7 Asymmetric Skill Substitution, Labour Market Flexibility, and the Allocation of Qualifications

Adriaan van Zon, Joan Muysken and Huub Meijers

Introduction

One of Europe’s most pressing problems of today is the relatively high level of unemployment especially among low-skilled workers, who generally suffer more and longer from unemployment than the rest of the working population.\(^1\) The generic cure for this problem, which economists have a habit of promoting, is cuts in wage costs\(^2\) and social benefits. The result of a reorganisation of the welfare state along these lines, however, may be high social costs. Before implementing such a reorganisation, one should at least be fairly sure about the size of the benefits involved, as well as their distribution over different skill groups in society. This would require at least a transparent labour market with respect to education and schooling.

This transparency of the labour market is related to an important, although often neglected, reason to believe that the benefits of wage-policy measures in terms of the envisaged effects on employment growth may be over-estimated.\(^3\) This reason is that effective substitution possibilities between different skill levels of labour may be overrated by the inadvertent use of standard economic concepts such as (multi-level) production functions. These functions are often included in models used to quantify the expected employment effects of certain policy prescriptions, such as selective (i.e. skill-specific) cuts in wage costs.\(^4\) It is then assumed that standard substitution features as they presumably apply at the aggregate level between labour and capital, also hold with regard to substitution between various skill groups of labour. These models therefore essentially assume that any skill group can be made to provide the services of any other skill group, if only the incentives for producers to make them do so are right. The logical conclusion following from this assumption is that the unemployment differences between skill groups, which can be observed in practice, exist only because of imperfections in the functioning of the wage-price system and/or other rigidities in the labour market. The remedy then is simple: remove (wage and other) rigidities.

This policy recommendation is completely in line with the assumption of symmetric substitution possibilities between skill groups, which actually means that one skill group can be interchanged for another. In our opinion, this implicit symmetry assumption neglects the fact that high-skilled people can generally perform
a larger set of tasks at a positive efficiency level than low-skilled people. More in particular, high-skilled people could probably perform the tasks which low-skilled people could perform too, but not necessarily the other way around. Hence, a given number of tasks (i.e. jobs) offers more employment opportunities to the group of high-skilled people than to the group of low-skilled people. This feature of the labour market should be acknowledged in formulating employment policies.

In order to do so, we have constructed a model with asymmetric skill-substitution. This asymmetry has several important implications. (1) First, it is obvious that the number of jobs which can efficiently be performed by low-skilled people is essentially an upper boundary to employment of low-skilled workers. (2) The creation of jobs for the low-skilled does not necessarily have to coincide with an increase in employment for the low-skilled. High-skilled workers find their employment opportunities increased too, and may compete directly with low-skilled workers for the jobs of the latter, but not the other way around. (3) On the other hand, the creation of high-level jobs requires the input of high-skilled workers and hence this would free jobs to be filled by low-skilled workers, insofar as the high-skilled workers were taking up low-level jobs. This is the so-called 'chimney effect' of the creation of high-level jobs which draws the 'still smoking' low-skilled workers from the flames of unemployment. (4) As a consequence, substitution between skills takes place in two different ways: direct substitution by means of the one skill replacing the other in low-level jobs, and indirect substitution of employment opportunities by means of changes in the job composition of employment. With respect to the latter one should realise that the job composition of employment is fixed to a large extent not only by the characteristics of the capital stock but also by the organisation of the production process as a whole. (5) This means that cuts in wage costs may have limited effects on the average job composition of employment, and mainly work through direct substitution. The resulting net effect on employment may therefore be limited.

These features of the model imply that in the short term, the employment effects of selective cuts in wage costs are bound from above, as far as old jobs are concerned. It also means that net creation of low-level jobs is required when direct substitution possibilities have been exhausted, in order to improve employment perspectives for the low-skilled. In times of a relatively large supply of skilled workers, however, the creation of high-level jobs may be more effective in alleviating unemployment problems for the low-skilled.

In this Chapter we will not focus on macroeconomic issues, but rather on the way in which the existence of asymmetric substitution possibilities influences the scope for wage cuts as an instrument to influence the distribution of unemployment over different skill groups. To do so, we define a framework with asymmetric substitution possibilities and add this to an otherwise standard (multi-level) production model, where we use made-up data and made-up parameters to implement the model. Before engaging in model simulations, however, we start with a description of the match
between skills and jobs in the Netherlands in the next Section. The third Section describes the skill-substitution framework, while the fourth Section is devoted to a description of the production function framework and the way in which the skill-substitution framework is integrated with the rest. The fifth Section provides a description of the simulation experiments we have conducted with the model, while the final Section contains a summary and some concluding remarks.

The match between skills and occupations in the Netherlands, 1988-1994

In this Section we will analyse data for the Netherlands to get an impression of the utilisation of skills in that country and the mechanisms underlying some developments over time. Unfortunately the period for which data are available is rather short, 1988-1994. Nonetheless some interesting observations can be made, which are also useful as a background for our theoretical analysis in the remaining part of this Chapter. We will first present some general features of the utilisation of skills in the Netherlands and then concentrate on the relation between the skill structure and job structure.4
To get an impression of the general situation in the period 1988-1994, aggregate employment and the rate of unemployment are presented in Figure 1. One can see that until 1992 the economic situation is prosperous: the rate of unemployment is declining and employment is increasing. From 1992 the economic situation deteriorates: the growth in employment slackens, while the rate of unemployment increases from 6 per cent to almost 8 per cent in 1994.

For our research we are particularly interested in a breakdown of these data with respect to skills and occupations. In the data available, skills refer to the highest level of education attained by workers.9 We distinguish between a high, medium and low level of acquired education. The high educational level corresponds with high vocational and university education. The low educational level corresponds to primary school and low secondary school (up to age 16). Medium education is everything in between. The employment shares for these three skills are presented in Figure 2. Remarkable features are the high share of medium-level skills, about 60 per cent over the period and the low share of low skills, below 20 per cent over the period. As can be seen from Figure 3, the share of low skills in unemployment is very high, however, of over 50 per cent. And as a consequence, the rate of unemployment amongst low-skilled persons is over 25 per cent. This stresses the precarious situation of low-skilled people.

The data also allow us to look at occupational categories. Actually, two dimensions are given: occupational types and levels. We concentrate on the latter, since our analysis typically applies to the relation between skill levels and job levels.8 The job level refers to the functional content of the jobs, and we distinguish between high-, medium-, and low-level jobs. The high job level requires professional or scientific skills at the level of high vocational or university education, whereas the low job level is characterised by simple routine tasks, which mainly require the use of hand-held tools and often some physical effort. Everything in between is labelled ‘Medium’, which means that at least some skill level is required.

<table>
<thead>
<tr>
<th>Skill level</th>
<th>Low</th>
<th>Occupational level</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>2.5</td>
<td>11.0</td>
<td>3.5</td>
</tr>
<tr>
<td>Medium</td>
<td>3.5</td>
<td>35.0</td>
<td>22.5</td>
</tr>
<tr>
<td>High</td>
<td>0.2</td>
<td>2.8</td>
<td>18.0</td>
</tr>
</tbody>
</table>

Source: Own calculations, based on Labour Force Survey data, Eurostat; cf. note 7
Figure 4 shows that over 50 per cent of employment is in jobs with a medium level, while the share of high-level jobs is almost 40 per cent. Employment in low-level jobs remains at a share of under 10 per cent over the whole period. Hence comparing this result with Figure 2 learns that low-skilled employment must also be found in higher-level jobs. The share of workers with a certain skill level in a certain occupational level is presented in Table 1.

![Figure 3 Unemployment shares by skill in the Netherlands, 1988-1994](image1)

![Figure 4 Employment share by job-level in the Netherlands, 1988-1994](image2)

It is remarkable that about 35 per cent of all jobs are both of a medium level and occupied by persons with a medium skill. Moreover, over 10 per cent of all jobs are also of a medium level but occupied by workers with a low skill. This is inconsistent with our notion that higher level jobs cannot be occupied by lower level skills. The same holds for the substantial share of medium-skilled people in high-level jobs, which is over 20 per cent of all jobs.10

A possible explanation for the occurrence of these ‘incorrect’ shares is that skills are not properly measured by the highest level of education, and that job experience should also be taken into account. Actually, we have data on average job experience in all relevant dimensions, as can be seen from Table 2.

From these data we observe that low-skilled workers in high-level jobs have six more years of experience on average compared to those employed in medium-level
jobs, i.e. 23.9 versus 18.0. And they have also three more years of experience on average, when compared to medium-skilled workers in high-level jobs. This partly explains why relatively many low-skilled workers are employed in higher level jobs, as we observed above. Hence a better measurement of skills, which would take working experience into account, might lead to observations more consistent with our assumptions. Moreover, one should realise that the average level of schooling has increased rapidly over time. Hence, older generations of workers probably started in higher level jobs in their youth, than they would have nowadays with a similar age and level of schooling. Therefore, there may also be a vintage effect in the high share of low-skilled workers in higher-level jobs. In addition, the shedding of in particular these jobs and workers could also be an explanation for the observation that over 40 per cent of total unemployment is concentrated in low-skilled workers that where employed in medium-level jobs, i.e. about 80 per cent of low-skilled unemployment is in this situation.

