Partial versus full system modelling of cointegrated systems
An empirical illustration

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Abstract

In this paper we discuss and illustrate the relative merits and pitfalls of modelling cointegrated systems in partial or in complete system set-up. We follow both the complete VAR based approach advocated by Hendry and Mizon (1993) and the partial system approach based on so-called structural error correction models proposed by Boswijk (1991, 1992). The issue of exogeneity is discussed and illustrated using quarterly data for Belgian aggregate imports. Several economically interesting questions related to price effects in international trade are raised in the empirical analysis.

Key words: Cointegration systems; VAR models; Weak exogeneity; Error correction models; Import demand

JEL classification: C22; C32; C51; C52

1. Background

Since the publication of the paper by Engle and Granger (1987), the study of cointegrated time series has become one of the more prolific and promising
fields of research in theoretical and applied econometrics. However, numerous different approaches have been advocated for the efficient estimation of, and testing for, cointegrating relationships among nonstationary economic variables. In their overview, Phillips and Loretan (1991) investigate different approaches for the estimation of cointegration relationships and study their relative merits both in terms of asymptotic efficiency and finite sample performances. While such a paper has the benefit of stating problems in terms of asymptotic efficiency, it still leaves the applied researcher confronted with a variety of possible alternative approaches without providing a real guide for empirical research. In this respect, it is also worth noting the Monte Carlo studies by inter alia Gonzalo (1994), Gregory (1991), Reimers (1992), Inder (1993), the survey by Johansen (1992b), and those by Campbell and Perron (1991, 1992) who propose rules the applied researcher has to be aware of when studying relations among nonstationary macroeconomic times series. These rules cover inference on the presence of unit roots in univariate times series as well as in multivariate time series. Nevertheless, even in this extensive review of alternative estimators, tests, and model representations, there remain at least two important questions for the empirical researcher: (i) Should the short-run dynamic be explicitly modelled or not? (ii) Should cointegrated processes be investigated in closed multivariate systems, in open multivariate systems (or partial systems where the generating model of some variables is not modelled), or in a single-equation framework?

Johansen (1992b) clearly favours the use of a system modelling framework based on complete vector autoregressive Gaussian models in which a set of interesting statistically and economically meaningful hypotheses can be raised (see also Johansen and Juselius, 1992). Although the optimality of full systems based approaches has been pointed out for example by Phillips (1991), conditional (partial) models have similar optimal properties when exogeneity conditions are satisfied. In this case, lower-dimension systems are, from a practical point of view, easier to handle and standard limiting distribution results apply to the cointegrating vector estimators (see Johansen 1992a; Boswijk, 1992). It should however be stressed that incorrect exogeneity assertions invalidate any subsequent inference so that testing for exogeneity assumptions should be an integral and unavoidable step in any partial system modelling exercise.

The empirical researcher is thus often confronted with the difficult choice of running the risk of losing efficiency and facing invalid inference when using a partial system analysis or losing empirical tractability when using a complete system framework. In this paper, we shall be concerned with the practical aspects of this apparent dichotomy and illustrate this in an empirical study of Belgian aggregate imports.

The outline of the paper is as follows. In Section 2 we introduce the empirical problem studied in this paper and the economically interesting hypotheses we are facing. In Section 3 we discuss the framework that is used in empirical
In Section 4 we report empirical results for Belgium using quarterly data for the period 1970-1991. Some conclusions and general remarks on dynamic empirical modelling are reported in last section.

In the sequel, we take the following conventions: lower-case letters denote the natural logarithm of the corresponding variables and \( \Delta_k \) defines the \( k \)th difference, i.e., \( \Delta_k x_t = x_t - x_{t-k} \).

2. The econometric modelling of aggregate imports

The econometric modelling of aggregate trade flows has a long history in the economic and applied econometric literature (see the survey of Goldstein and Khan, 1985). One of the reasons for this important amount of published studies is certainly the fact that the underlying (economic) theoretical framework for the determination of prices and trade volumes is usually quite simple and familiar from standard consumer demand or production theory. On the other hand, the effectiveness of international trade policies is dependent on the size of price and activity effects on trade flows so that policy makers show important interest in reliable estimates of these parameters. From an econometric point of view it has been however surprising, as already pointed out by Goldstein and Khan (1985), that among all fields of applied econometrics, researchers dealing with the modelling of trade flows have been quite reluctant to integrate the recent advances in time series econometrics. Exceptions include among others the contributions by Gagnon (1988), Husted and Kollintzas (1987), and Clarida (1991) who develop theoretical based dynamic stochastic models for aggregate imports, while Urbain (1990, 1992a) and Asseery and Peel (1991) use cointegration theory for the empirical modelling of aggregate imports.

