Interrelation, Structural Changes and Cointegration in a Model for Manufacturing Demand in the Nederlands

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1 Introduction

This paper is concerned with dynamic factor demand systems that are based on the adjustment cost model of the firm. It makes three contributions. First, the solution of the stochastic control problem of a firm assumed to maximize the expected present value of current and future profits under rational expectations (RE) given the production technology and quadratic adjustment costs (see also Hansen and Sargent, 1981) is used to get more insight into the restrictions on the resulting multivariate flexible accelerator models (see e.g. Treadway, 1971 and Epstein and Denny, 1983). Compared with Epstein and Yatchew (1985), we obtain the optimal production factor trajectories without previously specifying an expectations formation process. We show that independently of the assumptions on the expectations formation, the dynamic factor demand equations have a stable adjustment matrix which is diagonal. Interrelations in the adjustment of quasi-fixed production factors result if the innovations of the production technology have specific characteristics.

Second, in line with Lucas's (1976) critique, we analyze the impact of a structural change in the process for input prices on the dynamic factor demand equations. Unanticipated shocks in the environment of the firm result into innovations of the factor demand equations whereas a structural change in the process of the exogenous variables recognized as such by the firm leads to a step change of the parameters of the demand schedules.

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Third, in contrast to the existing literature (for instance on real business cycles, see e.g. Danthine, 1989) where the non-stationarity of the series of the production factors is viewed as a nuisance that is beyond the scope of the theoretical model and that has to be remedied by e.g. detrending the variables prior to the analysis, we model the non-stationarity of factor demand as being induced by the non-stationarity of input prices. In other words, factor demand is assumed to be cointegrated with factor prices, an assumption that is found to be in accordance with the information in the quarterly data for the manufacturing sector in the Netherlands for the period 1971.I–1984.IV.

The paper is organized as follows. In section 2, the theoretical model and closed-form solution decision rules for the rational expectations (RE) factor demand are given, with parameters expressed in terms of those of the underlying theoretical model. The impact of both temporary innovations and structural changes in the process of the input prices on factor demand is also discussed.

Section 3 contains an empirical analysis of the Dutch manufacturing sector for the period 1971.I–1984.IV. Capital and labor are found to be cointegrated with the real price of capital and real labor costs. The model is estimated using a two step estimation procedure which is asymptotically equivalent to maximum likelihood (ML) estimation. The impacts of the oil price shocks in 1973.IV and 1979.II are modeled as a structural change in the process of the factor prices. This approach leads to an improvement of the model. Section 4 concludes the paper.

2 The Model

A representative firm produces output \( y \) using an \( 2n \)-vector of inputs \( x = (l', k')' \) where \( l \) is an \( n \)-vector of employment and \( k \) is an \( n \)-vector of capital stocks depreciating through time. At time \( t \), the production function of the firm is defined as

\[
y_t = y(x_t, s_t) = (\alpha + s_t)'x_t - \frac{x_t'Ax_t}{2},
\]

where \( \alpha \) is a \( 2n \)-vector of positive constant parameters, \( A \) is a symmetric, positive definite \( (2n \times 2n) \)-matrix and \( s_t \) is a \( 2n \)-vector of exogenous stochastic shocks of the production technology. The quadratic specification is a Taylor series approximation for a more general production function. The advantage of the quadratic functional form is that it leads to linear decision rules. We prefer to use the exact solution of a quadratic optimization problem instead of approximating the first order condition of a non-quadratic optimal program.
When the firm wants to alter the factor inputs, it faces adjustment costs which reflect the quasi-fixedness of inputs. Costs of search, training, market research, reorganization are examples of adjustment costs of factor inputs. The adjustment cost function, \( ac_t \), is given by

\[
ac_t = ac(\Delta x_t) = \frac{\Delta x_t' B \Delta x_t}{2},
\]

where \( \Delta \) is the difference operator, \( B \) is a regular \((2n \times 2n)\)-matrix with off-diagonal elements reflecting the trade-off of costs resulting when several inputs are altered simultaneously. The variable costs, \( vc_t \), consist of wage and investment costs

\[
vc_t = vc(x_t, \omega_t, q_t) = w'_t \ell_t + q'_t (k_t - (I_n - \delta)k_{t-1}),
\]

where \( w_t \) and \( q_t \) are \( n \)-vectors of stochastic real labor costs and real prices of investment goods respectively and \( \delta \) denotes a diagonal \((n \times n)\)-matrix of constant depreciation rates. All prices are normalized by the price of output and assumed to be given for the firm in the sense that they are not Granger-caused by factor demand of the firm.

The firm's objective is to maximize its real present value of profits, that is

\[
\maximize \ E \left[ \sum_{t=0}^{\infty} \tau^t (y_{t+i} - ac_{t+i} - vc_{t+i})|\Omega_t \right],
\]

where \( \Omega_t \) is the information set available to the firm at time \( t \), and \( \tau \) is a constant real discount factor. At each period \( t \), the firm chooses contingency plans for \( \ell \) and \( k \), by solving the first order conditions

\[
E[x_{t+1}|\Omega_t] = -(\tau B)^{-1} \alpha + ((1 + \tau^{-1})I_{2n} + (\tau B)^{-1}A)x_t - \tau^{-1}x_{t-1}
\]

\[
- (\tau B)^{-1}s_t + (\tau B)^{-1}p_t - (\tau B)^{-1}dE[p_{t+1}|\Omega_{t}],
\]