<table>
<thead>
<tr>
<th>Skill level</th>
<th>Years of experience</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>Medium</td>
</tr>
<tr>
<td>Low</td>
<td>21.0</td>
<td>18.0</td>
</tr>
<tr>
<td>Medium</td>
<td>16.0</td>
<td>16.2</td>
</tr>
<tr>
<td>High</td>
<td>6.2</td>
<td>11.3</td>
</tr>
</tbody>
</table>

Source: Own calculations, based on Labour Force Survey data, Eurostat; cf. note 7

In conclusion, if we look at years of schooling as an indication of skill level, we do not find the asymmetries assumed by our model, whereas casual observation of our own workplace, for example, indicates that we should. In general, however, working experience is definitely an important factor in measuring skills. And taking this into account will probably lead to a picture which is much more consistent with the asymmetries assumed in our analysis. Moreover, one should be aware of the fact that the high, medium and low levels in the classifications of jobs and skills are actually ordinal measures rather than absolute ones. Therefore, the empirical correspondence between the two classifications does not necessarily have to be one-to-one. We note these observations as questions for further research, which, however, requires the availability of much more detailed data than we have at this moment.
Jobs, skills and asymmetries in substitution possibilities

Introduction

High-skilled people may in general be expected to perform better in low-level jobs than low-skilled people in high-level jobs. Hence, in terms of the assignment of skills to jobs there must be an intrinsic (more or less technical) bias against allocating low-skilled people to high-level jobs. Nonetheless, some kind of substitution between higher skills and lower skills should be possible, since empirical investigations in this area point to a substitution set-up where high-skilled people are complements of capital, and where low-skilled people in turn can be substituted for this high-skilled people/capital complex.\(^4\) Broadening the skill spectrum to include intermediate skills, one finds relatively high elasticities of substitution between low and medium skills on the one hand, and a fair amount of substitution possibilities between the lower skill echelon and the high-skills/capital complex on the other.

In our model, a job is a set of tasks which need to be performed and which requires the people engaged in that job to have a certain minimum skill level in order to be able to perform the tasks concerned. These specific skill requirements define the level of the job. Moreover, a low-level job requires only low-level skills in order to be able to execute the tasks relating to that job.\(^4\) Hence, everybody who has the minimum required level of skills available can actually be hired for that job. Therefore, generally speaking, high-skilled people can be hired for more levels of jobs than low-skilled people. This provides an asymmetry in employment opportunities for high-skilled and low-skilled people, which, given a certain lack of compensating asymmetries in wage formation, might lead to a bias in employment opportunities in favour of high-skilled people rather than low-skilled people.

Ex ante behaviour: choosing jobs

Let us assume that the design of a production process entails the definition of certain packages of tasks which require at least low skills and high skills, respectively – i.e. the combination of low-level and high-level jobs that are associated with the production process. Let us furthermore assume that technical constraints are such that a decrease in the number of high-level jobs must be compensated by increasing numbers of low-level jobs, and vice versa. These are the ‘standard’ substitution assumptions in the neo-classical production theory, and they are reflected in an isoquant like \(J/J\) in Figure 5. In this figure the curve \(J/J\) defines the number of low-level jobs and high-level jobs which are required to generate one efficiency unit of labour. Since the axes of this figure actually refer to the skills of workers, the isoquant is depicted as if all low-level jobs are filled by low-skilled people \((L)\) and all high-level jobs are filled by high-skilled people \((H)\).
In our set-up the layout of the production process has been defined in terms of the number of high-skilled and low-level jobs required to generate one efficiency unit of labour, but the actual assignment of people to those jobs is a separate issue. Given the asymmetries described above, we know that low-skilled people can not efficiently perform in high-level jobs, whereas high-skilled people can perform efficiently in both low-level and high-level jobs. This means that the number of low-level jobs constitutes an upper limit for the number of low-skilled people which may actually be employed, while the number of high-level jobs is a lower limit to the number of high-skilled people which may actually find employment. Assuming that the efficiency of a high-skilled person relative to a low-skilled person in a low-level job is the constant number $e'$, it follows that the framework sketched above can be pictured as in Figure 5.

Let us assume that point $Q$ represents the optimum choice, where the optimum combination of jobs requiring only low skills and jobs requiring high skills is represented by the point $(L^*, H^*)$. The dotted line $eQH'$ then represents the
combinations of low-skilled and high-skilled people which can generate an efficiency unit of labour. The slope of the line segment $QH'$ represents the relative efficiency $e'$ of high-skilled persons relative to low-skilled ones. The relative efficiency of low-skilled people in high-level jobs is equal to zero by assumption, as is indicated by the vertical line segment of the dotted line in Figure 5. Hence, on the unit-job isquant, $H$ cannot fall below $H'$. When $H$ is equal to $H'$, then all low-skilled labour $L$ is replaced by high-skilled labour $H$ and still one labour efficiency unit is generated, i.e. $H' - H' + L'/e'$. Actually, all combinations of $L$ and $H$ on the line segment connecting points $H'$ and $Q$ are combinations of low-skilled and high-skilled people which generate one labour efficiency unit.

Given the location of $H'$ and $L'$ on the unit-job isquant, and $e'$, the location of the line segment $QH'$ is determined too. Hence, the choice of $H'$ and $L'$ also defines the combinations of high-skilled and low-skilled people which may actually be observed to be employed. If we, for reasons of simplicity, were to ignore the possibility of supply constraints and costs of hiring and firing in this particular setup, then the combinations of low- and high-skilled people which may actually be observed at time $t$, for given levels of $L'$ and $H'$ which have been chosen at time zero, are given by:

$$L_t = L' \exp(-\delta t) - e'(H_t - H' \exp(-\delta t))$$

(1)

where $\delta$ enters the analysis because we assume that the job design is embodied in machinery and equipment which is subject to (exponential) technical decay at rate $\delta$. $H'$ in turn is the maximum number of $H$ people required to generate one labour efficiency unit. It can easily be determined from (1) by setting $L=0$:

$$H'_t = \left(\frac{L'}{e'} + H'\right) \exp(-\delta t)$$

(2)

It follows directly that for given wages by skill $w_H$ and $w_{L'}$, entrepreneurs can influence the cost of operating a labour efficiency unit in two different ways. Firstly, they can do so by choosing a certain job combination $(H', L')$ from the isquant $j'$ in Figure 5. This also determines the position of the line between $Q$ and $H'$ in the figure. Second, they can choose workers with certain skills to fulfill this job combination $(H', L')$ by selecting a skill combination $(H, L)$ from the line between $Q$ and $H'$ in Figure 5 (cf. equation (1)).

We assume that entrepreneurs can determine the job layout of their production process (i.e. $(H', L')$) only when new capacity is installed. They will do so in such a way that the expected present value of the total operating cost of one labour
efficiency unit installed at time zero is minimised. Given a nominal rate of discount of \( \rho \), this present value declines with the progress of time at rate \( \delta + \rho \). Hence, it is given by:

\[
T = \int_{0}^{\infty} \left( w_{Ht}^e H_t + w_{L}^e L_t \right) \exp\left( -(\rho + \delta) t \right) dt
\]

(3)

where \( w_{Ht}^e \) and \( w_{L}^e \) are the expected values of the wage rate for high-skilled and for low-skilled people at time \( t \), respectively. For reasons of simplicity, we assume that wages can be expected to grow at constant rates, \( \dot{\bar{w}}_H \) and \( \dot{\bar{w}}_L \), respectively.

We now will approach the problem of the minimisation of (3) subject to (1) recursively in two steps, analogous to the two possibilities sketched above. That is, we will first look at the allocation of skills to jobs, given a certain job combination. Then we will look at how this job combination is determined.

The allocation of skills to jobs

For a given job combination, i.e. given values of \( L^* \) and \( H^* \), (3) describes the evolution over time of expected labour costs. In the absence of supply constraints and hiring and firing costs, there are no intertemporal connections between labour costs. Hence (3) can only be minimised when instantaneous labour costs are minimised at each moment in time, including the present, for a given initial job combination \((L^*, H^*)\). In terms of Figure 5, this comes down to the choice of skill combinations from the line between \( Q \) and \( H^* \).

It can easily be seen from Figure 5 that the skill combination \((H^*=H^*, L=0)\) is the solution when \( w_{Ht}^e/w_{L}^e < e^* \), while the combination \((L=L^*, \exp(-\delta t), H^*=H^*, \exp(-\delta t))\) is the solution when the opposite is the case, because in these cases instantaneous labour costs, and hence total expected labour costs over the infinite lifetime of the jobs \( L^* \) and \( H^* \) under consideration, will be minimised. Using the shorthand notation \( w_0 = w_{Ht}^e/w_{L}^e \) for time zero, i.e. the present time, and \( \dot{\bar{w}} = \bar{w}_H - \bar{w}_L \), for given and constant expected exponential growth rates \( \dot{\bar{w}}_H \) and \( \dot{\bar{w}}_L \), there are now four distinct possibilities to consider, depending on whether \( w_0 < e^* \) or \( w_0 > e^* \) and \( \dot{\bar{w}} < 0 \) or \( \dot{\bar{w}} > 0 \). These four cases are depicted in Figure 6.

