S_T \) will also depend on the estimation of the breakpoint in the variance parameter \( \sigma^2 \). The approximations provided by Andrews (1993) and Gombay and Horváth (1996) should be modified to take the uncertainty in the estimation of the variance break into account.

Estimating all three breakpoints jointly is computationally very demanding and we opt for a sequential procedure to estimate the variance and dependence breaks. The copula decomposition of a joint distribution allows us to first estimate the marginal distributions, including the breakpoint in variance, followed by the estimation of the copula, which greatly reduces the computational burden. Therefore we apply a conditional test in the second step, estimating a breakpoint in the copula parameter, conditional on a break in the variance. The breaks in variance as well as the 95\% confidence intervals \( I_{p_1} \) and \( I_{p_2} \) are estimated in a first step using the method introduced in section 2.4.\(^{13}\) In a second step both series are transformed into uniform variables \((\tilde{u}, \tilde{v})\) such that \( \tilde{u} = \tilde{F}(x) \) and \( \tilde{v} = \tilde{F}(y) \). \( \tilde{F}(\cdot) \) is the empirical probability integral transform which has a different form before and after the estimated breaks \( p_1 \) and \( p_2 \):

\[
\hat{F}_{p_1}(x) = \frac{1}{p_1} \sum_{t=1}^{p_1} 1\{X_t \leq x\} \tag{8}
\]

\[
\hat{F}_{p_1^*}(x) = \frac{1}{(T - p_1)} \sum_{t=p_1+1}^{T} 1\{X_t \leq x\}
\]

and

\[
\hat{F}_{p_2}(y) = \frac{1}{p_2} \sum_{t=1}^{p_2} 1\{Y_t \leq y\} \tag{9}
\]

\[
\hat{F}_{p_2^*}(y) = \frac{1}{(T - p_2)} \sum_{t=p_2+1}^{T} 1\{Y_t \leq y\}.
\]

Finally, the structural break test proposed by Dias and Embrechts (2004) is applied to the

\(^{13}\) \( p_1 \) and \( p_2 \) refer to the breaks in the first and second series, respectively.
transformed data ($\tilde{u}, \tilde{v}$). Thus, we compute a similar test statistic with the following form

$$S_T = \max_{1 \leq k < T} LR_k.$$ 

Assuming that the variance breaks are consistently estimated the resulting estimate (of the break in the correlation coefficient) is also consistent. However, the nuisance caused by the estimation error in the first step must be taken into account when obtaining the critical values for the sup $LR$ statistic in the second step. Furthermore, the stability may exhibit a size bias when the breaks are close to the trimming value (see Candelon and Lütkepohl, 2001). To tackle these potential problems the following parametric bootstrap algorithm is set up:

1. Sequentially estimate the variance and correlation breaks $p_1, p_2$ and $k$ and store the confidence intervals $I_{p_1}$ and $I_{p_2}$ for the variance breaks.

2. Estimate the time constant copula parameter $\bar{\theta}$

3. Randomly draw two points $p'_1, p'_2$ for the variance breaks from the confidence intervals $I_{p_1}$ and $I_{p_2}$.

4. Estimate GARCH parameters $\omega_{p'_i}$ and $\omega_{p''_i}$ before and after the drawn breaks for both series.

5. Generate two random series $(u, v)$ from a time constant copula $C(u, v; \bar{\theta})$ and transform the marginal series into heteroscedastic variables using the estimated GARCH parameters before and after that break.

6. Apply the sequential breakpoint test to this series and compute the sup $LR Z^*$ statistic for the correlation break.

7. Repeat steps 3-6 $m$ times ($m$ being sufficiently large) and obtain the desired empirical quantile from the bootstrapped test statistics.

Until now, for the sake of generality no specific type of copula is assumed. Nevertheless, in empirical work we suggest using the simple and well known Gaussian copula. Its distribution function is given by

$$C_{\text{Gaussian}}(u, v) = \int_{-\infty}^{\phi^{-1}(u)} \int_{-\infty}^{\phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{s^2 - 2\rho st + t^2}{2(1-\rho^2)}} ds dt,$$
where $\rho$ is the linear correlation coefficient of the corresponding bivariate normal distribution.\textsuperscript{14} The simulation results in the next section will show that breakpoints in the dependence parameter can be found reliably using the Gaussian copula even when the data are issued from data generating processes producing tail dependence or volatility clusters. ST procedure is then quite robust to misspecification. Restricting the attention to the Gaussian copula is also justified by the fact that the asymmetric dependence often encountered in financial data and modeled by more flexible copulas seems to be partly be taken into account by time varying dependence.