where \( I_{2n} \) is the \((2n \times 2n)\)-identity matrix, \( p_t = (w'_t, q'_t)' \) and \( d \) is a diagonal \((2n \times 2n)\)-matrix \[
\begin{bmatrix}
0 & 0 \\
0 & \tau(I_n - \delta)
\end{bmatrix}
\]. We can rewrite (2.5) as follows

\[
\begin{bmatrix}
x_t \\
E[x_{t+1}|\Omega_t]
\end{bmatrix} = 
\begin{bmatrix}
0 & I_{2n} \\
-\tau^{-1}I_{2n} & \tilde{A}
\end{bmatrix}
\begin{bmatrix}
x_{t-1} \\
x_t
\end{bmatrix}
+
\begin{bmatrix}
0 \\
(\tau B)^{-1}z_t
\end{bmatrix},
\]

where \( \tilde{A} = ((1 + \tau^{-1})I_{2n} + (\tau B)^{-1}A) \) and \( z_t = -\alpha - s_t + p_t - dE[p_{t+1}|\Omega_t] \).

Hansen and Sargent (1981) have proved that for a quadratic objective function the number of stationary solutions to (2.5) is equal to the number of non-stationary solutions. Kollintzas (1985) generalized this finding by proving that a multivariate symmetric adjustment costs model has structural parameters that are real.
If $\tilde{A}$ has $2n$ eigenvalues stored in two diagonal $(n \times n)$-matrices $M_1$ and $M_2$, then the characteristic equation associated with (2.6) becomes

$$(-\Lambda^2 + M_1 \Lambda - \tau^{-1}I_n)(-\Lambda^2 + M_2 \Lambda - \tau^{-1}I_n) = O. \quad (2.7)$$

If all eigenvalues of $\tilde{A}$ differ, then a unique stationary forward looking solution of the linear difference equation set (2.5) exists (see Blanchard and Kahn, 1980; Palm and Pfann, 1990)

$$x_t = \Lambda_1 x_{t-1} - C^{-1} \sum_{i=0}^{\infty} (A_2)^{-i-1} C(\tau B)^{-1} E[x_{t+i} | \Omega_t], \quad (2.8)$$

where $A_2$ is a diagonal $(2n \times 2n)$-matrix of solutions of (2.7) which are greater than one in absolute value, and $\Lambda_1$ is a diagonal $(2n \times 2n)$-matrix that contains the eigenvalues of (2.7) that do not lie outside the unit circle.

We note that $\Lambda_1 A_2 = \tau^{-1}I_{2n}$, according to (2.7). If $\tilde{A}$ and $\Lambda_1$ are partitioned as follows

$$\tilde{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad \text{and} \quad \Lambda_1 = \begin{bmatrix} \Lambda_{11} & 0 \\ 0 & \Lambda_{12} \end{bmatrix}$$

then

$$C = \begin{bmatrix} A_{12} & A_{12} \\ M_1 - A_{11} & M_2 - A_{11} \end{bmatrix} \begin{bmatrix} \tau \Lambda_{11} (\tau \Lambda_{11}^2 - I_n)^{-1} & 0 \\ 0 & \tau \Lambda_{12} (\tau \Lambda_{12}^2 - I_n)^{-1} \end{bmatrix},$$

where all submatrices are of order $n \times n$.

By definition the autoregressive part of (2.8) is stationary. This important restriction implies that a non-stationarity of factor inputs arises through current and expected future levels of the variables in $x_t$, such as factor prices.

In the multivariate flexible accelerator type models (e.g. Treadway, 1971, and Epstein and Denny, 1983), the interrelated adjustment path for factor inputs, is as follows

$$\Delta x_t = M(x_{t-1} - x^*), \quad (2.9)$$

where $x^*$ is a steady state equilibrium and $M$ denotes a stable adjustment matrix of order $2n$. However, from (2.8) we get that $I_{2n} + M = \Lambda_1$, which implies that $M$ must be diagonal if the structural model is quadratic. We note that Epstein and Yatchew (1985), hereafter denoted as EY, argue that $M$ has $2n$ eigenvalues lying between $-1$ and $0$. 
EY solve a deterministic (certainty equivalence) version of (2.4) replacing all random variables by their conditional expected values. EY “avoid the difficulty of solving” (2.4) (p.240) to find the closed-form solution (2.5) with $\Lambda_1 (= I_{2n} + M$ in EY) expressed as a function of $\alpha$, $A$ and $B$, by reparametrizing the production function $f$ in terms of $\alpha$, $B$ and $\Lambda_1$.

This technique, however, cannot be applied if $f$ contains a multiplicative disturbance term $s_t x_t$ as in (2.1). Moreover, by reparametrizing, information on $\Lambda_1$ is lost which appears to be crucial when modeling interrelated factor demand given a quadratic objective function, since then $\Lambda_1$ is diagonal.

Although none of structural parameter matrices, $A$ and $B$, were assumed to be diagonal, we find that independently from assumptions on expectations formation processes the adjustment matrix is diagonal. EY estimate $P = BM$, assuming $B$ is diagonal. Consequently, $P$ is diagonal, which is confirmed by the empirical results of EY, where unrestricted estimates of the off-diagonal parameter of the symmetric matrix $P$ are $-0.01$ and $0.00000$ respectively for different periods (see EY, table 5, p.249).