In order to understand the difference between the four cases, one should realise that, for instance, we have \( w_0 < e^* \) and \( \dot{\bar{w}} < 0 \), the optimal solution will always be \((H^*=H^*, L=0)\). In this case, low-level jobs will be filled by high-skilled persons from the beginning. When \( w_0 < e^* \) and \( \dot{\bar{w}} > 0 \) there will be a moment when the other solution \((H^*=H^*, L=L^*)\) will be chosen. We assume the change to take place at time \( \tau^* \). A switch between states will also occur when \( w_0 > e^* \) and \( \dot{\bar{w}} < 0 \) which gives rise to the allocation of first \((H^*=H^*, L=L^*)\) and then \((H^*=H^*, L=0)\). The final case will always have \((H^*=H^*, L=L^*)\) since \( w_0 > e^* \) and \( \dot{\bar{w}} > 0 \).
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Figure 6  Expected wage growth and skill switching

Table 3
Optimum Job-skill combinations

<table>
<thead>
<tr>
<th>Case</th>
<th>wage/</th>
<th>growth rates</th>
<th>switching moment</th>
<th>(L_1,H_1),(L_2,H_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>efficiency ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>( w \leq e' )</td>
<td>( \dot{w} \leq 0 )</td>
<td>( t'(L,H)=0 )</td>
<td>( (0,H'),(0,H') )</td>
</tr>
<tr>
<td>2</td>
<td>( w \leq e' )</td>
<td>( \dot{w} &gt; 0 )</td>
<td>( t'(H,L)=\ln(e'/w_0)/\dot{w} )</td>
<td>( (0,H'),(L',H') )</td>
</tr>
<tr>
<td>3</td>
<td>( w &gt; e' )</td>
<td>( \dot{w} \leq 0 )</td>
<td>( t'(L,H)=\ln(e'/w_0)/\dot{w} )</td>
<td>( (L',H'),(0,H') )</td>
</tr>
<tr>
<td>4</td>
<td>( w &gt; e' )</td>
<td>( \dot{w} &gt; 0 )</td>
<td>( t'(H,L)=0 )</td>
<td>( (L',H'),(L',H') )</td>
</tr>
</tbody>
</table>
In two of the four cases a switch from the one corner solution to the other will take place at time \( t' \). Having determined \( t' \), the minimisation of the present value of expected labour costs over an infinite horizon can be defined as the solution to the problem of minimizing unit operating costs over two consecutive horizons, namely \( 0-t' \) and \( t'-\infty \). The solution for the optimal combination of skills, depending on the chosen job combination \((L', H')\), is summarised for each case in Table 3.

**The choice of jobs**

Given the results in Table 3, the question remains what the optimal job combination \((L', H')\) should be. Of course, this combination is chosen in such a way that equation (3) is minimised subject to equation (1), and given the results of Table 3. It can be shown that the solution in each case has the general form:

\[
T_i = \alpha_i H' + \beta_i L' 
\]  

(4)

\( T_i \) denotes the value of the objective function given by (3) for case \( i \) as described in Table 3. The coefficients \( \alpha_i \) and \( \beta_i \) differ for each case, but are always positive. The resulting values of \( \alpha \) and \( \beta \) are summarised in Appendix A.

Note that the objective function \( T_i \) is linear in \( L' \) and \( H' \). Hence, maximisation of \( T_i \) by choosing \((L', H')\) constrained by the unit job-isocost \( j/j' \) should lead to a straightforward determination of \( L' \) and \( H' \) in terms of \( e' \), relative wages, relative growth rates in wages and the parameters of the job-isocount. Since the \( \alpha \) 's and \( \beta \) 's are positive, the intertemporal cost minimisation problem has a meaningful solution in all four cases – cf. Appendix A. This solution is implicitly given by the first-order condition:

\[
\text{MRS}(L'_i, H'_i) = \frac{\alpha_i}{\beta_i} 
\]  

(5)

where \( L'_i \) and \( H'_i \) reflect the optimum values of \( L' \) and \( H' \) when case \( i \) is expected to hold, and \( \text{MRS} \) is the marginal rate of substitution. Note that equation (5) is totally comparable with the first-order conditions of a static cost minimisation problem, and hence we can conclude that a fall in the ratio \( \alpha_i/\beta_i \) corresponds to a decrease in the slope of the isocost lines and hence to a movement down the unit job isocount (see Figure 5) and therefore to a decrease in the \( L'/H' \) ratio. It is easy to show from the \( \alpha/\beta \) ratios that case 1 will have the lowest value of the \( L'/H' \) ratio, while case 3 will have a larger \( L'/H' \) ratio than case 4. Case 2, in turn, will have a larger \( L'/H' \) ratio than case 1.
Switching thresholds

So far, we have implicitly assumed that switching workers on a job is costless. It seems reasonable, however, to assume that the act of switching entails costs, i.e. the costs of hiring and firing. These may act as a break on switching behaviour. Moreover, if expectations regarding the development of relative wages are uncertain, an unexpected reversal in the growth rate of relative wages may cause entrepreneurs to incur switching costs twice – instead of not at all if only one had waited a while. Hence, it stands to reason to introduce thresholds in order to account for uncertainties in expectations in combination with (implicit) switching costs. However, only if entrepreneurs have already decided on a certain allocation of skills towards jobs, potential switching costs come to bear on the decision-making process: in the set-up we have chosen, switching costs are only incurred ex post. We assume that switching is always perceived as costly regardless of the direction of switching. We may implement this idea by defining ‘threshold wage rates’ at which a switch may actually be implemented. In case 2, for instance, in which a switch from \((H=H', L=0)\) to \((H=H', L=L')\) is expected to occur at time \(t'\), it stands to reason to switch only when the wage ratio has risen above the efficiency ratio by a certain percentage \(\Delta\) : \(w_r = e'(1 + \Delta)\). Similarly, case 3 may be redefined in terms of switching costs by assuming that a switch takes place only when the relative wage ratio falls below the efficiency ratio by a certain margin: \(w_r = e'(1 - \Delta)\).

Only cases 2 and 3 are affected by the introduction of switching thresholds, for only in these cases a switch is expected to be necessary in the future relative to the initial situation which is optimal at the ruling \(w_r/e'\) ratio. So, only \(\beta_2\) and \(\beta_3\) can change and it is fairly easy to show that for \(0 < \Delta < 1\), all \(\beta\)'s are positive again so that meaningful solutions to the intertemporal cost minimisation problem can be obtained, as is shown in van Zon and Muysken (1996).

The introduction of a switching threshold implies that, ceteris paribus, switching takes place at a later date. This implies too that the initial situation will have a greater weight in determining the present value of unit labour costs. This by itself would provide an incentive to change the job composition in a direction which makes the initial job composition more cost-effective relative to the ‘end composition’ after switching. This is elaborated in van Zon and Muysken (1996). With respect to case 3, similar conclusions can be drawn.

Threshold distributions

We now assume that entrepreneurs differ only with respect to their switching thresholds, and that these thresholds are uniformly distributed over the range \(0 - \Delta\). A high value of \(S\) signals a producer which is somewhat reluctant to switch, whereas a low value signals the opposite inclination. However, after a switch has taken place, we assume that the remaining entrepreneurs will become uniformly distributed over
the entire threshold range at their side of the \( e' \)-mark (see Figure 6), in order to avoid the occurrence of 'gaps' in the threshold distribution.  

Given these assumptions, the population \( \alpha \)'s and \( \beta \)'s can be obtained by aggregation over all individual entrepreneurs, leaving the \( \alpha \)'s unchanged, while only \( \beta_2 \) and \( \beta_3 \) change as indicated in Appendix A. The resulting parameters can be used to determine the demand for low- and high-level jobs by the entire population of entrepreneurs, while taking account of the impact of differences in risk aversion and perceived switching costs between entrepreneurs on the skill composition of employment.

The skill composition of employment

Our analysis now enables us to derive the development over time of the skill composition of employment. We define this composition by the ratio of employment by skill level over the number of jobs of the corresponding level. It is clear that at any point in time such a skill composition is essentially given by the initial skill composition and the effects of a series of skill switches (when applicable), between the present and the moment of the 'creation' of \( L^* \) and \( H^* \). Hence, both \( L/L^* \) and \( H/H^* \) are essentially defined in a time-recursive way.

The assumption that entrepreneurs are uniformly distributed with respect to their switching thresholds is useful when specifying these time-recursive relations, because this assumption enables us to subdivide the population of entrepreneurs at any time into a subset which will change the skill composition existing at a certain point in time, and a subset which will not. This essentially depends on the ratio \( w/e' \). More specifically, given the assumption of uniform distributions of switching thresholds, it follows that for a situation in which \( w/e' > 1 \) and \( w_w/w_{w-1} \), a fraction \( (w_w/e')/(1 + \Delta e' - e') = (w_w/e')/(\Delta e') \) of entrepreneurs would be satisfied with the state \((L=L^*, H=H^*)\), and hence the part of that fraction which had not yet switched from \((L=0, H=H')\) to \((L=L^*, H=H^*)\) would do so, while the part of that fraction which had already attained the state \((L=L^*, H=H^*)\) in the past, would be satisfied to stay in that state. Because of our assumption that the threshold distributions at both sides of the \( e' \)-mark remain uniform, individual entrepreneurs who are switching between states only change the densities of the distributions at both sides of the \( e' \)-mark rather than the distributions themselves. Moreover, as long as the wage rate is above the \( e' \)-mark, there is an incentive for the population in the state \((L=0, H=H')\) to switch to the other state.

Denoting the fraction of the population of entrepreneurs which is in the state \((L=L^*, H=H^*)\) at time \( t \) by \( \mu^* \), it follows that at any point in time:
\[
\mu^*_t = \mu^*_{t-1} + \frac{w_r - \bar{e'}}{\Delta \bar{e'}} (1 - \mu^*_{t-1}) \quad \text{if} \quad w_r/\bar{e}' > 1
\]
\[
\mu^*_t = \mu^*_{t-1} - \frac{e' - w_r}{\Delta \bar{e'}} \mu^*_{t-1} \quad \text{if} \quad w_r/\bar{e}' \leq 1
\]

The skill-composition of employment, as given by \(L/L^*\) and \(H/H^*\), can now readily be obtained by using (6):

\[
\frac{L_t}{L^*_t} = \mu^*_t
\]

\[
\frac{H_t}{H^*_t} = \frac{\mu^*_t H^*_t + (1 - \mu^*_t) H'_t}{H^*_t} = 1 + \frac{(1 - \mu^*_t)(L^*_t/\bar{e}')}{H^*_t}
\]

Equation (6) shows the importance of thresholds in determining lags in switching behaviour. The higher the average switching thresholds (\(\Delta\)) are, the lower the impact is of the current wage rate on switching and the higher will be the influence of the employment situation in the past. Note that a rise in \(\Delta\) would tend to defer the moment of switching, thus increasing the relative weight of the present value of the initial skill allocation in the objective function. Hence there should be an incentive to change the job composition in such a way that the costs of using those jobs with the initial skill allocation would become lower. And so, a change in average switching thresholds will have an impact on the job composition of employment too. Finally, equations (7) and (8) show the asymmetry in skill employment because \(H\) can never fall below \(H^*\), whereas \(L\) can decline to zero.