3 Monte Carlo Study

The purpose of this section is to study the performance of our testing procedure in small samples. To this aim, we retain three of the most frequent data generating processes (DGP’s) encountered in empirical works.

The first DGP (DGP(1)) corresponds to a simple Gaussian copula with i.i.d. Gaussian marginals, i.e. a bivariate normal distribution. Thus the series $Y_{1,t}, Y_{2,t}$ have the distribution

$$F(Y_{1,t}, Y_{2,t}) = C_{\text{Gaussian}}(\Phi(Y_{1,t}), \Phi(Y_{2,t}), \rho),$$

where $\Phi(\cdot)$ denotes the CDF of the standard normal distribution. This distribution is the same as the one used in the sequential approach. Therefore, it is expected that under this DGP, the LR approach for detecting a break in variance and dependence performs the best.

The Monte Carlo experiments should also analyze the behavior of this testing procedure in front of DGP’s relaxing the normality assumption. To this aim, DGP(2) assumes volatility clusters in stock market returns series, whereas DGP(3) allows for asymmetric dependence structures and tail dependence. To be more precise, in DGP(2) let $(\Phi(\epsilon_{1t}), \Phi(\epsilon_{1t})) \sim C_{\text{Gaussian}}$. Then $Y_{i,t}$ follows a GARCH process:

$$Y_{i,t} = \sqrt{h_{i,t}} \epsilon_{i,t},$$

where

\textsuperscript{14}In general the dependence between two variables having a Gaussian copula is the correlation the variables would have if they had normal margins.
Such a process is frequent for financial series, which exhibits volatility clustering.

$DGP(3)$ corresponds to Clayton Copula with i.i.d. normal marginals and has the following form. The Clayton copula is given by

$$C_{\theta}^{Clayton}(u,v) = \max[(u^{-\theta} + v^{-\theta} - 1)\frac{1}{\theta}, 0],$$

with $\theta > 0$. To ensure comparability of the degree of dependence with the DGP’s using the Gaussian copula, the parameter for the Clayton copula is chosen such that the two copulas have the same value for the concordance measure Kendall’s $\tau$.\(^{15}\) In a first experiment, the DGP’s are simulated under the null hypothesis of no break in the dependence parameter (i.e. size analysis). Nevertheless, it is assumed that a single break in the variance occurs at date $p$, where $p = (1/4, 1/2, 3.4) \cdot T$ and $T$ is the sample size. The unconditional standard deviations before and after the breakpoint are set to $\sigma_l = 1$ and $\sigma_h = 3$, corresponding more or less to the estimates found in our empirical part. The sample sizes investigated are 500, 1000 and 2000.

Table (1) reports the rejection frequency of the test for a structural break in the dependence parameter in three cases. First, when the break in variance is ignored and applying the original Dias and Embrecht’s (2004) test (DE hereafter). Second, when this break is taken into account and following the sequential approach (ST hereafter) developed in section 2.5. Third, when the volatility regimes are known and applying the famous Forbes and Rigobon (2001)’s test (FR hereafter). In all cases, the nominal size is fixed at 5%.

It turns out that ignoring the break in variance leads to severe size distortion whatever the DGP considered. The rejection frequencies are always above the nominal size, indicating that the presence of a break in the dependence parameter is too often supported by DE in presence of volatility break. The importance of the size distortion increases with the sample

\(^{15}\)Closed form expressions of the link between the copula parameter and Kendall’s $\tau$ are available in the case of a Gaussian and a Clayton copula in Nelson (2006).
size, but is not affected by the location of the variance break. This result corroborates the initial findings of Forbes and Rigobon (2001), who show that tests for dependence stability are heavily biased in the presence of volatility regimes. Nevertheless, FR does not appear as an adequate alternative when DGP’s are copula based. Columns 5 to 7 of Table (1) show that the rejection frequency of the FR test is strongly undersized. The null hypothesis of stability in the dependence is then too often supported, leading to reject almost anytime the presence of shift contagion.