In order to derive explicit factor input decision rules we specify the stochastic processes for $s_t$ and $p_t$. Stochastic movements of factor productivity are assumed to follow a stationary first order Markov process $E[s_{t+1} | \Omega_t] = \bar{R} s_t$, where $\bar{R}$ is a $(2n \times 2n)$-matrix. Let $S$ be a $(2n \times 2n)$-matrix, such that $S s_t$ is a stationary AR(1) process with innovation $\xi_t^*$

$$S s_t = RS s_{t-1} + \xi_t^*, \quad (2.10)$$

where $R$ is a $(2n \times 2n)$-matrix satisfying the stability conditions and let $\text{vec}(S) = (I_{2n} \otimes C^{-1}(\tau \Lambda_1)^{-1}C - \bar{R}' \otimes I_{2n})^{-1} \text{vec}((\tau B)^{-1})$ then

$$C^{-1} \sum_{i=0}^{\infty} (\tau \Lambda_1)^i C(\tau B)^{-1} E[s_{t+i} | \Omega_t] = S s_t. \quad (2.11)$$

The proof of this result is fairly straightforward and is given in Palm and Pfann (1990).

The stochastic process for $p_t$ is assumed to be autoregressive of order $r$

$$p_t = c^p + Q_1 p_{t-1} + Q_2 p_{t-2} + \cdots + Q_r p_{t-r} + \xi_t^p, \quad (2.12)$$

where $c^p$ is a $2n$-vector of constants, $Q_1$ to $Q_r$ are $(2n \times 2n)$-matrices of constant parameters and $\xi_t^p$ is the innovation of prices. Then
\[
C^{-1} \sum_{i=0}^{\infty} (\tau \Lambda_1)^{i+1} C(\tau B)^{-1} E[p_{t+i} - p_{t+i+1} | \Omega_t] = \mathcal{P} + \mathcal{Q}_0 p_t + \mathcal{Q}_1 p_{t-1} + \cdots + \mathcal{Q}_{r-1} p_{t-r+1},
\]

(2.13)

where

\[
\mathcal{P} = C^{-1}((\tau \Lambda_1)^{-1} - I_{2n})^{-1} C((\tau B)^{-1} - (\tau B)^{-1} d)c^f,
\]

\[
\mathcal{Q}_0 = C^{-1} \Lambda_1 CB^{-1} V,
\]

\[
\mathcal{Q}_i = \sum_{j=2}^{r-i+1} \frac{(C^{-1} (\tau \Lambda_1)^{j-1} C)(C^{-1} \Lambda_1 CB^{-1} - (\tau B)^{-1} d)VQ_j}{j-i},
\]

\[i = 1, \ldots, r - 1\]

and where \(V\) is a \((2n \times 2n)\)-matrix such that \(\sum_{j=1}^{r} C^{-1} (\tau \Lambda_1)^j CVQ_j = I_{2n}\).

Substituting (2.11), (2.12) and (2.13) into (2.8), and applying a Koyck transformation to eliminate the autocorrelation in (2.10), the reduced form of the factor demand equations becomes

\[
x_t = \mathcal{P} + (R + \Lambda_1)x_{t-1} - R\Lambda_1 x_{t-2} + U_1 p_{t-1} + \cdots + U_r p_{t-r} + \xi_t^f
\]

(2.14)

with

\[
c^f = c^f(c^f, \alpha, A, B, R, \tau, \delta),
\]

\[
\Lambda_1 = \Lambda_1(A, B, \tau),
\]

\[
U_j = R^{\mathcal{Q}_{j-1}} - \mathcal{Q}_0 Q_j - \mathcal{Q}_j = U_j(A, B, R, Q_1, \ldots, Q_r, \tau, \delta), \quad j = 1, \ldots, r - 1,
\]

\[
U_r = R^{\mathcal{Q}_{r-1}} - \mathcal{Q}_0 Q_r = U_r(A, B, R, Q_1, \ldots, Q_r, \tau, \delta),
\]

and

\[
\xi_t^f = (\xi_t - \mathcal{Q}_0 \xi_t^f)\] being the innovations in factor inputs.

Equation (2.14) is a closed-form solution of (2.5). The overidentifying restrictions and relationships between the parameters of the structural equations (2.1), (2.2), (2.3), (2.10) and (2.12), and the factor demand equations (2.14) can be made explicit. An important restriction is the diagonality of the adjustment matrix \(\Lambda_1\): in a RE model with a quadratic objective function the occurrence of trade-offs of costs (the non-diagonality of matrix \(B\)) or of complementarity in the production (the non-diagonality of matrix \(A\)) does not lead to interrelations in the adjustment process. Interrelated multivariate adjustment, however, results from serial cross-correlation of the unforeseen shocks in the firm's technology (the non-diagonality of matrix \(R\)).

A second important finding is that the stability of the autoregressive part of equation (2.14) follows from the properties of \(\Lambda_1\) and \(R\). In this model a non-stationarity in \(x_t\) results from non-stationary factor prices.
Third, notice that as a result of intertemporal optimization under uncertainty the parameters of the conditional model for input demand given prices and of the marginal process for factor prices are not variation-free. According to the theoretical model, factor prices are not weakly exogenous with respect to the parameters of the demand equations. Joint estimation of the demand and price equations is expected to lead to an efficiency improvement. Moreover, a structural change in the process for prices is expected to affect the parameters of the demand equations so that super-exogeneity is not likely to occur (see e.g. Engle et al., 1983).