A multi-level putty-clay production model with substitution asymmetries

Introduction

As we have discussed above, the job composition of employment can only be altered by investing in new machinery and equipment. We should therefore include the investment decision in our analysis, in order to properly model the determinants of
the skill composition of employment. For this purpose we will use a putty-clay vintage model based on Bischoff (1971), which by its very nature introduces an asymmetry in time of substitution possibilities between capital and labour. It should be noted that this production model, which we will use to run a number of simulation experiments, is highly stylised: we have left out every detail which is not directly connected to the representation of production technology as such, since we are at this stage only interested in the principal working of asymmetries in substitution possibilities between different types of labour given the asymmetries in substitution possibilities in time (i.e. differences in substitution possibilities ex ante and ex post) between capital on the one hand and labour on the other. More in particular, we will use a linear homogeneous production function, where labour and capital are the only factors of production. With regard to technological change, we only consider purely labour augmenting technical change, possibly biased against low-skilled labour.

**Behavioural assumptions**

Apart from the production function features mentioned above, we assume that producers will try to minimise production costs while producing an amount of output which is exogenously given. Since we use a linear homogeneous production function, this implies that producers try to minimise average production costs. As we have explained in the previous Section, producers making investment decisions take into account expectations regarding price developments and the consequences for future (discounted) production costs of the choice of a production technique. However, we assume that producers do not consider the consequences of their current investment plans for the formulation and realisation of plans for the not too distant future, which in turn influence the realisation of plans still further in the future, and so on. The solution to such an optimisation problem would have to be obtained by means of a dynamic optimisation procedure which takes into account all future effects of the decisions entrepreneurs have to make today. This goes beyond the illustration purposes which the vintage model is intended to serve.

We also assume that the supply of physical capital is infinitely elastic at the ruling price of capital goods. Furthermore, entrepreneurs share the same expectations regarding the development of prices.

Note that in the previous Section we have assumed that labour is a composite factor (or a 'complex' of different types of labour) rather than a purely homogeneous factor. The 'price' of a unit of such a complex then depends on its composition, but for the purpose of defining the production framework it can be assumed as given for the moment.
The putty-clay vintage production model

For the purpose of finding the cost minimising factor proportions on the newest vintage, it should be noted that factor proportions ex post are assumed to be fixed. Hence we assume, very much as in the job composition problem described in the previous Section, that entrepreneurs try to minimise the expected present value of the total costs associated with buying and using a new vintage over an infinite horizon. We furthermore assume that the ex ante production function is a linearly homogeneous CES function and that only embodied labour augmenting technical change occurs. The production structure of the vintage model can then be described by:

\begin{equation}
1 = B_K (e^{\gamma T} v_{t,T})^{-\sigma} + B_K \kappa_{t,T}^{-\rho} \\
\kappa_{t,T} = \kappa_{t,T} \\
v_{t,T} = v_{t,T} \\
I_{t,T} = (1 - \delta)^{T-t} I_{T,T}
\end{equation}

where \( \sigma = 1/(1 + \rho) \) is the elasticity of substitution between capital and labour (i.e. jobs), \( B_K \) and \( B_K \) are distribution parameters and \( \gamma \) represents the rate of embodied labour augmenting technical change. \( v_{t,T} \) and \( \kappa_{t,T} \) represent the labour/output ratio and the capital/output ratio at time \( t \) of the vintage installed at time \( T \), respectively.\(^{25}\) Minimisation of the expected present value of the total cost per (initial) unit of output on the newest vintage over an infinite horizon then yields the optimum factor proportions on the newest vintage as a function of factor prices.\(^{27}\) In the absence of disembodied technical change, total capacity output \( X_t \) as well as total capacity labour demand \( N_t \) can now be obtained from:

\begin{equation}
X_t = \frac{I_{t,t}}{\kappa_{t,t}} + (1 - \delta) X_{t-1} \\
N_t = \frac{v_{t,t}}{\kappa_{t,t}} I_{t,t} + (1 - \delta) N_{t-1}
\end{equation}

The vintage model now works as follows. Output is exogenously determined, as are wages. Together with prices, the latter determine optimum factor proportions on the newest vintage and therefore unit total cost on the newest vintage. Given the volume of capacity output associated with ‘old’ equipment which has not been completely worn down yet, this determines how ‘large’ the newest vintage should be. Total capacity labour demand then follows as the capacity labour demand associated
with the existing capital stock and capacity labour demand associated with the new vintage.

*Adding heterogeneous labour to the production function framework*

In the previous Section we have postulated that employment is the result of a match between the demand for labour as reflected by the number of jobs on the one hand and the supply of labour in terms of skills on the other. We now assume that capacity labour demand within the context of the vintage model, refers to labour demand in terms of high-level and low-level jobs, which are combined into one job complex. That is, \( \nu_p \), in the ex ante production function (9) represents the labour/output ratio in terms of job efficiency units per unit of output: it comprises both low-level and high-level jobs.

In order to model substitution possibilities between low-level and high-level jobs within the job complex, we have taken a linear homogeneous job-CES-function, with an elasticity of substitution equal to 0.5. With regard to substitution between capital on the one hand, and jobs on the other, we nested the job-CES-function in another linear homogeneous CES function with elasticity of substitution equal to 0.25.

In the absence of skill supply constraints, the optimum composition of the labour complex in terms of jobs and the allocation of skills to these jobs defines the expected present value of the total cost associated with using a job efficiency unit over an infinite horizon. Hence, this 'present value' price of a job efficiency unit serves the same function as the 'present value wage rate' in equations (4) and (5).

With regard to allocating skills to jobs, a distinction should be made between existing capacity and new capacity. The reason is that, although ex post substitution is possible by assumption, switching costs may prevent this. By assumption the average employment state on old capacity is a weighted average of the states \( (L=L', H=H') \) and \( (L=0, H=H') \) with weights \( \mu' \) and \( (1-\mu') \), respectively. New equipment starts out either in the state \( (L=L', H=H') \) or in the state \( (L=0, H=H') \), depending on whether \( w_{\theta} \leq e' \) or \( w_{\theta} > e' \). Obviously, unit variable cost on the old vintages depends on this 'mix' of states too.

*Adding wage and price responses*

In standard macro-economic models, wage formation is influenced by labour productivity, inflationary expectations and labour market conditions. Usually an increase in the rate of unemployment decreases the rate of growth of wages, which in turn increases the demand for labour, ceteris paribus, which then lowers unemployment again. In our case direct substitution between \( L \) and \( H \) will occur as long as \( w_{\theta} < e' \), i.e. as long as the state which entrepreneurs are leaving has a cost disadvantage relative to the 'target' state.
For illustrative purposes we have specified a very simple wage adjustment equation consisting of a constant trend (equal to zero in most experiments) and a Phillips effect which is linear in the unemployment rate. We have added a uniformly distributed random component to the growth of wages in the range from -1 to 1 per cent. Let \( \hat{w}_i \) refer to the value of the growth rate of the wage rate associated with skill \( i \), then:

\[
\hat{w}_i = \hat{\tau}_i - \theta_i \mu_i + \hat{\varepsilon}_i
\]  

(11)

where \( \hat{\tau}_i \) refers to the constant trend term, and \( \hat{\varepsilon}_i \) refers to the random component in the rate of growth of relative wages. Assuming that the trend growth of the wage rates of both skills are the same, it follows from this specification that the ratio of the steady-state unemployment rates by skill is defined in terms of the relative strength of the Phillips effect. More in particular, a stable ratio of \( w_H/w_L \) for a given supply of labour implies that:

\[
\bar{u}_L = \frac{\hat{\tau}_L}{\theta_L} \frac{\hat{\tau}_H}{\theta_H} \bar{u}_H
\]  

(12)

where \( \bar{u}_i \) denotes the steady-state rate of unemployment of skill \( i \). Note that this equation implies that for symmetric trend growth of wages and symmetric speeds of adjustment of wage growth to unemployment, the steady-state unemployment rates for low-skilled and high-skilled workers are identical. However, if the symmetry were broken by, for instance, a decreasing low-skilled trend growth of wages for a given supply of low- and high-skilled workers, this would lead to an immediate disparity in the long-term rates of unemployment of low-skilled and high-skilled workers.

With regard to the price of output, we assume that it is equal to unit total cost on the newest vintage. The price of investment is assumed to be equal to the price of output in turn.

*Supply constraints and the chimney effect*

In order to be able to see the 'chimney effect' at work, it is necessary to define explicit supply constraints with respect to low- and high-skilled labour. Moreover, supply must be distributed over the various entrepreneurs who are in either of the two states associated with 'old jobs'. This is because one should realise that the skill composition of employment derived in the previous Section (cf. equations (7) and
(8)), essentially holds only for the old jobs. Hence the preferred skill for old low-level jobs may be either \( H \) or \( L \), which are different by definition for the two groups of entrepreneurs. With respect to the latter, we assume that the distribution of supply by skill level over these two groups is proportional to the group sizes. That is, when total supply of low and high skills is \( L' \) and \( H' \), respectively, the relevant supply for the group preferring low-skilled labour on low-level jobs is \( \mu^*L' \) and \( \mu^*H' \), while the relevant supply for the group preferring high-skilled labour on low-level jobs is \((1-\mu^*)L' \) and \((1-\mu^*)H' \).