Although the nature of the goods being traded, the end-use and the geographical orientation are factors which are important in modelling disaggregated trade flows, we restrict attention to total aggregate import flows so that the basic underlying economic theory can be quite simple. Traditionally, the basic question is whether imports (and exports) are, or are not, perfect substitutes for domestically produced goods. Since two-way trade is usually observed, i.e., imports, exports, domestic production, and intra-industry trade, a perfect substitute model is ruled out for the modelling of international aggregate trade flows. Within the framework of the imperfect substitutes model (and under the assumption of infinite supply elasticity), a prototypical long-run import demand model reads as

\[
m_t = f(y_t, pm_t/pd_t),
\]

where \( m_t \) is import volume, \( y_t \) some activity or demand variable (usually real domestic income), \( pd_t \) the domestic price of tradable goods, and \( pm_t \) the import
prices expressed in the domestic currency. Usually, price homogeneity is imposed so that price effects are captured using a relative price ratio defined as \( pr = \frac{pn}{pd} \). Such an homogeneity assumption is however questionable, especially in short-run aggregate import demand models; see the discussion in Murray and Ginman (1976) and Urbain (1992c). Based on this underlying long-run specification various models have been proposed, ranging from static to partial adjustment models, all of these derived in a single-equation framework. However, several authors, including Urbain (1992a, 1993), Gagnon (1988), and Marquez (1991), have argued in favour of alternative and less restrictive specifications or at least a less restrictive modelling framework.

Since the seminal paper by Orcutt (1950), most researchers working in the field of trade flows modelling have been aware of the statistical problems underpinning single-equation modelling of aggregate trade flows. The empirical relevance of Orcutt's critiques, which include simultaneity, instability, ..., has been extensively analysed over the last decades with more or less fortune. An account of these is given in Goldstein and Khan (1985). In this paper, we shall implicitly be concerned with several of these, and especially the well-known simultaneity bias critique which is central to Orcutt's paper. It stems from the idea that single-equation estimates are biased due to the simultaneity of prices and quantities, resulting in the lack of identification of the true demand equation, in which case a single-equation specification implies estimates which are weighted averages of demand and supply elasticities. To overcome this problem one usually assumes an infinite supply elasticity. Tentative work based on simultaneous models has been pursued, without convincing empirical success (see Goldstein and Khan, 1985). As pointed out by Haynes and Stone (1985), most supply studies use supply–quantity models where the quantity supplied is a function of both present and past prices, resulting in the well-known lack of identification. When the firms act in a uncertain world, they will probably pursue a pricing policy which is based on past market conditions so that the supply model becomes more naturally a price-supply equation. Even in this case, exogeneity of the prices is open to empirical analysis if the derivation of price and income effects in international trade has be to investigated. Another interesting question raised in Orcutt's (1950) paper is the so-called quantum effect, as labelled by Magee (1975), which implies that trade flows react differently to small changes in prices than to large changes. In particular, it is conjectured that adjustment to exchange rates will be more rapid and more significant than adjustment to (usually small) price changes. Although the empirical results on the quantum effect have been quite inconclusive, we note the results of Wilson and Takacs (1979) supporting this view, at least for relatively open economies.

It appears thus clear that several important topics are of interest in order to obtain reasonably reliable estimates of income and price effects in international
trade:
- the potential simultaneity between prices and import volume;
- the dynamic specification and recognition of the trending character of the series;
- the use of a relative price and the underlying homogeneity assumption;
- the potential differences in responses of trade flows to prices and exchange rate changes.