The closed-form solution (2.14) enables us to identify the impact of a change in the process for prices on factor demand. For instance, a (structural) step change in the real price level, $c^p$, which is immediately recognized as such, will lead to a step change in factor demand, $c^x$. The magnitude of the step change in factor demand, however, depends also on the size of adjustment costs ($B$) and the firm’s technology ($\alpha, A$).

A structural change in the autoregressive part of the process for real input prices, $Q_1, \ldots, Q_r$, influences the factor input decisions through price effects $U_t$. However, the agent may not immediately correctly assess size and sign of a change in the parameters of $Q_t$, in which case the firm’s uncertainty about the environment increases. As a result the (subjective) variances of the innovations increase. When more information about the impact of the shock on prices becomes available, the uncertainty about the structure of the process for $p_t$ will decrease again.

Consequently, the process of learning about the nature and size of the structural change in $Q_t$ may possibly induce autoregressive conditional heteroskedasticity (ARCH) in the disturbances of the dynamic factor demand equations (2.14).

The explicit closed-form solution (2.14) also throws light on the identification problem of the structural parameters. EY argue that the parameters ($\alpha, \delta, \tau, A, B$) and the parameters of the processes for the explanatory variables ($R, Q_1, \ldots, Q_r$) can only be identified if the output supply equation is included in the system to be estimated. We find that under RE’s this is only true for the off-diagonal parameters of $A$ and $B$, the real discount rate $\tau$, and the capital depreciation rate $\delta$.

Finally, innovations in technology and prices lead to innovations in factor demand, since $\xi_t^x = \xi_t^c - \widetilde{Q}_0 \xi_t^p$. Accordingly, the disturbances of the systems (2.12) and (2.14), $\xi_t = (\xi_t^c, \xi_t^x)'$, are contemporaneously correlated. In the sequel we assumethat $\xi_t$ is independently normally
distributed with zero mean and an unrestricted symmetric positive definite covariance matrix $\Sigma$.

3 Empirical analysis

In this section, we apply the models (2.12) and (2.14) to quarterly aggregate manufacturing data in the Netherlands for the period 1971.I–1984.IV. Figure 1 shows time series for aggregate employment ($\ell$), capital ($k$), real labor costs ($w$) and real prices of investments ($q$), the base year for all data being 1980. A description of the data sources is given in appendix A. In the empirical analysis $n$ equals 1.

Unfortunately, time series on branches of industry are not available for the Netherlands. Therefore we have to rely on sectoral aggregates. We cannot take into account the shifts within the sector. In the last two decades some branches of the sector were threatened to collapse under increasing production costs and growing foreign competition (e.g. textile, paper, shipbuilding) whereas capital and know-how intensive industries, such as chemical and electrotechnological industries, experienced a substantial expansion.

![FIG 1a: REAL PRICE INDEX MANUFACTURING LABOR COSTS](image)
Energy supply and prices were strongly affected by the world oil crises which occurred in 1973.IV (OC₁) and in 1979.II (OC₂). OC₁ reduced the profitability of firms. Heavy foreign competition prohibited the price increases necessary to cover the increase of production costs. Sales fell off as a result of depressed demand. Bankruptcy was a major cause of a rising unemployment.

Anticipating on price increases due to OC₂ the export of energy intensive industries expanded. In our analysis, however, we do not account for the effects of a foreseen price change before it occurred, but for the impact of OC₂ on wage costs and factor demand after it had taken place.

The slow-down of the growth of domestic sales was caused by the decrease of real disposable income. The market share of imported goods increased again. Employment decreased and profit shares deteriorated in the years after OC₂. Wage restraints reduced the ratio of wage costs to total production costs (see figure 1).

We start our empirical investigation with an analysis of the single data series. First, we test for the order of integration of the series. We assume that each time series can be adequately represented by the univariate model

\[ V_t = \beta_0 + \beta_1 OC_1 + \beta_2 OC_2 + \alpha_1 V_{t-1} + \alpha_2 \Delta V_{t-1} + \varepsilon_t \]  \hspace{1cm} (3.1)
where $OC_1$ and $OC_2$ are oil shock dummies being 1 for the periods 1973.IV–1984.IV and 1979.II–1984.IV respectively and zero otherwise, $\epsilon_t$ is an i.i.d. $N(0, \sigma^2)$ random variable, and $V_t \in \{w_t, q_t, \ell_t, k_t\}$. Three dummies were added to the equation to account for seasonality in the data.

To test the hypothesis $H_0 : \alpha_1 = 1$ that the second order process for $V_t$ has a unit root, we computed the Dickey-Fuller test yielding $-1.38, -1.99, -.79$ and $-1.15$ for $w_t, q_t, \ell_t$ and $k_t$ respectively, with $R^2$ being $.987, .897, .997$ and $.998$, and the $DW$ being $2.05, 1.95, 2.08$ and $2.01$ respectively. Consequently, on the basis of a comparison with the values in table 8.5.2 of Fuller (1976) or with the values given by Perron (1989) for models with dummies, we cannot reject the hypotheses of $w_t$, $q_t$, $\ell_t$ and $k_t$ being $I(1)$.