Columns 5 to 7 of Table (1) report the rejection frequency using ST approach, which takes into account the variance break. It turns out that the size is close to the nominal size of the test when considering DGP(1). Such a result is not surprising for DGP(1), as the Gaussianity hypothesis is at the basis of the sequential procedure. More interestingly, the size is also quite correct, even if slightly higher than 5% in the case of the Clayton copula (DGP(3)). It shows that ST is not too much affected by the presence of lower tail dependence. Intuitively, the dependence parameter corresponds to the center of a distribution. It is thus likely that stability tests are not affected by symmetric heavy tail and only marginally by asymmetric lower or higher tails. To conclude, the type of copula does not constitute a limiting factor for the ST approach, and thus the Gaussian copula will be used in the rest of the paper. The size of ST in the case of DGP(2) is also close to 5%. Such an outcome is also expected as the critical values used in the test for structural break in unconditional volatility are obtained by simulations allowing for GARCH dynamics.

All these conclusions hold whatever the sample size and the location of the variance break.

In a second experiment, the three DGP’s and designs are kept identical except that the dependence parameter is subject to a structural break, occurring at date \( k \), where \( k = (1/4, 1/2, 3.4) \cdot T \). Before the break, \( \rho_l = 0.3 \), and then increases to \( \rho_h = (0.5, 0.6, 0.7) \). We are thus now investigating the power of the ST approach.

---

Insert Table 2

---

\(^{16}\)It is nevertheless noticeable that the size bias is the lowest in the case of a variance break located at \( p = 3/4 \cdot T \).

\(^{17}\)The test for structural break in unconditional volatility can also be modified accordingly if one suspects asymmetric or fat tailed GARCH process. Critical values can be obtained by modifying the simulation technique and are available upon request for the case of a GARCH with Student-t innovations.
Table (2) reports the rejection frequency of the test for structural break in the dependence parameter using the ST approach.

The power\textsuperscript{18} of the ST approach turns out to be quite high and of the same magnitude for the three DGP’s under scope. Besides, as already noticed, it does not seem to depend on the location of the variance break. Finally, as expected, the power increases with the sample size $T$ and is maximum for breakpoints in the dependence parameter located at the middle of the sample.

Finally, from the previous experiment it is possible to determine the 95% confidence bound for $k$, the break date in the dependence parameter. They are reported in Table (3).

Insert Table 3

ST appears to perform quite well in detecting the date of break in the dependence parameter. Not surprisingly, the width of the confidence bounds around the true value of the break date decreases with the sample size $T$.

To conclude, the ST approach based on a Gaussian copula appears as an adequate approach to test for a structural break in the dependence parameter, while taking into account for the possible structural break in volatility.

4 Empirical Application

The financial turmoil that has affected Asian countries in 1997 has fueled the empirical literature on contagion (see Dungey et al or Candelon et al, 2008). Its main feature is that Asian countries were assumed to be well behaved, i.e. to possess good macroeconomic fundamentals, before the occurrence of the crisis, leaving market analysts or modelers of the first generation without any voice. The importance of the transmission of the crisis from Thailand, which was first hit, to the rest of Asia is therefore at the core of this global crisis. The conclusion of Forbes and Rigobon (2002) rejects the presence of contagion in this crisis, and has left analysts skeptical.\textsuperscript{19} However, this test tends to be biased if the

\textsuperscript{18}It is noticeable that power is not size corrected. Nevertheless, as the size distortion are really small (see Table (1)) using ST, such a conclusion will remain valid when considering size adjusted power.

\textsuperscript{19}Some other empirical papers extending the Forbes and Rigobon (2002)’s test reach a different conclusion. See Corsetti et al, 2005 and Candelon et al 2005.
break in variance and correlation are erroneously assumed to be identical. In such a case the sequential procedure based on time-varying copula presented in section 2 turns out to be the best alternative.

4.1 data

In this empirical application, stock returns for eight Asian countries\textsuperscript{20} are considered. Series are daily, extracted from Datastream market indices, labeled in US$ and cover the period 01/01/1996 – 30/06/1998, i.e. 652 observations. Returns of the market indices are plotted in Figure (1).

To shed new light on the question of the possible contagious characteristic of the Asian crisis, the ST approach, presented in Section 2.5, is applied. We consider both Thailand and Hong Kong as the "ground-zero" countries at the origin of the financial crisis. It is well-documented in the literature (see for example Dungey et al. 2006) that the Asian crisis was first initiated by the Thai Baht devaluation, and then by the crash of the Hong Kong stock market. These shocks have to be analyzed separately to draw a global picture of the shift-contagious process during the Asian crisis.