What we will point out in this paper, is that cointegrated system analysis provides an appealing framework for the investigation of these various aspects. As simultaneity is one of the key issues in this field (see also the motivation underlying the recent paper of Marquez, 1991), it already implicitly raises the question of single-equation versus system analysis. Notice that we focus on Error Correction Models (ECMs) which have been found in Urbain (1990, 1992a) and Asseery and Peel (1991) to provide well-specified models for aggregate imports by clearly separating short-run from long-run effects in a single parameterisation.

3. Partial or full system analysis of aggregate imports?

3.1. Full system approach

Among the various system based approaches to cointegration, we confine ourselves to the Gaussian maximum likelihood framework based on complete VARs proposed and advocated by Johansen and Juselius in a number of important papers (Johansen, 1988, 1991; Johansen and Juselius, 1990, 1992, 1994). Consider a vector autoregressive error correction model of order $p$ for the $(k \times 1)$ vector time series $\{x_t\}$:

$$\Delta x_t = \sum_{i=1}^{p-1} \Gamma_i \Delta x_{t-i} + \Gamma x_{t-1} + \mu + \Phi D_t + \epsilon_t, \quad t = 1, \ldots, T, \quad (2)$$

where $\epsilon_t$ denotes a $k$-dimensional normal variate with mean zero and nonsingular, p.d.s. covariance matrix $\Sigma$. $\mu$ is a vector of constant terms and $D_t$ is a vector of other deterministic variables. We assume that:

1. $\text{rank } (\Gamma) = r < k$ so that $\Gamma$ can then be written as $\alpha \beta'$, where both $\alpha$ and $\beta$ are $(k \times r)$ matrices of full column rank;

2. $\alpha' (I - \Gamma_1 - \ldots - \Gamma_{p-1}) \beta$ has full rank, with $\alpha_\perp$ and $\beta_\perp$ $k \times (k - r)$ matrices of full rank which are orthogonal to $\alpha$ and $\beta$.

Under these conditions, $\{x_t\}$ is $I(1)$ and there are $k - r$ nonstationary linear combinations and $r$ stationary linear combinations, i.e., there are $r$ cointegrating
relationships (see Johansen, 1991). The \( r \) rows of \( \beta' \) and the \( r \) cointegrating vectors while the elements of \( \alpha \) are the factor loadings, i.e., the weights of the different cointegrating vectors in the different equations. As shown in Johansen (1988, 1991), the maximum likelihood estimate of a basis of the cointegrating space is then given by the empirical canonical variates of \( x_{t-p} \) with respect to \( \Delta x_t \), corrected for the short-run dynamic and the deterministic components. The number of cointegrating relationships is given by the number of significant canonical correlations and their significance can be tested by means of a sequence of Likelihood Ratio (LR) tests whose limiting distribution is expressed in terms of vector Brownian motions (see Johansen, 1988, 1991). Once the number of cointegrating relationships has been determined, particular hypotheses on \( \alpha \) and/or \( \beta \) can be tested using \( \chi^2 \) LR tests. Inference on the presence of deterministic components can also be conducted as pointed out in Johansen and Juselius (1990, 1992) and Johansen (1992c).

At this stage it is interesting to mention the issue of identification of the different cointegrating vectors.\(^1\) Without further restrictions or normalization, the ML approach does only identify the space spanned by the columns of \( \beta \), but the different cointegrating vectors are not yet identified. For any nonsingular \((r \times r)\) matrix \( P \) we can define \( \beta^* = \beta P' \) and \( \alpha^* = \alpha P^{-1} \) so that \( \alpha^* \beta^* = \alpha \beta' \).

Therefore identification of the different cointegrating relations is done \textit{a posteriori} by imposing restrictions on the \( \beta \) matrix; see for example Johansen and Juselius (1994) and Mosconi (1991). By analogy with traditional simultaneous equation models (as discussed in Johansen and Juselius, 1994), we impose (in addition to the normalization) at least \( r - 1 \) restrictions of the form \( R_i \beta_i = 0 \), with \( R_i \) a \((g_i \times k)\) matrix of known elements (with \( g_i \geq r - 1 \)), on each cointegrating vector \( (\beta_i) \) in order for the system of long-run (stationary) relationships to be identified. In other words, if we define a matrix \( B_i \) orthogonal to \( R_i \), i.e., \( B_i = R_{i+} \), we require the \( i \)th cointegrating vector to satisfy \( \beta_i = B_i \phi_i \), where \( \phi_i \) is composed of unknown parameters. The rank condition for identification requires

\[
\text{rank } (R_i \beta) \geq r - 1, \quad i = 1, \ldots, r.
\]