In the RE-model (2.14) the autoregressive part of $\ell_t$ and $k_t$ is restricted to be stationary. Yet, figure 1 clearly indicates that both series are non-stationary. In the literature, a non-stationarity is often viewed as being beyond the scope of the economic model, and is remedied by incorporating deterministic trends. However, when factor demand and real input prices are cointegrated, the non-stationarity of the latter may capture that of the input demand. To test for cointegration we compute the cointegrating regressions for $\ell_t$ and $k_t$ and the corresponding augmented Dickey-Fuller regressions for the period 1971.I–1984.IV, and Johansen's likelihood ratio (LR) (see Johansen and Juselius, 1990 table A.1) test on the number of cointegrating vectors $m$. We find the following results where asymptotic t-statistics are given within parentheses and seasonal dummies have been omitted.

$$
\begin{align*}
\hat{\ell}_t &= .464 + .085 OC_1 + .034 OC_2 - 1.164 w_t + 1.563 q_t (+u_\ell^t) \\
& (2.85) \quad (4.77) \quad (2.21) \quad (-15.35) \quad (8.76) \\
\hat{\ell}_t &= \Delta \hat{u}_\ell^t \quad = - .317 u_{\ell-1}^t - .001 \Delta u_{\ell-1}^t \\
& (-3.14) \quad (-.22)
\end{align*}
$$

LR(0) = 20.09(29.5); LR(1) = 1.39(15.2); LR(2) = .64(3.9)
\[ k_t = 1.740 - 0.047 OC_1 - 0.029 OC_2 + 0.917 w_t - 1.580 q_t \]
\[ (14.21) \]  \[ \begin{array}{c}
\text{(-3.91)} \\
\text{(-2.69)} \\
\text{(17.22)} \\
\text{(-11.70)}
\end{array} \]
\[ R^2 = 0.976; \text{ DW} = 0.774 \]

\[ \Delta u^k_t = -0.375 u^k_{t-1} - 0.001 \Delta u^k_{t-1} \]
\[ (3.29) \]  \[ \text{(-9.3)} \]

\[ LR(0) = 33.01(29.5); \text{ LR}(1) = 4.66(15.2); \text{ LR}(2) = 0.92(3.9) \]

The CRDW-statistics (Cointegrating Regression Durbin-Watson statistic) are .620 and .774 for \( \ell_t \) and \( k_t \) respectively. The critical value at the 5% level given by Engle and Granger (1987) is .386 for \( T = 100 \), so that the null hypothesis of no cointegration can be rejected. The CRDW-statistic, however, has low power against the alternative hypothesis. A similar conclusion can be drawn from the augmented Dickey-Fuller (ADF)-statistic, with values -3.14 and -3.29 respectively for which Fuller (1976) gives a critical value of -3.21 at the 5% level and \( T = 50 \); using tables II and III of Engle and Yoo (1987), the null is rejected at a 10% level for \( \ell_t \) and \( k_t \). Johansen's LR(m)-test indicates that the null hypothesis of no cointegration, i.e. \( m = 0 \), is rejected against the alternative hypothesis \( H_1 : m > 0 \text{LR}(0) \) in the trivariate system for \( (k_t, w_t, q_t) \). The 5 percent critical values are given between parentheses. For the system for \( (\ell_t, w_t, q_t) \), there is evidence in favor of no cointegration. When we test \( H_0 : m = 1 \) against \( H_1 : m > 1 \text{LR}(1) \) or \( H_0 : m = 2 \) against \( H_1 : m = 3 \text{LR}(2) \), the null hypotheses do not have to be rejected for both trivariate systems.

Excluding \( OC_1 \) and \( OC_2 \) from the cointegrating regressions, we find .16 and .35 for the CRDW-statistics for \( \ell_t \) and \( k_t \) respectively and -1.33 and -1.57 for the ADF-statistics. Johansen's LR-statistic drops to 4.96 and 14.20 respectively for the test of \( H_0 : m = 0 \) against \( H_1 : m > 0 \) leading to the conclusion that the null hypothesis cannot be rejected at conventional significance levels. This finding indicates that the direct effect of \( OC_1 \) and \( OC_2 \) on \( \ell_t \) and \( k_t \) given factor prices has been possibly non-stationary.

The existence of one cointegrating relationship between factor demand and all factor prices can be interpreted in terms of the theoretical model of section 2 as follows. Basically, the model for \( x_t \) can be written as a partial adjustment model (2.9). As \( x_t \) and \( p_t \) were found to be \( I(1) \), equation (2.9) reduces to an error correction mechanism (see Nickell, 1985 and appendix B) and the Granger Representation Theorem (see Engle and Granger, 1987) implies that \( x_t \) and \( p_t \) are cointegrated.
To summarize, the disturbance of the long-run relationship between the demand for labor and capital respectively, and factor prices represents the effect of shocks to the technology represented by $s_t$ and unexpected variations in factor prices. This disturbance also picks up approximation errors of the model, and in particular the effects of left-out variables such as, for instance, variations in product demand. The above analysis suggests that these effects have been stationary once we control in a simple way for seasonal variation and for changes resulting from the oil crises. In other words, the impacts of technological change, that is of the so-called Solow residual, of variations in product demand and of other omitted variables seem to average out for the stock of capital. For labor, one cannot exclude the possibility of non-stationary productivity and other shocks.