With regard to 'new jobs', we have assumed that all entrepreneurs face the same hiring costs.\(^{32}\) Hence, all entrepreneurs will have the same 'preferred' allocation with respect to new low-level jobs, which again may be either \( L \) or \( H \). However, the preferred allocation may not be feasible due to the possible existence of supply constraints, i.e. supply of the 'preferred skill' may fall short of demand, in which case we assume that second best allocations will be made. As a consequence, we can distinguish between four different situations, which cannot occur simultaneously, however. The first two situations, \( A \) and \( B \), occur when the preferred skill for new low-level jobs is \( L \). In situation \( A \) the preferred skill for old low-level jobs is \( L \) too, while in situation \( B \) the preferred skill is \( H \). The next two situations, \( C \) and \( D \), occur when the preferred skill for new low-level jobs is \( H \). The difference between \( C \) and \( D \) lies in the preferred skill for old low-level jobs which is \( L \) and \( H \), respectively. Hence, denoting the skill combination which is preferred for old and new jobs by \((\text{Old}, \text{New})\), where both Old and New can be \( L \) and \( H \), it follows that the situations \( A-D \) are given by \( A=(L,L), B=(H,L), C=(L,H) \) and \( D=(H,H) \).

The influence of supply constraints on the allocation process can now be introduced in a straightforward manner. First it should be noted that after installation of each vintage, the number of jobs on that vintage is fixed except for technical decay, which affects both types of jobs to the same extent. Secondly, because of the job complementarity ex post and because high-level jobs can only be filled by high-skilled workers, it follows that the maximum degree of 'high-level-job-utilisation' (i.e. \( q_H \)) which will be realised for a given supply of high-skilled labour \( H' \), is equal to:

\[
q_H^* = \min \left( 1, \frac{H'}{H^*} \right)
\]  

(13)

where \( H^* \) denotes the available supply of high-skilled workers. Obviously, if supply is larger than needed to fill \( H' \) jobs, high-skilled workers may be used to fill \( L' \) jobs as well in combination with the available supply of low-skilled workers. The maximum degree of low-level job utilisation is then equal to:
If \( q_{L} < q_{H} \) then, because of the complementarity effect between low- and high-level jobs, the maximum overall degree of job-utilisation is between \( q_{L} \) and \( q_{H} \), and this level can be reached by reallocating some of the \( H \) workers who were initially allocated to \( H \) jobs to \( L \) jobs instead. The latter leads to a fall in \( q_{H} \) and a rise in \( q_{L} \), in accordance with equation (14) above. This reallocation should continue until \( q_{L} = q_{H} \), which, when substituted into (14), gives rise to the overall job-utilisation rate \( q^* \) as given by:

\[
q^* = \frac{(H^* - q_{H} \cdot H^*) e^* + L^*}{L^*}
\] (14)

The allocation should now be such that first-best allocations are realised to the largest extent possible, whereas second-best allocations are used to fill the gaps in the first-best allocations resulting from the existence of supply constraints. Consider for instance the group A entrepreneurs, i.e. \( A=(L,L) \). Denoting the amount of skills \( S=L,H \) allocated to jobs \( J=L^*,H^* \) by \( S_J \), it follows that for group A entrepreneurs:

\[
LL^* = \min(q^* L^*, \mu L^*)
\]

\[
HL^* = \min \left( \frac{q^* L^* - LL^*}{e^*}, \mu H^* - HH^* \right)
\] (16)

Something similar holds for group D entrepreneurs, who prefer high-skilled workers on both old and new low-level jobs. By contrast, group \( B=(H,L) \) and group \( C=(L,H) \) entrepreneurs have different preferences for old and new jobs. Group B entrepreneurs first fill new jobs with low-skilled workers, and use further low-skilled workers only when low-level jobs cannot be filled by the preferred high-level skill. The opposite holds for group C entrepreneurs.\(^{34}\)
Simulation results

Introduction

In this Section we will present the results of some simulation experiments in order to illustrate the working of our model and to emphasise some interactions which might be relevant from a policy point of view. The experiments are:

1. a decrease in the barriers to firing (lower switching thresholds);
2. lowering the relative wage of low-skilled workers;
3. a lower demand for high-level jobs relative to low-level jobs; and
4. a higher demand for high-level jobs.

These experiments will be discussed after we have introduced the simulation set-up and presented the base run.

The simulation set-up

In this Section we will present the results of the simulations we ran with the putty-clay vintage model. To this end we have defined a very simple general framework with output growing at a fixed rate of 2.5 per cent per year. This also holds for the supply of both low- and high-skilled workers. We assume that at zero rates of unemployment, nominal wage rates grow with an average rate of 2 per cent per year with random fluctuations of 0.5 per cent on average. The sensitivity of the growth of wages with respect to the rate of unemployment by skill has been set to 0.25, which is roughly in line with the findings by van Zon et al. (1995) for the Netherlands. The interest rate is assumed to be equal to the nominal rate of discount $\rho$, which in turn is equal to 10 per cent. Technical decay has been set at 5 per cent. The elasticity of substitution between jobs and capital is 0.25, and 0.5 between low- and high-level jobs, as has been elaborated above. The price of output (which is equal to the price of capital goods by assumption) is set equal to marginal production cost. The parameter $e'$ has been set at 1.25, which is in line with the wage ratio for high-skilled and low-skilled workers given in Nickell and Bell (1995). Hence, at this particular wage ratio entrepreneurs are assumed to be indifferent between hiring either low-skilled workers or high-skilled workers on low-level jobs. Finally, the threshold value of $\tilde{\Delta}$ has been set at 0.75.

Starting values for the endogenous and exogenous variables were chosen in such a way that the distribution of jobs is 35 per cent in low-level jobs and 65 per cent in high-level jobs (in the sense of not low). This is an exaggeration of the actual distribution (compare for instance Figure 4 for the Dutch situation), but is used for illustrative purposes only.
In order to exclude the influence of the choice of initial values for the capital stock on the outcomes of the model, we let it run for 225 years and then applied a shock in accordance with the experiment under consideration. The simulation experiment always ends in year 300, although the shock is ended after 50 years, i.e. in year 275.

*The base run*

In Figures 7-9 below, we present the outcomes of the base run. The average rate of unemployment for both types of skill \( u_l \) and \( u_h \), respectively, is about 7 per cent as can be seen from Figure 7. We see that the overall rate of unemployment \( u \) is

![Graph showing unemployment and ratio of high/low-skilled wages over time.](image)

**Figure 7** Unemployment of high- and low-skilled in base run  
**Figure 8** Ratio high/low-skilled wages in base run

more stable than \( u_h \) and \( u_l \), while in turn the fluctuations in \( u_l \) are roughly twice as large as those in \( u_h \). Moreover the fluctuations in \( u_h \) and \( u_l \) are almost perfectly negatively correlated. The reason is that changes in employment are caused primarily by entrepreneurs switching between states rather than changing the job composition on the newest vintages. And since \( H^* \) (and \( L^* \) employment) is about twice as large as \( L^* \) (and \( L \) employment), replacement of a low-skilled worker on a low-level job by a high-skilled worker leads to a fall in the rate of high-skilled unemployment which is about half the corresponding rise in the rate of low-skilled unemployment. Note
that on average both rates of unemployment are equal. This follows directly from equation (12), since we have \( t_{11} = t_{21} = 2.5 \) and \( \theta_{11} = \theta_{21} = 0.25 \), as mentioned above.

![Graph showing percentage of low-skilled workers in low-skilled jobs over time.]

**Figure 9  Fraction low-skilled workers in low-skilled jobs in base-run**

The ratio of high-skilled wages and low-skilled wages, \( w_{o} \), is on average equal to 1.25 as is shown in Figure 8. Fluctuations in this ratio are caused by random shocks on the one hand and fluctuations in \( u_{H} / u_{L} \) on the other. The share of employment in ‘old jobs’ in the state \( (L=L', H=H') \) is given by \( \mu^{*} \) which is depicted in Figure 9. \( \mu^{*} \) fluctuates around 0.75 for most of the period 200-300 except for a 10 per cent drop of \( \mu^{*} \) during the last 20 years of the simulation period. Note that there is a positive correlation between fluctuations in \( \mu^{*} \) and fluctuations in \( w_{o} \), although the former fluctuations are less volatile, due to the partial adjustment of \( \mu^{*} \) to deviations between \( w_{o} \) and \( e^{*} \) (cf. equation (6) above).
Figure 10 Changes in unemployment of high- and low-skilled with diminishing barriers to firing

Figure 11 Changes in employment of high- and low-skilled jobs with diminishing barriers to firing

**Diminishing barriers to firing**

In experiment 1, we have changed the threshold value of $\bar{A}$ from its base-run value of 0.75 to a value of 0.375. One would expect a fall in switching thresholds to increase the intensity of the flows of labour from the one state into the other. Hence, the more flexible allocation of skills to jobs would tend to lower the effective user cost of a job-efficiency unit, and would therefore increase the labour/capital ratio. However, only the job composition on new vintages may profit from less costly reallocations, and hence the effects on the creation of new jobs are only marginally positive, as can be seen from Figure 11. Because of that, the overall employment rate is hardly affected at all (see Figure 10), while changes in unemployment rates by skill are far less outspoken than in the wage experiment. Moreover, these changes vary over time, sometimes increasing unemployment vis-à-vis the base run and sometimes decreasing unemployment. Hence, we conclude that a reduction in the cost of firing does not seem to have any net effects for particular groups of workers in the longer run.
Lowering the growth of low-skilled wages

In experiment 2, the trend growth of low-skilled wages is lowered from 2.5 per cent by 1 per cent, while that of high-skilled wages remains at 2.5 per cent. This reduction in trend wage growth leads to a decrease of the steady-state equilibrium rate of low-skilled unemployment by about 4 percentage points (i.e. the reduction in trend growth divided by the unemployment elasticity of wages, c.f. equation (12)). Although we do not show this here, the results are very similar to the ones obtained in an experiment with a higher wage flexibility where we doubled the speed of adjustment of low-skilled wages to unemployment conditions. However, this similarity does not result from the specification of the model, but is a result of the chosen numerical values.