4.2 The Sequential Approach (ST)

For each bivariate system of the other 6 Asian countries with both Thailand and Hong Kong (i.e. 13 models), we estimate the VAR given in model (5) and then the steps 1 – 7 are performed on the residuals.\textsuperscript{21} Note that following the earlier discussion and the outcomes of the Monte-Carlo experiments, only Gaussian copulas are considered for sake of simplicity. Table (4) reports the dates for the structural breaks found in all univariate systems, and Table (5) gathers the dates for the structural breaks in the conditional copula, together with

\textsuperscript{20}These countries are Thailand, Malaysia, Japan, Hong-Kong, Taiwan, Indonesia, Korea and the Philippines.

\textsuperscript{21}The study is limited to bivariate systems in order to avoid the possible effect of indirect contagion or third-country effects. The Johansen Cointegration tests reject the presence of cointegration relationship for the bivariate systems composed by the indices, allowing us to specify the VAR with the return series. Furthermore, optimal lag length is determined according to the Schwarz information criterion.
confidence intervals, which together should allow for a conclusion whether contagion has occurred.

Insert Tables 4 and 5

It first turns out that point estimates for the dates of the break in variance generally precede the ones from of the conditional correlations.\textsuperscript{22} There are two cases, one where the confidence intervals of the variance and correlation breaks overlap and one where they do not. The assumption of concomitance between the change in volatility and dependence is thus not always supported. It supports the intuition that some time exists between the occurrence of the crisis, the increase in volatility and its spill-over developments. This delay finds its justification in the time for information to be spread over the region. Typically, tensions occurring on the ”ground zero” reduce the dependence with the other markets until the transmission of the crisis. Then, linkages would rise to become higher than before the turmoil. Moreover, such a result justifies our sequential testing procedure, which tends to support contagion for all the systems except (Hong Kong; Japan) and (Hong Kong; Korea), for which no significant change in conditional correlation, is detected. It is nevertheless clear that these two systems are a minority compared to the 11 other exhibiting a significant break. Thus, the main picture delivered by our study is clearly in favor of the existence of contagious characteristics of the Asian 1997 crisis.

4.3 Modeling time-varying dependence

Next, a loglikelihood ratio test is used to evaluate the relative significance of the volatility and the correlation regime (characterized by two different dummies) in the time-varying copula parameter. To this aim the conditional mean is modeled via a VAR model given in (2). The conditional variance \( h_t \) of the errors \( \epsilon_t \) is modeled by a t-GARCH, which can be stated as

\[
h_t = \omega_t + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1},
\]

\textsuperscript{22} Asides from comparison of break dates in the variance and the conditional correlation, the ”J-shape” form of the conditional correlations, which is plotted in Figure 3 constitutes another support for this conclusion.
where the standardized innovations $\nu_t = \sqrt{\frac{\kappa}{\bar{h}_t(\kappa-2)}} \nu_t \sim t_\kappa$. The $\nu_t$’s are modeled by a semi-parametric gaussian copula model, where the correlation parameter is allowed to vary over time conditional on a set of variables. Formally,

$$
\begin{pmatrix}
\nu_{1t} \\
\nu_{2t}
\end{pmatrix} | \Omega_{t-1} \sim C^{Gaussian}(\hat{F}(\nu_1), \hat{F}(\nu_2); \rho_t),
$$

where $\hat{F}$ denotes the empirical probability integral transform. Recall that $p$ is the breakpoint in variance of the ”ground-zero” country and $k$ is the break of the copula parameter estimated previously. Define two dummies $D_t(\sigma)$ and $D_t(\rho)$ as: $D_t(\sigma) = 0$ for $t < p$, $D_t(\sigma) = 1$ for $t \geq p$, $D_t(\sigma) = 0$ for $t < k$ and $D_t(\sigma) = 1$ for $t \geq k$. Then $\rho_t$ evolves over time according to

$$
\rho_t = \Lambda\left(\alpha + \beta_1 \cdot D_t(\sigma) + \beta_2 \cdot D_t(\rho)\right),
$$

where $\Lambda(x) = \frac{1-e^{-x}}{1+e^{-x}}$ is the modified logistic transformation to keep $\rho$ in $(-1,1)$.\textsuperscript{23} We test the two restrictions that one of the two dummies is not significant, given the other is included in the model. Formally, the sequence of hypotheses is the following one:

$H_0^a$: $\beta_2 = 0$

and

$H_0^b$: $\beta_1 = 0$

against:

$H_1$: $\beta_1 \neq 0$ and $\beta_2 \neq 0$

Insert Table 6

The p-values for the tests are given in table (6). In almost all the cases, $H_0^a$ is rejected against $H_1$, indicating that the correlation dummy improves the model when the variance dummy is included. However, $H_0^b$ is rejected in favor of $H_1$ only in a few cases. The presence of a dummy for the variance hence is not found to be significant when a correlation break

\textsuperscript{23} The conditional variance estimated from a GARCH model can also be used as the measure for volatility instead of the dummy. The results were very similar to the ones presented here and are available upon request.
dummy is already in the model. Taken together these results support the previous finding of shift-contagion, irrespective of the presence of a change in volatility.

As a final step in our analysis we use conditional copulas to model the path of the correlation coefficient of a Gaussian copula for the standardized VAR-tGARCH residuals, transformed by the empirical distribution function. To this end the correlation coefficient evolves as proposed in Patton (2006a):

\[ \rho_t = \Lambda(\alpha + \beta_1 \cdot \rho_{t-1} + \beta_2 \cdot \frac{1}{p} \sum_{j=1}^{p} \Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j}) + \gamma \cdot Z_t), \]

where, as before, \( \Lambda(\cdot) \) is the modified logistic transformation. The number of lags of cross products is chosen such that the fit of the model is best. \( Z \) is a set of additional conditioning variables. If \( Z \) is exogenous the distribution of the marginals must also be conditioned on it. In our case, however, it will be the dummies capturing the breakpoint in correlation and variance we detected with our procedure above. Dummies are included depending on the results obtained with the likelihood ratio tests given in table (6).

Insert Figures 2 and 3

The GARCH-variance are reported in Figure 2, whereas the conditional correlations are plotted in Figure 3. One can see that conditional correlation vary quite a lot over time, increasing after the "correlation" break. The evolution of the conditional correlation after the "volatility break" show a more shadowed picture: On one hand, it turns out that dependence decreases for some system leading to a "J-shape" evolution of the conditional correlations. Such a shape remains difficult to explain, even if it is confirmed by other studies using different techniques (Cappiello et al, 2005). On the other hand, some pairs of countries exhibit a constant increase in the conditional correlation, stressing that dependence begins to increase with the volatility break. Nevertheless, Figures (2) and (3) support our previous conclusions that the break in variance generally precedes the one in correlation. This is thus further evidence that high volatility is not always concomitant to an increase in correlation.
4.4 Robustness check

The robustness of the previous results vis-à-vis the data and the estimation method of the conditional copula has been checked. We consider filtering the series only by AR models (instead of VAR) (See appendix A), with or without GARCH filtering, and a fully-parametric estimation of the copula (i.e. specifying the marginal distributions parametrically), without finding different qualitative results.\textsuperscript{24} The qualitative results are also quite robust to the choice of copula.\textsuperscript{25} In some particular cases, applying a different type of copula appears to outperform the Gaussian copula. However, as the conclusions are not modified and the computing time explodes, we only report results for the Gaussian copula.

5 Conclusion

In this paper we propose a new sequential procedure using time varying copula to test for the presence of an increase in stock market dependence after a financial crisis, i.e. contagion process. We show that it is very important to consider simultaneously separate breaks in the volatility and dependence, otherwise the contagion test would be severely biased. In order to offer a better approach, we develop a sequential algorithm, which allows for different breakpoints in the variance and the conditional correlation. Moreover, the proposed contagion test is a sequential ”all-in-one” procedure which takes into account the uncertainty in the determination of the variance regime. The formal stability test is elaborated from the one proposed by Dias and Embrechts (2004) and a bootstrap procedure is implemented in order to tackle the distortion in the asymptotic distribution due to the presence of breakpoint in the nuisance parameters. Applied to the recent 1997 Asian crisis, the results produced by our sequential algorithm support that assuming the same break date for the variance and the conditional correlation is an erroneous assumption: Breaks in variances are generally preceding those in conditional correlation. Nevertheless, the Asian crisis turns out to have been characterized by a regional contagious transmission of the Thai shock.