The cointegrated system must have a causal ordering in at least one direction (Engle and Granger, 1987). In section 2, we assumed unidirectional causal ordering in the sense of Granger from prices to factor demand. We use a fourth order VAR of $w_t$, $q_t$, $\ell_t$, and $k_t$ in levels, including a constant, $OC_1$ and $OC_2$, and three seasonal dummies, to test the hypothesis that the coefficients of all lagged factor inputs in the price equations are zero. The WALD statistic associated with the null hypothesis of no causality from demand to prices equals 24.38 which is not significant when compared with the $\chi^2(16)$-value at the 5% level whereas that for no causality from prices to demand equals 38.26 and is significantly different from zero at a 1% level (df = 16). Notice that the size of the WALD test may be smaller than the nominal size because of the possible occurrence of unit roots. Consequently, a WALD test, being computed from a regression in levels, that rejects the hypothesis of Granger causality, implicitly rejects the hypothesis when the estimations were carried out in first differences. In conclusion we can say that unidirectional causality from input prices to factor demand is found to be in accordance with the information in the data.

Under the null hypothesis of unidirectional causality from input prices to factor demand, we can examine the process for the price series independently from the factor input data. Bivariate identification of the order of $p_t = (w_t, q_t)'$ has been carried out using the SCA statistical system provided by Liu et al. (1986). The results for Akaike’s Information Criterion (AIC) and a statistic, CHI$^2$, testing the significance of an additional lag in the bivariate lag-polynomial, are given in table 1. CHI$^2$ is based on the logarithmic difference of the residual covariance matrix ML-estimates of autoregressions with and without an additional lag respectively and is asymptotically $\chi^2(4)$ distributed. On the basis of the results in table 1 and the tests for unit roots we
assume that the process of $p_t$ can be adequately modeled as follows

$$\Delta p_t = c_0^p D_t + c_1^p OC_1 + c_2^p OC_2 + Q \Delta p_{t-1} + \xi_t^p$$

(3.3)

where $c_0^p$ is a $(2 \times 4)$-matrix, $D_t$ is a 4-vector including a constant and three seasonal dummies, $c_1^p$ and $c_2^p$ are 2-vectors and $Q$ is a $(2 \times 2)$-matrix of constant parameters. When first differences of $p_t$ are modeled, the oil shock dummies $OC_1$ and $OC_2$ are 1 in 1973.IV and 1979.II respectively and zero otherwise.

<p>| TABLE 1 |</p>
<table>
<thead>
<tr>
<th>Bivariate order selection of $p = (w, q)'$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td>CHI$^2$</td>
</tr>
<tr>
<td>AIC</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>CHI$^2$</td>
</tr>
<tr>
<td>AIC</td>
</tr>
</tbody>
</table>

For reasons explained in the previous section we assume that the structural parameter matrices $A$ and $B$ are diagonal, without loss of generality. Then the dynamic factor input demand equation (2.14) becomes

$$x_t = c_0^x + c_1^x OC_1 + c_2^x OC_2 + (R + \Lambda_1) x_{t-1}$$

$$- R \Lambda_1 x_{t-2} + U_1 p_{t-1} + U_2 p_{t-2} + \xi_t^x,$$

(3.4)

where

$$U_1 = (I_2 - R) \Lambda_1 B^{-1} V + (I_2 + d - \tau \Lambda_1) \Lambda_1 B^{-1} V Q,$$

$$U_2 = (I_2 - R) (d - \tau \Lambda_1) \Lambda_1 B^{-1} V Q,$$

$V$ is a $(2 \times 2)$-matrix solving $V - \tau \Lambda_1 V (I_2 + Q) - (\tau \Lambda_1)^2 V Q = (\tau B)^{-1} I_2$,

and

$\xi_t^x$ is a 2-vector of innovations.

Note that $C^{-1} \Lambda_1^i C = \Lambda_1^i$ since $C = I$ if $A$ and $B$ are diagonal. The 4-vector of disturbances $(\xi_t^p, \xi_t^x)'$ is assumed to be i.i.d. $N(0, \Sigma)$ distributed. Seasonal dummies were added to the regression equations to take account of the seasonality in the data.
Estimates of the structural parameters of the system (3.3) and 
(3.4) are obtained using the method of asymptotic least squares (ALS).
The ALS-estimation procedure is a two step procedure, proposed by 
Chamberlain (1982) (see also Gourieroux et.al., 1985, and Kodde et.al., 
1990) which minimizes the distance between the unrestricted reduced 
form estimates and the reduced form coefficients expressed as functions 
of the identified structural parameters. The ALS method is asymptotically 
equivalent to MLE, provided the unrestricted estimates in the 
first step are obtained by ML and an optimal weighting matrix is used. 
In order to obtain the weighting matrix we need the derivatives of 
the nonlinear relationships between the unrestricted parameters and 
the structural parameters. The nonlinear functions are known when 
a closed form solution has been obtained, as described in section 2. 
The ALS-estimation results are given in table 2. In order to reduce the 
nonlinearity of the estimation procedure, we assume the quarterly real 
discount factor τ to be .98 (see e.g. Danthine, 1989, p.226 for a similar 
approach in real business cycle modeling) and set the depreciation 
rate δ for capital equal to .047. In a sensitivity analysis we found that 
small variations of these parameters hardly affect the estimates of the 
remaining parameters.

FIG 2a: RESIDUALS MANUFACTURING LABOR COSTS

+0.05
+0.04
+0.03
+0.02
+0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05


TIME ** 1971.III - 1984.IV **
FIG 2b: RESIDUALS MANUFACTURING CAPITAL COSTS

FIG 2c: RESIDUALS MANUFACTURING EMPLOYMENT
We have to note that as shown by Phillips and Durlauf (1986), when the regressors are integrated of order one, estimators and test statistics do in general not have standard asymptotic distributions. However, when estimating the model we impose the unit roots instead of implicitly estimating them along with the other parameters. As shown by Phillips (1991) full system maximum likelihood brings the problem of inference within the family that is covered by the locally asymptotically mixed normal asymptotic theory provided that all unit roots in the system have been eliminated by specification and data transformation. Therefore, to the extent that we have imposed all unit roots in the system, standard asymptotic theory of inference applies.