![Diagram](image)

**Figure 12** Changes in unemployment of high- and low-skilled with lower growth of low-skilled wages

**Figure 13** Changes in employment of high- and low-skilled with lower growth of low-skilled wages

The background of the similarity of the results of both experiments is highlighted in Figures 14 and 15 which depict deviations of $\mu^*$ from its base-run value and the percentage deviations of $w_m$ respectively. From Figure 14 one sees that fluctuations in $\mu^*$ hardly occur in experiment 1, as might be expected, whereas $\mu^*$ rises by roughly
10 percentage points in experiment 2. This is a reflection of what happens to the relative wages in experiments 1 and 2 as is shown in Figure 15.

Figure 14 Comparison of the fraction of high-skilled workers in high-skill jobs in two scenarios

Figure 15 Comparison of the change in the relative wage rate in two scenarios

The relative wage rate, \( w_0 \), rises rapidly in a few years by about 5 per cent compared to the base run. After this year \( w_0 \) falls to its original value in roughly the same amount of time. Note that this leads the timing of the peaks in the changes in the unemployment rates, which reach their largest deviations in about period 240, when \( w_0 \) has more or less returned to its original level. But although changes in relative wages have led to fairly large changes in \( \mu^* \), one can observe again that there is hardly any net job creation and hence the overall employment effects are very limited indeed – although low-skill workers will outcompete high-skilled workers on low-level jobs, for a while at least. This is apparent from Figure 12, where the overall rate of unemployment remains relatively unaffected, whereas changes in unemployment for the individual classes of low- and high-skilled workers are far more outspoken. The same picture emerges from Figure 13, where the job composition of employment changes only slightly in favour of low-level jobs. Actual employment, however, changes much more in favour of low-skilled workers, but at the expense of employment opportunities for the high-skilled.
The overall conclusion to be drawn is then that a change in the growth rate of low-skilled wages does not change the job composition of employment to a large extent, but affects the allocation of skills to jobs instead. More in particular, low-skilled workers are used more intensively in low-level jobs, and high-skilled workers who had displaced them become unemployed in turn. This is a consequence of the relatively low employment share of low-skilled workers, which causes induced substitution effects between capital and labour to be of minor importance from an employment perspective. This is aggravated by the fact that the job composition of employment is for a large part embodied in already existing machinery and equipment, which causes ‘on-the-job substitution’ to be more relevant in the short and medium run than either substitution between jobs or substitution between capital and jobs.

*Experiments concerning the working of the chimney effect*

In order to simulate the working of the chimney effect, we have introduced a bias in job augmenting technical change, thus changing the job composition of the demand for labour. In experiment 3 we have assumed a 1 per cent high-level job saving rate of technical change and a 1 per cent low-level job using technical change. Hence, fewer high-level and more low-level jobs are needed to generate one job efficiency unit, and it is obvious that the job composition of labour demand shifts in favour of low-level jobs. Since the demand for high-skilled workers on account of high-level jobs is now diminished for a given supply of high-skilled workers, relatively more high-skilled workers will be available for low-level jobs. In this case, the creation of low-level jobs does not necessarily have to lead to an increase in low-skilled employment. Rather, the ‘excess availability’ of high-skilled workers may reduce the employment opportunities for low-skilled workers.

By contrast, experiment 4 assumes only a 1 per cent high-level job using technical change, thus creating the conditions for the chimney effect to occur. Now a given supply of high-skilled labour will be confronted with an ever growing number of high-level jobs, thus reducing the amount of high-skilled labour available for employment in low-level jobs.

Figure 16 shows the principal difference between the two experiments. In experiment 3, relative wages fluctuate around their base-run values, where the amplitude of the cycles increases during the experimental period 225-275 and decreases again after the experiment has ended. The reason is that the 1 per cent rate of high-job saving technical change increases the availability of high-skilled workers for low-level jobs. Moreover, the initial increase in \( u_H \) decreases \( w_h \), but \( w_l \) does not decrease enough to fully counter the increase in \( u_H \). Also \( u_L \) starts to rise, which depresses low-skilled wages and hence increases \( w_l \) again. The main effect is that changes in \( u_L \) and \( u_H \) fluctuate around an upward trend in the overall rate of unemployment, as can be seen from Figure 17. The reason for this upward trend is
that the additional availability of high-skilled workers makes them replace low-skilled workers in low-level jobs. However, the increase in the number of low-level jobs, as shown in Figure 18, still does not favour employment of low-skilled workers because high-skilled workers take over their jobs.

![Graph showing changes in demand for low-skilled jobs and unemployment](image1)

**Figure 16** Comparison of change in relative wages in two scenarios

![Graph showing changes in unemployment](image2)

**Figure 17** Change in unemployment of high- and low-skilled due to a shift in demand towards low-skilled jobs

The bias in technical change is illustrated quite clearly in Figure 18, where low-level jobs increase and high-level jobs decrease. Nonetheless, both high-skilled employment and low-skilled employment decrease (with fluctuations around this decreasing trend). This conclusion is corroborated by inspection of Figure 19, where \( q_t \) exceeds a value of 1, while \( q^* \) is equal to 1, indicating that there is no quantitative shortage of supply of both high-skilled and low-skilled workers. Hence, experiment 3 shows that the effectiveness of the creation of low-level jobs in alleviating low-level unemployment is negatively affected by the existence of an excess supply of high-skilled workers.
By contrast, experiment 4 shows that the introduction of high-level job using technical change, leading to the creation of additional high-level jobs, may be much more effective in creating employment opportunities for low-skilled workers than the creation of low-level jobs did in the context of experiment 3.

![Graph 18](image1.png)

**Figure 18** Change in employment due to shift in demand towards low-skilled jobs

![Graph 19](image2.png)

**Figure 19** Utilization of high- and low-skill jobs in experiment 3 which shifts demand towards low-skilled jobs

Figure 21 shows that the introduction of high-level job using technical change does not only increase the number of high-level jobs, but also the number of low-level jobs. The latter is caused by the phenomenon of substitution of low-level jobs for high-level jobs, which is induced by the increase in high-skilled wages following the reduction of high-skilled unemployment. Nonetheless, low-skilled employment is only positively affected for part of the experimental period, i.e. from 225-245 as indicated by the fall in $q_e$, which was the maximum rate of low-level job utilisation attainable for a given supply of low-skilled workers and supply of high-skilled
workers available for employment on low-level jobs. Hence the increase in demand for high-skilled workers due to the increase in high-level jobs, extends employment opportunities for low-skilled workers. From Figure 20 one can see that this leads to a drop in the rates of unemployment for both skills until year 245. In 245 something interesting happens: $q_L$ falls below a value of 1. This means that the supply of labour available for employment in low-level jobs is insufficient to meet the demand. Hence, in the period between 245 and 265, we have a situation of a shortage of supply of low-skilled workers. However, from period 265 $q_H$ falls below $q_L$, which indicates that from period 265 the supply constraint with respect to high-skilled labour becomes more binding (i.e. high-skilled labour becomes relatively scarce again), and we see the rate of unemployment of low-skilled labour rising again from period 265. The overall unemployment rate rises too, but falls again after resetting the rate of high-level job using technical change in period 275. The latter would tend to alleviate the bottleneck character of the supply of high-skilled workers from period 265, thus allowing employment for low-skilled workers to increase too.
The main conclusions of experiment 4 are therefore firstly that the creation of high-level jobs may favour employment opportunities for low-skilled workers, and secondly that the occurrence of bottlenecks in the supply of high-skilled workers may also diminish employment prospects for low-skilled workers. Both conclusions point to the paradox of the need to monitor the high-skilled side of the labour market, especially in times of low-skilled unemployment.

![Figure 22 Utilization of high- and low-skilled jobs in the experiment concerning the chimney-effect](image)

**Summary and conclusion**

In this Chapter we have presented the outlines of a labour allocation model which focuses on asymmetries in substitution possibilities between different types of labour. The general idea underlying this model is that high-skilled people can take over the
jobs of low-skilled people but not necessarily the other way around. This implies that the number of low-level jobs is an upper limit to low-skilled employment, while the number of high-level jobs is a lower limit to high-skilled employment. We assume that a high-skilled person has a non-zero efficiency on a low-level job, whereas a low-skilled person has zero efficiency on a high-level job, which is an exaggeration of the actual immobility of skills between jobs.

We have assumed forward-looking behaviour for the firm. Entrepreneurs determine the job composition of employment, as well as the capital intensity of production, based on the expected growth of relative wages. Moreover, in deciding about the job composition of labour demand, they take account of future changes in the allocation of skills to jobs—when relative wages and relative efficiencies indicate that this would be the most profitable thing to do. However, we also introduced switching thresholds in order to model adjustment costs and uncertainty, and assumed that entrepreneurs are heterogeneous with respect to these thresholds. By using the assumption of a uniform distribution of these thresholds, we have been able to derive a simple skill allocation mechanism. This mechanism reacts to wage formation and to average threshold sizes, given the job composition of employment. However, the latter also depends on the average size of thresholds, although in a far more complex way than the skill allocation process itself.