Beyond the separate analysis of the effect of the volatility regime on the evolution of asset market dependence, tail-dependence may be also interesting as a complementary measure for contagion (Rodriguez, 2007). Future research would include another step in the sequential

\textsuperscript{24}Tables are not reported to save space but can be requested from the authors on request.
\textsuperscript{25}Further copulas considered are the Student, Clayton, Frank and Gumbel copulas and mixtures of them.
procedure that would allow for lower tail dependence (see Joe, 1997, for a definition) as well as changing types of dependence over time (as studied by Rodriguez, 2007). This could be performed using a more flexible copula model that additionally allows for conditional tail dependence. Even if such an analysis would bring a complementary insight on the tail dependence time path, it would not modify the previous results.

References


Table 1: Size of the stability test using DE and ST approaches.

<table>
<thead>
<tr>
<th>break date* T</th>
<th>1/4</th>
<th>1/2</th>
<th>3/4</th>
<th>1/4</th>
<th>1/2</th>
<th>3/4</th>
<th>1/4</th>
<th>1/2</th>
<th>3/4</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>break in variance (p)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DGP1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td>0.601</td>
<td>0.605</td>
<td>0.543</td>
<td>0.051</td>
<td>0.059</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T=1000</td>
<td>0.731</td>
<td>0.758</td>
<td>0.667</td>
<td>0.061</td>
<td>0.052</td>
<td>0.064</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T=2000</td>
<td>0.850</td>
<td>0.860</td>
<td>0.788</td>
<td>0.053</td>
<td>0.048</td>
<td>0.055</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DGP2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td>0.528</td>
<td>0.484</td>
<td>0.335</td>
<td>0.049</td>
<td>0.049</td>
<td>0.053</td>
<td>0.001</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T=1000</td>
<td>0.598</td>
<td>0.565</td>
<td>0.466</td>
<td>0.053</td>
<td>0.035</td>
<td>0.036</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T=2000</td>
<td>0.670</td>
<td>0.651</td>
<td>0.547</td>
<td>0.049</td>
<td>0.062</td>
<td>0.054</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DGP3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td>0.611</td>
<td>0.656</td>
<td>0.564</td>
<td>0.073</td>
<td>0.061</td>
<td>0.083</td>
<td>0.002</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T=1000</td>
<td>0.752</td>
<td>0.767</td>
<td>0.699</td>
<td>0.079</td>
<td>0.06</td>
<td>0.075</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T=2000</td>
<td>0.832</td>
<td>0.857</td>
<td>0.798</td>
<td>0.083</td>
<td>0.074</td>
<td>0.068</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: Size of the copula breakpoint test by Días and Embrechts (2004) ignoring changes in volatility (columns 2 to 4) and the size of our copula breakpoint test procedure (columns 5 to 7). Nominal size is 5% and the number of replications is equal to 1,000.
Table 2: Power of the stability test using ST approach.

<table>
<thead>
<tr>
<th>Break</th>
<th>$\sigma^\ast T$</th>
<th>$1/4$</th>
<th>$1/2$</th>
<th>$3/4$</th>
<th>$1/4^\ast T$</th>
<th>$1/2$</th>
<th>$3/4$</th>
<th>$1/2^\ast T$</th>
<th>$3/4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DGP1</td>
<td>$\rho_h = 0.5$</td>
<td>0.509</td>
<td>0.528</td>
<td>0.532</td>
<td>0.642</td>
<td>0.646</td>
<td>0.654</td>
<td>0.496</td>
<td>0.464</td>
</tr>
<tr>
<td></td>
<td>$\rho_h = 0.6$</td>
<td>0.917</td>
<td>0.938</td>
<td>0.928</td>
<td>0.973</td>
<td>0.977</td>
<td>0.975</td>
<td>0.917</td>
<td>0.917</td>
</tr>
<tr>
<td></td>
<td>$\rho_h = 0.7$</td>
<td>0.999</td>
<td>1.000</td>
<td>0.999</td>
<td>1.000</td>
<td>0.999</td>
<td>0.999</td>
<td>1.000</td>
<td>0.999</td>
</tr>
<tr>
<td>T=1000</td>
<td>$\rho_h = 0.5$</td>
<td>0.838</td>
<td>0.831</td>
<td>0.829</td>
<td>0.922</td>
<td>0.920</td>
<td>0.944</td>
<td>0.843</td>
<td>0.833</td>
</tr>
<tr>
<td></td>
<td>$\rho_h = 0.6$</td>
<td>0.999</td>
<td>0.999</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.999</td>
<td>0.997</td>
</tr>
<tr>
<td></td>
<td>$\rho_h = 0.7$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>T=2000</td>
<td>$\rho_h = 0.5$</td>
<td>0.995</td>
<td>0.992</td>
<td>0.994</td>
<td>1.000</td>
<td>1.000</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td></td>
<td>$\rho_h = 0.6$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>$\rho_h = 0.7$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