The assumptions underlying our model appear to be corroborated. The technological shocks follow a stationary process, and the structural matrices $A$ and $B$ are positive definite. The adjustment costs of labor are smaller than those of capital ($B_{11} < B_{22}$), and $B_{11}$ is not significantly different from zero. This finding is intuitively plausible, and is consistent with findings from similar empirical research (see e.g. Pindyck and Rotemberg, 1983). The insignificance of several structural parameters points into directions in which the model could be possibly further simplified. One could also check the statistical properties of the model after imposing a unit root on labor productivity by restricting the first diagonal element of $R$ to be equal to one. We have not pursued these directions any further.
The LR-statistic for the overidentifying restrictions implied by the theoretical model, is significant at the 5 percent level but not at the 1 per cent level. The significance of the LR-statistic may be due to the rather simple functional form of the firm’s technology and costs structure. Other potential reasons for the LR-test to be significantly different from zero are small sample bias and the non-stationarity of the regressors in the model. The multivariate portmanteau test statistic of Ljung and Box (1978) does not point to any higher order significant residual autocorrelation.

<table>
<thead>
<tr>
<th>TABLE 2</th>
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<tbody>
<tr>
<td>ALS-estimation results and tests of the RE-factor demand model for the period 1971.III-1984.IV*</td>
</tr>
</tbody>
</table>

\[
p = (w, q)' : \\
\begin{align*}
p' & = \begin{bmatrix}
.019 & (5.95) \\
-.002 & (.63) \\
-.022 & (1.90) \\
-.018 & (2.24)
\end{bmatrix} \\
OC_1 & = \begin{bmatrix}
.001 & (.13) \\
.004 & (.45)
\end{bmatrix} \\
Q & = \begin{bmatrix}
.394 & (2.02) & .146 & (6.66) \\
.147 & (.85) & .293 & (1.51)
\end{bmatrix}
\end{align*}
\]

\[
x = (\ell, k)' : \\
\begin{align*}
x^* & = \begin{bmatrix}
.006 & (.34) \\
.005 & (.49) \\
-.007 & (2.06) \\
.001 & (.20)
\end{bmatrix} \\
OC_1 & = \begin{bmatrix}
.003 & (1.32) \\
-.005 & (2.41)
\end{bmatrix} \\
R & = \begin{bmatrix}
.886 & (1.58) & .127 & (.97) \\
-.173 & (.189) & .232 & (1.35)
\end{bmatrix} \\
B & = \begin{bmatrix}
.354 & (.39) & 0 \\
0 & .730 & (4.62)
\end{bmatrix}
\end{align*}
\]

\[
A = \begin{bmatrix}
1.774 & (.28) & 0 \\
0 & 1.450 & (2.50) \\
\end{bmatrix}
\]

\[
\Sigma_4 = \begin{bmatrix}
12.23 \\
7.65 & 9.88 \\
-1.20 & -2.21 & 2.82 \\
-.24 & .33 & .21 & 1.21
\end{bmatrix}
\times 10^{-5} \log L = 795.55
\]

TESTS

Ljung & Box: \( \chi^2(16s - 12) \) ARCH: \( \chi^2(s) \)

<table>
<thead>
<tr>
<th>s</th>
<th>( \chi^2(.95) )</th>
<th>LB</th>
<th>( \chi^2(.95) )</th>
<th>s</th>
<th>w</th>
<th>q</th>
<th>l</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>68.25</td>
<td>88.25</td>
<td>.35</td>
<td>.09</td>
<td>.24</td>
<td>.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>144.62</td>
<td>177.39</td>
<td>2</td>
<td>.94</td>
<td>.11</td>
<td>2.33</td>
<td>2.87</td>
<td></td>
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<tr>
<td>15</td>
<td>226.28</td>
<td>264.23</td>
<td>3</td>
<td>1.99</td>
<td>.55</td>
<td>2.49</td>
<td>5.65</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>300.68</td>
<td>349.93</td>
<td>4</td>
<td>3.40</td>
<td>5.70</td>
<td>2.99</td>
<td>5.56</td>
<td></td>
</tr>
<tr>
<td>LR(8)</td>
<td>19.94</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

* The absolute asymptotic t-values are given within parentheses. The estimated coefficients of seasonal dummies have not been presented in the table.

In figures 2 the residuals of the estimated model are given. After both oil crises the uncertainty as measured by the variances of real factor prices increased. However, the ARCH-statistics in table 2 (see
Engle, 1982), do not indicate any significant residual autoregressive conditional heteroskedasticity. This finding suggests that the agents’ learning in the presence of a structural change has indeed been modeled in a way that is not in contradiction with sample evidence, by adding dummy variables for the effects on demand of a step change in the process of input prices. $OC_1$ leads to a significant scrapping of capital. $OC_2$ significantly reduces the demand for labor, thereby increasing unemployment in the Netherlands.

4 Concluding remarks

This paper has been concerned with the multivariate adjustment rational expectations model. A solution of the stochastic control problem of the firm has been given and the implications of the theoretical model for the factor demand equations have been discussed. In particular, it has been shown that interrelation between factor demand does not result from possibilities for substitution between inputs or trade-off in the adjustment costs. It can result from the cross-correlation between the shocks affecting the production technology of the firm. In line with the Lucas critique, special attention has been paid to the implications of a structural change in the processes generating relative factor prices. Finally, the model has been applied to quarterly data for the manufacturing sector in the Netherlands for the period 1971-1984.