Apart from specifying the model, we have also looked at labour force survey data for the Netherlands available from Eurostat. We found that the substitution asymmetries between high-skilled and low-skilled people are far less outspoken in the data than in our model. Since we measure skill levels as the educational level attained, this seems to point to on-the-job learning as a possibility to cross skill boundaries. The Eurostat data allow us to check this surmise in a tentative way. We noticed that the mobility of low-skilled people between low-skilled and medium-level jobs is considerable indeed. This is also the case with medium-skilled people between medium- and high-level jobs, which in our view stresses the necessity of defining skills in human capital terms rather than the last level of schooling attained. It is obvious that this requires a more transparent labour market, that is, one cannot solely rely on diplomas as an indicator of the relevant skills. Finally, the data did show us that the problem of unemployment of low-skilled people in the Netherlands is severe— as it is in most West-European countries.

A typical solution to the problem of the uneven distribution of unemployment across skills is to rely heavily on wage flexibility to alleviate this problem. However, we have run some simulations with our model using plausible parameter values—although output growth is assumed to be exogenous. One result is that asymmetries in substitution possibilities between low-skilled and high-skilled people imply that the employment response in terms of a change in the job composition in favour of low-level jobs may require quite a fall in low-skilled wages. But even then the largest impact on low-skilled unemployment is to be expected from diminishing the incentives for having high-skilled people replace low-skilled people on low-level
jobs. Another result concerns the removal of barriers to firing. It turns out that this may not lead to any significant net benefits for a particular skill group. Rather, fluctuations in unemployment rates become more outspoken due to more frequent and more intensive direct substitution of low-skilled and high-skilled labour on low-level jobs.

Finally we presented a simulation experiment to illustrate the working of the so-called chimney effect. It was shown that the creation of low-level jobs in a situation of excess supply of high-skilled workers may not be an effective way of creating employment opportunities for low-skilled workers. Instead, the creation of high-level jobs which require reallocations of high-skilled workers from low-level jobs to high-level jobs may be much more effective in generating effective job openings for low-skilled workers than the direct creation of low-level jobs would be (this is the chimney effect). Moreover the creation of high-level jobs may lead to the creation of low-level jobs as a 'side-effect', which would essentially increase the effectiveness of the creation of high-level jobs in alleviating low-skilled unemployment problems.

These last conclusions point to the paradox of the need to monitor the high-skilled side of the labour market, especially in times of low-skilled unemployment. That is, transparency of the labour market is an important element – not just in identifying relevant human capital, as mentioned above – in the combat against low-skilled unemployment.

Notes

1 For the Netherlands this is recognised in, for instance, CPB (1996), pp. 77-79, and CPB (1996), Ch. 6.
2 In Nickell and Bell (1995, p. 51), the ratio of high-skilled wages and low-skilled wages for the US and the Netherlands are 1.51 and 1.22 respectively, which implies a 20 per cent drop in low-skilled wages in the Netherlands, if the ratio of high-skilled wages and low-skilled wages in the US was considered to be the benchmark.
3 A more common objection is of course that an increase of labour market flexibility insofar as such would lead to lower levels of private consumption, may be less effective in promoting employment for the low-skilled unemployed than expected.
4 Examples of such studies are Hamermesh and Grant (1979), Broer and Jansen (1989) and Hebbink (1990).
5 Cf. also van den Rijen (1996). Interestingly enough, there is a magnification effect involved in the creation of high-level jobs with respect to low-level employment opportunities when high-skilled workers would be more efficient in low-level jobs than low-skilled workers, since the creation of one high-level
job filled by a high-skilled worker who previously performed low-level tasks would offer employment opportunities for more than one low-skilled worker. Another possible 'magnification' effect could be present when productivity increases are indeed connected with the use of human capital, as suggested by Lucas (1986) and Romer (1990), for instance. The creation of an additional high-level job may then lead to overall efficiency increases, which may in turn favour employment in general and hence employment for the low-skilled too.

A similar, somewhat more elaborate analysis has also been performed for Germany, 1988-1994, in van Zon and Muysken (1996). In particular, more attention has been paid to the sectoral differentiation and the differentiation between types of jobs. It turned out that these dimensions were not very relevant for the analysis of the match between skills and occupations. Since the same holds for the Netherlands – cf. also Draper and Manders (1996), Appendix 1 – we ignore these dimensions below.

We have used Eurostat's Labour Force Survey data for employment and unemployment. The data are available by country (European Union), by sector of industry, occupation (ISCO 68/88), age and highest level of education attained. The latter dimension corresponds to the ISCED educational classification. (Further information on the Dutch educational system is provided in the Appendix to this book.)

The data have been provided only for the period 1988-1994, because Eurostat did not perform an educational desaggregation of (un-)employment data for earlier years. In 1992, two major changes occurred in the classification of the data. First, the classification of sectors of industry changed from NACE-CLIO to NACE-Revision 1. Second, the classification of occupations changed from ISCO 68 to ISCO 88.

In the empirical version of this model we distinguish between three different types of occupations: administrative-commercial, personal services and technical, however.

Similar observations can be inferred from Figure III.5.1 from CPB (1996), p. 77.

The observation that more higher-skilled workers than low-skilled workers are employed in low-level jobs is also made in CPB (1996), Figure III.5.2, p. 78. This is also seen as evidence for the 'chimney effect', mentioned in the first Section above. Cf. SoZaWe (1996), pp. 57-58.

A comparison with 21.0 years of average experience in low-level jobs not useful, since working experience of low-skilled people in low-level jobs may be quite irrelevant for the formation of human capital, when compared to working experience in medium- and high-level jobs.

This is also an explanation for the low participation rate of low-skilled persons, since most of the males in this category are either early retired (VUT) or disabled (WAO) – cf. SoZaWe (1996), p.59.
Many writers have investigated the possibility of direct substitution between skill-categories. A survey for the US is provided by Hamermesh and Grant (1979), while Hebbink (1990) and Broer and Jansen (1989) provide some results for the Netherlands. Kugler et al. (1990) do the same for Germany. Mincer (1989) provides additional results for the US. The general conclusions which emerge from these studies are firstly that capital and high-skilled labour complements, while secondly low-skilled labour and the capital/high-skilled labour complex are substitutes.

From our discussion in the second Section it is obvious that skills should be interpreted in a broad sense, including such aspects as experience.

Note that due to Hicks-neutral disembodied labour augmenting technical change, the effective discount rate of job operating costs rises with the value of the rate of disembodied technical progress. However, for the moment we disregard the influence of technical change.

Since high-level jobs always need to be filled by high-skilled workers, the four distinct possibilities can actually be defined in terms of the allocation of skills to low-level jobs only.

\((L_n, H)\) is the combination of \(L\) and \(H\) during sub-period \(i\). \(i=1,2\) denotes the period before and after switching, respectively.

\(t^* = t(x^*, y)\) defines the moment in time when a switch from type \(x\) employment to type \(y\) employment should be realised. Note that a switch with \(t'(L, H) = 0\) implies that entrepreneurs would expect to allocate high-skilled workers to low-level jobs from the beginning.

Although this has not been indicated in Figure 6, \(t^*\) can of course be different for these cases. \(t^*\) itself is easily determined from the condition that: \(w^* = w_{L^*}/w_{L^*}\) \(\exp{(\hat{\psi}_H - \hat{\psi}_L)}\). \(t^* = e^*\). Hence: \(t^* = \ln(e^*/\hat{\psi})\).

This assumes, of course, that there are no initial switching costs, which means that these costs should be interpreted in terms of the costs of firing rather than hiring. Note however, that when switching costs are defined in terms of on-the-job learning, switching costs may be incurred at each moment in time when a certain allocation of skills to jobs has to be made. This means that asymmetries in learning potentials/speeds between skills now determine how large the sunk costs will be for a certain allocation, and hence how these costs should actually be sunk. Indeed, for a simple implementation of costs in a switching setting, it is easy to show that the case of switching thresholds is but a particular case of a more general (but still simple) learning model (see van Zon and Muysken (1996), and van Zon, Meijers and Muysken (1997)). For reasons of simplicity we stick to the simple threshold case in this model, however.

i.e. switch at time \(t^*\) as it is implicitly defined by \(w^* = e^*(1+\Delta)\), instead of at \(t\) as given by \(e^* = w^*\). Cf. also note 19.
Note that it is not absolutely necessary for the switching thresholds to be symmetric around $e^*$. Rather, the $A$'s could be different, indicating asymmetries in the costs of hiring and firing, depending on which skill is hired and which is fired. We will not pursue this potential asymmetry here, however. See van Zon, Meijsers and Muysken (1997) on this subject.

These gaps would make it impossible to describe aggregate behaviour without tracking the behaviour of individual entrepreneurs. Obviously, we want to avoid the latter.

Note that substitution of equation (7) into (8) still implies that $H_i = H_i^* + (L_i^* - L_i)/e^*$, which follows from the assumption that high-skilled people can replace low-skilled people, but not vice versa.

See Appendix D of van Zon and Muysken (1996).

The Bischoff approach disregards economic obsolescence.

The expected present value of the total cost per (initial) unit of output on the newest vintage over an infinite horizon ($\Lambda_T$) is equal to:

$$\Lambda_T = \frac{w_T}{\rho + \delta - \tilde{w}} \cdot v_{T,T} + q_T \cdot \kappa_{T,T}$$

where $q_T$ represents the price of investment at time $t$, $w_T$ the wage rate and $\tilde{w}$ the growth rate of wages. See van Zon (1990) for a derivation of this result. $v_{T,T}$ and $\kappa_{T,T}$ can now be obtained by minimising $\Lambda_T$ conditional on the ex ante production function as given in equation (9).