If the equality holds, then just-identification is achieved, otherwise the system of long-run relationships will be overidentified.\(^2\) A simple choice for exact identification is to consider (Johansen, 1991) the transformation \( \tilde{\beta}_e = \beta (C' \beta)^{-1} \) with the matrix \( C' = (I_r, 0) \). Such a transformation usually relies on the implicit

\(^{1}\) On the problem of identification, see inter alia Bauwens and Lubrano (1993), Boswijk (1992), Campbell and Perron (1992), and Johansen and Juselius (1994).

\(^{2}\) Notice however that the rank condition is not sufficient for identification. Conditions for restrictions to be formally identifying without necessitating the use of generic (usually) unknown coefficients are given in Theorem 1 in Johansen and Juselius (1994).
assumption that we can partition the vector of variables into a $r$-dimensional subset of 'endogenous' and $k - r$ 'exogenous' (in an economic sense) variables. The resulting system for $\{x_t\}$ is then interpreted as a reduced form model, both with respect to the long-run and short-run structures (see Johansen and Juselius, 1992b). Naturally, economic theory should help in imposing (just) identification restrictions through the specification of $R_i$, in which case the resulting cointegrated VAR becomes a short-run reduced form model with a long-run structural form (Johansen and Juselius, 1994). When the short-run structure has to be jointly investigated, it is nevertheless preferable to reformulate the VAR-ECM in the form of a dynamic structural simultaneous equation model (SEM) of the general form

$$
A_0 d_{t+1} x_t = \sum_{i=1}^{p-1} A_i d_{t+1} x_{t-i} + \alpha^+ \beta^+ x_{t-i-1} + \mu^* + \Phi D_t + u_t,
$$

where $A_0$ is a nonsingular $k \times k$ matrix with $\text{diag}(A_0) = \text{diag}(I_k)$, $u_t = A_0 \varepsilon_t$, $\alpha^+ \beta^+ = \alpha \beta^*$, while the parameters contained in $(A_0, \{A_i\}, \alpha^+, \beta^+)$ are functions of those of (2) such that are assumed to be economically well-defined.$^3$ In the framework of cointegrated $I(1)$ variables, Hendry and Mizon (1993) have proposed a set of reductions for the modelling of structural econometric models which arise as reparametrisations/reductions of cointegrated VARs. One first performs a cointegration analysis using Johansen's approach. Once the cointegrating relationships have been determined and identified, the analysis is divided in three major steps. First, the unrestricted VAR (UVAR) model is reparametrised in order to be an acceptable congruent constant parameter representation of the data. The analysis is then mapped from the space of $I(1)$ variables to the space of $I(0)$ variables where a parsimonious VAR (PVAR) model is formulated. If the resulting model is congruent with the data evidence, it is considered as the maintained model against which each subsequent simplification is tested. A traditional SEM is specified and the final (complete) SEM, estimated by full information maximum likelihood (FIML), compared to the PVAR. The validity of the overidentifying restrictions implied by the SEM are tested against the PVAR. The SEM will be considered as an adequate characterisation of the data if the reductions of the PVAR are valid. Similar reduction sequences in a stationary framework are considered in Monfort and Rabemanajara (1990) and Palm (1983).

3.2. Conditional and structural subsystems

Another framework in which cointegration can be analysed is that of open systems where some variables are considered as strongly, or at least weakly, correlated.
exogenous for the parameters of interest. The motivation for the use of condi-
tional subsystems stems for the traditional way one usually partitions the set of
variables under investigation between so-called endogenous variables and those
whose generating model will not be modelled, the 'exogenous' variables. Al-
though proceeding this way involves the risk of imposing invalid exogeneity
assumptions, and hence invalidating any subsequent inference, it has the ad-