As there is empirical evidence that factor demand series and relative factor prices are cointegrated, the non-stationarity of the factor demand series has been modeled by including the levels of the relative prices in the demand schedules, instead of detrending the variables prior to the analysis.

To summarize, the following conclusions were reached.

1. A quadratic objective function does not imply interrelated adjustments of production factors. Cross-correlation of innovations in the production technology, however, does.

2. In line with the RE hypothesis, the non-stationarity of the factor demand series can be accounted for by the non-stationarity in the explanatory variables.

3. A structural step change in the constant term of the process of the explanatory variables leads to a step change in the factor demand equations. The magnitude of the change depends on the size of adjustment costs and the firm’s technology.
4. The gradual learning about the nature of the structural change may induce patterns in the disturbances of the demand system that can be approximated by an ARCH-process.

5. For the Dutch manufacturing sector, labor and capital and their prices relative to the output price index are integrated of order one. Moreover, when including dummies to account for the oil crises we find cointegration between capital demand and factor prices. For labor demand one can not rule out the possible non-stationarity of the productivity shocks.

In order to prevent a misinterpretation of these results, some qualifications have to be made. First, the empirical analysis was based on aggregate data. Aggregation possibly hides the differences in the adjustments that have taken place in the various branches of the industry. Second, the rejection of the restrictions implied by the RE adjustment cost model at the 5 percent level may be the result of the rather simple functional form of the production function and of the adjustment cost structure, a criticism which has often been made in the literature (e.g. Morrison, 1986). More complicated functional forms lead to nonlinear first order conditions for the optimization problem and may be prohibitive for getting a closed form solution in which case the Euler equations can be analyzed using for instance an instrumental variables technique (e.g. Hansen and Singleton, 1982).

APPENDIX A

The Quarterly Data Series of the Dutch Manufacturing Sector

The base year of all data series is 1980. The variable \( \ell \) is the index number of total employment, \( PY \) is the producers price index of domestic sales, \( PL \) the price index of wage costs and \( PK \) the price index of investment goods. These data for manufacturing in the Netherlands have been obtained from the Central Bureau of Statistics (CBS). The variable \( k \) has been computed from annual data on capital stocks in Dutch manufacturing which are used in the VINSEC model of the Central Planning Bureau (CPB), and from the quarterly macro data of capital stocks used in model KOMPAS. Both CBS and CPB are kindly acknowledged for supplying the data. The data can be found in Kodde et.al. (1990). Real wage costs, \( w \), and the real price of capital, \( q \), were computed as \( w = PL/PY \) and \( q = PK/PY \) respectively. All data are seasonally unadjusted.
APPENDIX B

Partial Adjustment, Error Correction and Cointegration.

Let \( x_t = (\ell_t, k_t)' \) be a 2-vector of instruments available to the firm which maximizes its real present value of profits (2.4) and \( x_t \sim I(1) \). We can write \( x_t \) as a partial adjustment mechanism, say

\[
\Delta x_t = M(x_{t-1} - x^*) \tag{II.1}
\]

with \( M = \Lambda_1 - I \) and \( x^* = (\Lambda_1 - I) \sum_{i=0}^{\infty} (\Lambda_1)^{i+1} B^{-1} p_{t+i}^* \), where (for the ease of simplicity) the matrices \( A \) and \( B \) in (2.1) and (2.2) respectively are assumed to be diagonal and the real discount factor, \( \tau \), is assumed to be close to one (quarterly data), and \( p_{t+i}^* = E(p_{t+i} | \Omega_t) \) is a 2-vector of expectations at time \( t \) of future values of exogenous variables.

We assume that \( p_t \) follows an \( AR(2) \) process with a unit root and drift, say,

\[
\Delta p_{t+i} = p_0 + Q \Delta p_{t+i-1} + \xi_{t+i}^p. \tag{II.2}
\]

Then the expectation \( p_{t+i}^* \) is as follows (cf. Nickell 1985)

\[
p_{t+i}^* = p_{t-1} + \tilde{p}_0 + (I + Q^{i+1})(I - Q)^{-1} \Delta p_t \tag{II.3}
\]

where \( \tilde{p}_0 = \sum_{j=0}^{i-1} (i - j)Q^j p_0 \).

Substituting (II.2) and (II.3) into (II.1) we get

\[
\Delta x_t = x_0 + M x_{t-1} + B^{-1} \Lambda_1 M^{-1} p_{t-1} + \tilde{Q} \Delta p_t + \xi_t^x,
\]

where \( \tilde{Q} = B^{-1}[(\Lambda_1 Q)^{-1} - I_2]^{-1} - (\Lambda_1^{-1} - I_2)^{-1} \), and \( x_0 \) is a scalar.

Consequently, we have

\[
[I_2 : B^{-1} \Lambda_1 M^{-2}](x_{t-1}, p_{t-1})' \sim I(0).
\]

According to Granger's Representation Theorem (Engle and Granger (1987)) this implies that \( x_t \) and \( p_t \) cointegrate.
REFERENCES


Pindyck, R.S. and J. J Rotemberg (1983), Dynamic factor demands and the effects of energy price shocks, American Economic Review 73, 1066-1079.