DGP2

| $\rho_h = 0.5$ | 0.418 | 0.443 | 0.458 | 0.566 | 0.611 | 0.613 | 0.427 | 0.432 | 0.484 |
| $\rho_h = 0.6$ | 0.879 | 0.883 | 0.874 | 0.946 | 0.963 | 0.966 | 0.846 | 0.871 | 0.889 |
| $\rho_h = 0.7$ | 0.997 | 0.993 | 0.993 | 1.000 | 0.999 | 0.997 | 0.997 | 0.997 | 0.997 |
| T=1000| $\rho_h = 0.5$ | 0.783 | 0.776 | 0.751 | 0.880 | 0.896 | 0.888 | 0.755 | 0.786 | 0.817 |
|       | $\rho_h = 0.6$ | 0.997 | 0.992 | 0.994 | 0.999 | 1.000 | 0.989 | 0.998 | 0.997 | 0.997 |
|       | $\rho_h = 0.7$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| T=2000| $\rho_h = 0.5$ | 0.977 | 0.972 | 1.000 | 0.999 | 0.999 | 0.997 | 0.964 | 0.976 | 0.980 |
|       | $\rho_h = 0.6$ | 1.000 | 1.000 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
|       | $\rho_h = 0.7$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

DGP3

| $\rho_h = 0.5$ | 0.495 | 0.523 | 0.539 | 0.622 | 0.673 | 0.642 | 0.448 | 0.487 | 0.505 |
| $\rho_h = 0.6$ | 0.888 | 0.901 | 0.894 | 0.943 | 0.948 | 0.951 | 0.873 | 0.867 | 0.901 |
| $\rho_h = 0.7$ | 0.995 | 0.997 | 0.997 | 0.999 | 1.000 | 0.994 | 0.993 | 0.994 | 0.994 |
| T=1000| $\rho_h = 0.5$ | 0.804 | 0.818 | 0.822 | 0.911 | 0.911 | 0.925 | 0.773 | 0.811 | 0.821 |
|       | $\rho_h = 0.6$ | 0.992 | 0.995 | 0.997 | 1.000 | 0.999 | 0.998 | 0.998 | 0.995 | 0.995 |
|       | $\rho_h = 0.7$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| T=2000| $\rho_h = 0.5$ | 0.985 | 0.977 | 0.987 | 0.997 | 1.000 | 0.997 | 0.977 | 0.979 | 0.988 |
|       | $\rho_h = 0.6$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
|       | $\rho_h = 0.7$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

Note: Power of ST for all three DGP’s. Nominal size is 5% and the number of replications is equal to 1,000.
Table 3: 95% confidence bound for the estimated break date in the dependence parameter breakpoint using ST approach.

<table>
<thead>
<tr>
<th>Break σ^T</th>
<th>σ^T</th>
<th>1/4</th>
<th>1/2</th>
<th>3/4</th>
<th>1/4</th>
<th>1/2</th>
<th>3/4</th>
<th>1/4</th>
<th>1/2</th>
<th>3/4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DGP1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>T=1000</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>T=2000</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>DGP2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>T=1000</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>T=2000</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>DGP3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>T=1000</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>T=2000</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ = 0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** 95% confidence interval for the estimation of the location of the breakpoint in correlation based on 1,000 Monte Carlo simulations.
Figure 1: Returns series
Figure 2: GARCH Variances
Figure 3: Conditional Correlation

[Graph showing conditional correlations between countries]

Legend: Thailand vs Malaysia, Thailand vs Japan, Thailand vs Hong Kong, Thailand vs Indonesia, Thailand vs Taiwan, Thailand vs Korea, Thailand vs Philippines, Hong Kong vs Malaysia, Hong Kong vs Japan, Hong Kong vs Indonesia, Hong Kong vs Taiwan, Hong Kong vs Korea, Hong Kong vs Philippines.
Table 4: Breakpoints in variance.