This function is consistent with the isoquant $f_j$ in Figure 5. The value of the elasticity of substitution is consistent with that found in Hamermesh en Grant (1979) and Hebbink (1990). Note that we do not assume here that high-level jobs are complementary with capital. Rather, the job complex is a substitute for capital, while low-level and high-level jobs are substitutes within the job complex. The reason for doing so is that substitution reactions between high-skilled and low-skilled workers in response to changes in wage rates et cetera, should become more pronounced than if high-level jobs were complementary with capital.

The latter elasticity of substitution is the one found for the Netherlands by Kuipers and van Zon (1982) and Muysken and van Zon (1987), for instance.

The present value price of investment is equal to the price index of investment (see van Zon (1990) for a description of the assumptions underlying this result), while the present value price of labour is the initial value of the wage rate divided by the sum of the rate of discount and the rate of technical decay less the growth rate of wages.

Equation (12) follows from (11) and the assumption that $w_T/\omega_T$ is constant in the steady state.

See note 20.
The reader can easily verify that the value of $q^*$ given in (15) is consistent with $H'$ and $L'$ being completely exhausted at this value of the overall job utilisation rate, as required. Hence, $q^*$ is the maximum rate of job utilisation which can be realised given the available supply of skills.

See Appendix A for a more detailed description of the various allocation processes.

Roughly one third of all jobs is low-level in our simulation experiments.

In period 275 the rates of job-saving/using technical change are set equal to zero again, and one sees the rise in the overall rate of unemployment coming to an end.

References


J. van den (1996), 'Verdringing geen probleem op de Nederlandse smarckt', *ESB*, 5-6, pp. 508-510.


**Acknowledgement**

Thanks are due to Maurice Oude Wansink for his help in preparing the data.
Appendix A: Solving the intertemporal cost minimisation problem

For cases 2 and 3, there are two periods to consider, i.e. the pre-switching period and the post-switching period. Case 4 is the case where both L and H skills are used to L' and H' jobs. The various objective functions associated with the cases mention above are given in equations (A1)-(A3).

\[ T_2 = \int_{0}^{t'} w_{L'} (L' e' + H'). \exp\left( - (p + \delta) t \right) dt + \int_{0}^{t'} \left( w_{L} (L e + w_{H} H) \right) \exp\left( - (p + \delta) t \right) dt \]

\[ T_3 = \int_{0}^{t'} \left( w_{L} (L e' + w_{H} H') \right) \exp\left( - (p + \delta) t \right) dt + \int_{0}^{t'} \left( w_{H} (L' e' + H') \right) \exp\left( - (p + \delta) t \right) dt \]

\[ T_4 = \int_{0}^{t'} \left( w_{L} (L e + w_{H} H) \right) \exp\left( - (p + \delta) t \right) dt \]

The cost function parameters and presented in Table A.1, can be derived for a given moment of switching \( t' \). These are presented in the table below, when is defined as being equal to \( x = (w_{L} - \rho - \delta)/\bar{w}_{L} \).

In order for the cost-minimisation problem to have a meaningful solution, follows that the \( \alpha \)'s and \( \beta \)'s should be positive. This requires the growth rates of high skilled wages and low-skilled wages to be less than \( p + \delta \), which we have had assure in the first place in order to obtain finite objective function values. Hence, the \( \alpha \)'s are positive, while it is immediately clear that \( \beta_1 \) and \( \beta_2 \) are positive too. order for \( \beta_2 \) to be positive, it is necessary that \( (e' / \omega)_{z+1} < 0 \). Note that in case 3 \( z^* \) and \( e' / \omega < 1 \), which implies that \( (e' / \omega)_{z+1} < 0 \) for \( z > 0 \), as required. For case similar results can be obtained. Defining \( z^*_z = -z \), the requirement that \( \beta_2 > 0 \) impli that \( (e' / \omega)_{z^*_z} < 0 \), since \( z+1 < 0 \), by assumption. Because \( w_{L} / \omega < 1 \) in case 2, this or holds for \( z > 1 \), i.e. for \( z < -1 \). Hence \( \beta_2 > 0 \) when \( z < -1 \). Using the definition for \( z \) (above), this requires that \( \bar{w}_{L} - \rho - \delta < \bar{w}_{H} - \bar{w}_{H} \). Hence, \( \beta_2 > 0 \) \( 0 < z < 1 \).
### Table A.1
Cost function parameters

<table>
<thead>
<tr>
<th>Case</th>
<th>( \alpha )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{w_{H,0}}{\rho + \delta - \hat{w}_H} )</td>
<td>( \frac{w_{L,0}}{\rho + \delta - \hat{w}_L} \frac{w_0}{1 + z} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{w_{H,0}}{\rho + \delta - \hat{w}_H} )</td>
<td>( \frac{w_{L,0}}{\rho + \delta - \hat{w}_L} \left( \frac{w_0}{e'} \right)^z + \left( \frac{w_0}{e'} \right)^{-z} )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{w_{H,0}}{\rho + \delta - \hat{w}_H} )</td>
<td>( \frac{w_{L,0}}{\rho + \delta - \hat{w}_L} \left( 1 - \left( \frac{w_0}{e'} \right)^{-z} \right) \frac{1}{1 + z} )</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{w_{H,0}}{\rho + \delta - \hat{w}_H} )</td>
<td>( \frac{w_{L,0}}{\rho + \delta - \hat{w}_L} )</td>
</tr>
</tbody>
</table>

Cost function parameters and switching thresholds

When an individual entrepreneur, characterised by a certain value of, is assumed to take account of potential future switching costs, the revised cost function 'parameters' \( \beta_2 \) and \( \beta_3 \) are given by:

\[
\beta_2 = \frac{w_{L,0}}{\rho + \delta - \hat{w}_L} \left( \frac{w_0}{e'} \right)^z + \left( \frac{w_0}{e'} \right)^{-z} \left( 1 - \left( \frac{w_0}{e'} \right)^{-z} \right) \left( 1 + \Delta \right)^{\frac{1 - \Delta \z}{1 + z}} \tag{A.4}
\]

\[
\beta_3 = \frac{w_{L,0}}{\rho + \delta - \hat{w}_L} \left( 1 - \left( \frac{w_0}{e'} \right)^{-z} \left( 1 + \Delta \right)^{\frac{1 - \Delta \z}{1 + z}} \right) \tag{A.5}
\]

The requirement that all \( \beta \)'s are positive has the same implications as before, provided \( 0 \leq \Delta < 1 \), which seems to be a fairly unrestrictive assumption to make. Note that for \( \Delta = 0 \), the original no-threshold parameter values are obtained again.
Threshold distributions

Only $\bar{\beta}_2$ and $\bar{\beta}_3$ are changed due to the introduction of threshold distributions, since only in cases 2 and 3 a switch between states is expected to occur. Hence, only in these cases the population averages matter. The average values of the $\bar{\beta}$'s are given by:

$$\bar{\beta}_2 = \frac{\bar{\beta}_2(\Delta)}{\bar{\Delta}} d\Delta = f_{w_2,\rho,\delta,\bar{w}_2}\left(\frac{w_{2,e'}}{1+z} \frac{1+\bar{\Delta}y^{-1}(2-z\bar{\Delta})}{\Delta(z+2)}\right)$$  \hspace{1cm} (A.6)

$$\bar{\beta}_3 = \frac{\bar{\beta}_3(\Delta)}{\bar{\Delta}} d\Delta = f_{w_3,\rho,\delta,\bar{w}_2}\left(1 - \frac{(w_{3,e'})^2}{1+z} \frac{2-(1-\bar{\Delta})y^{-1}(2+z\bar{\Delta})}{\Delta(z+2)}\right)$$  \hspace{1cm} (A.7)

where a bar over a beta denotes a population average. Unfortunately, equations (A.6) and (A.7) cannot readily be interpreted.

Supply constraints and the allocation of skills to jobs

In the case of supply constraints, the preferred allocation of skills to jobs in old and new jobs may be different. This leads to the identification of four groups of entrepreneurs with their own preferences with respect to allocating skills to jobs. Group $A$ entrepreneurs have a preference for low skills on both new and old jobs, and the allocation process is relatively straightforward. This also holds for group $D$ entrepreneurs, who have a preference for high skills in both old and new jobs. Hence, the allocation process for group $D$ entrepreneurs is described by:

$$HL^* = \min \left\{ \frac{\bar{A}_{L}^*}{\bar{e}^*}, (1-\mu)^{H} - HH^{*} \right\}$$  \hspace{1cm} (A.8)

$$LL^* = \min (q' L^* - e'HL^*, (1-\mu)L^*)$$

Denoting 'new' low-level jobs by $\Delta L^*$, the allocation process for group $B$ entrepreneurs, who prefer high skills in new jobs and low skills in old jobs, can be described by:
\[ H\Delta L^* = \min (q^*, \Delta L^*/e^*, H^*-HH^*) \]
\[ LL^* = \min (q^*L^*/e^*, H\Delta L^*, L^*) \]
\[ HL^* = \min (q^*L^*/e^*, H\Delta L^*, LL^*, H^*-HH^*-H\Delta L^*) \]
\[ L\Delta L^* = q^*\Delta L^*/e^*H\Delta L^* \]

(A.9)

For group C entrepreneurs, who prefer low skills in new jobs and high skills in old jobs, the allocation process is given by:

\[ L\Delta L^* = \min (q^*, \Delta L^*, L^*) \]
\[ HL^* = \min ((q^*L^*/-L\Delta L^*)/e^*, H^*-HH^*) \]
\[ LL^* = \min ((q^*L^*/-L\Delta L^*)/e^*, HL^*, L^*-L\Delta L^*) \]
\[ H\Delta L^* = (q^*\Delta L^*/-L\Delta L^*)/e^* \]

(A.10)