<table>
<thead>
<tr>
<th>Country</th>
<th>Date</th>
<th>Confidence Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thailand</td>
<td>12-5-1997</td>
<td>[24-4-1997,2-6-1997]</td>
</tr>
<tr>
<td>Malaysia</td>
<td>30-7-1997</td>
<td>[28-7-1997,6-8-1997]</td>
</tr>
<tr>
<td>Japan</td>
<td>3-12-1996</td>
<td>[22-11-1996,26-12-1996]</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>15-8-1997</td>
<td>[14-7-1997,5-9-1997]</td>
</tr>
<tr>
<td>Indonesia</td>
<td>12-8-1997</td>
<td>[5-8-1997,18-8-1997]</td>
</tr>
<tr>
<td>Taiwan</td>
<td>18-7-1997</td>
<td>[24-1-1997,8-12-1997]</td>
</tr>
</tbody>
</table>

**Note:** This table reports the date of the structural break in volatility found using the sequential approach, described in Section 2.4. Confidence bounds are bootstrapped (1,000 replications) and indicated between brackets.
Table 5: Breakpoints in correlation.

<table>
<thead>
<tr>
<th></th>
<th>Malaysia</th>
<th>Japan</th>
<th>Hong Kong</th>
<th>Indonesia</th>
<th>Taiwan</th>
<th>Korea</th>
<th>Philippines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thailand</td>
<td>4-12-1997***</td>
<td>8-12-1997****</td>
<td>2-12-1997***</td>
<td>2-9-1997***</td>
<td>2-12-1997***</td>
<td>14-10-1997***</td>
<td>22-10-1997***</td>
</tr>
</tbody>
</table>

**Note:** This table reports the date of the structural break in conditional copula found using the sequential approach presented in Section 2.5 and 95% confidence bounds for the correlation break considering both Thailand and Hong Kong as the crisis country.

Table 6: Likelihood ratio tests for significance of variance and correlation break dummies.

<table>
<thead>
<tr>
<th></th>
<th>Malaysia</th>
<th>Japan</th>
<th>Hong Kong</th>
<th>Indonesia</th>
<th>Taiwan</th>
<th>Korea</th>
<th>Philippines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thailand</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.0011</td>
<td>0.0003</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0658</td>
<td>0.2827</td>
<td>0.0002</td>
<td>0.5368</td>
<td>0.2249</td>
<td>0.5668</td>
</tr>
<tr>
<td>Hong Kong</td>
<td></td>
<td>0.0169</td>
<td>0.3581</td>
<td>-</td>
<td>0</td>
<td>0.0027</td>
<td>0.0907</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5381</td>
<td>0.7328</td>
<td>-</td>
<td>0.0026</td>
<td>0.3171</td>
<td>0.0696</td>
</tr>
</tbody>
</table>

**Note:** This table reports the p-value for a LR test of a dummy capturing the correlation (variance) breakpoint, given a dummy for the variance (correlation) break is also included in the model.
Appendix A: Break in volatility and dependence considering an AR model
Table 7: Breakpoints in correlation AR specification.

|---------------|--------------|--------------|--------------|-------------|-------------|--------------|--------------|

Note: This table reports the date of the structural break in conditional copula found using the sequential approach presented in Section 2.5 and 95% confidence bounds for the correlation break considering both Thailand and Hong Kong as the crisis country. The conditional mean was filtered using an AR specification.

Table 8: Likelihood ratio tests for significance of variance and correlation break dummies for AR.

<table>
<thead>
<tr>
<th>Country</th>
<th>Malaysia</th>
<th>Japan</th>
<th>Hong Kong</th>
<th>Indonesia</th>
<th>Taiwan</th>
<th>Korea</th>
<th>Philippines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thailand</td>
<td>$H_0^\alpha$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.0011</td>
<td>0</td>
<td>0.0249</td>
</tr>
<tr>
<td></td>
<td>$H_0^\beta$</td>
<td>0.0388</td>
<td>0.4539</td>
<td>0.0033</td>
<td>0.3012</td>
<td>0.1733</td>
<td>0.9372</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>$H_0^\alpha$</td>
<td>0.0045</td>
<td>0.5023</td>
<td>-</td>
<td>0.1369</td>
<td>0.0012</td>
<td>0.068</td>
</tr>
<tr>
<td></td>
<td>$H_0^\beta$</td>
<td>0.1746</td>
<td>0.6394</td>
<td>-</td>
<td>0.9904</td>
<td>0.4902</td>
<td>0.0274</td>
</tr>
</tbody>
</table>

Note: This table reports the p-value for a LR test of a dummy capturing the correlation (variance) breakpoint, given a dummy for the variance (correlation) break is also included in the model. The conditional mean was filtered using an AR specification.