tantage that the analysis is conducted in open systems where some variables
may be added without requiring to be explicitly modelled and thus without
affecting the number of equations of the model. Note that when formulating
a VAR model, the selection of the variables is usually influenced by economic
theory or previous empirical stylised facts. Typical examples include the studies
of Johansen and Juselius (1990) and Hendry and Mizon (1993) where the
variables selected are those believed to enter long-run money demand equations.
If the variables appearing in the model are eventually sufficient to model money
demand, they may not be sufficient to explain the evolution, both in the long and
in the short run, of the remaining variables in the system, so that subsystem
analysis could be equally appropriate. This probably explains why some studies
(Clements and Mizon, 1991; Hendry and Mizon, 1993) based on full models had
to include several dummies to get parameter constancy and normality of
residuals. These dummies might reflect the insufficiency of the conditioning
information set which could be expanded to include more relevant variables
explaining these events. An alternative approach to deal with this problem is to
enlarge the VAR by jointly modelling variables that account for, or at least
explain, the parameter nonconstancy in the lower-dimensional VAR. Again, the
trade-off between increased dimensionality and invalid exogeneity applies.

For these reasons, we shall consider the partial system approach advocated
by Boswijk (1991, 1992, 1995) based on so-called structural ECMs. Under the
testable hypothesis of weak exogeneity (Boswijk, 1992; Johansen, 1992a) the
approach is equivalent to ML estimation and inference on the long-run para-

meters can be conducted by means of traditional \( \chi^2 \) statistics. It also allows
economically meaningful restrictions to be incorporated in the long-run rela-
tionships, while further (overidentifying) restrictions are easily introduced at
a later stage. We partition the vector of variables \( x_t = (y_t, z_t) \) with \( y_t \) a \((g \times 1)\)
vector of endogeneous variables and \( z_t \) a \((k - g \times 1)\) vector of (weakly) exo-
genous variables. We consider the following structural error correction model:

\[
\Gamma y_t = \kappa + \Pi_0 z_t + \lambda \beta' x_{t-1} + \sum_{i=1}^{p-1} (\Phi_i Ay_{t-i} + \Pi_i A z_{t-i}) + \Phi D_t + \eta_t, \quad t = 1, \ldots, T, 
\]

\[^4\text{For a more complete derivation of this specification as a reparametrisation of a conditional ECM see Boswijk (1991, 1992) or Urbain (1992b).}\]
where $\Gamma$ is a $(g \times g)$ nonsingular matrix and $\eta_i$ is an innovation sequence with respect to the $\sigma$-field generated by the set $(z, x_{i-1}, \ldots)$ whose covariance matrix is denoted by $\Omega$. We assume that $r$, the rank of the cointegrating matrix $\beta'$, is at most equal to $g$ so that all cointegrating relationships can be identified from the conditional model of $y_i$ given $z_i$. Following Boswijk (1991, 1992, 1995) we assume that $r = g$, i.e., the number of cointegrating relationships equals to the number of endogenous variables, and that the rank of $\lambda$ is equal to $r$ so that no cointegrating relationships drop out of the conditional model. In order to identify both the long- and short-run structure of (3), we normalize $\Gamma$ such that (i) $\Gamma_{ii} = 1$, (ii) $\beta$ such that $\beta_{ii} = 1$, (iii) we impose the error correction matrix $(\lambda)$ to be diagonal, (iv) we impose $g - 1$ restrictions of the form $R_i\beta_i = 0$, $i = 1, \ldots, r$, where $R_i$ is a $(g - 1 \times k)$ known matrix of full row rank and $\beta_i$ is the $i$th column of $\beta$, i.e., the $i$th cointegrating vector. For the model to be just-identified, we require that

$$\text{rank} \ (R_i\beta) = r - 1 = g - 1, \quad i = 1, \ldots, r.$$  

Under these (just) identifying restrictions, Boswijk (1991) proposes to test for the significance of the cointegrating relationships using a Wald test for the significance of the error correction term, based on OLS estimation (when $g = 1$) or IV when $g > 1$. The limiting distribution of the Wald test is expressed as a functional of a vector Brownian motion whose critical values are tabulated in Boswijk (1991, 1992). When the exogenous variables have linear trend components, the limiting distribution depends on unknown nuisance parameters unless a deterministic trend term is added to the regressors. Once the number of cointegrating relationships has been established, their estimates are obtained by using indirect estimators, equivalent to ML estimation under weak exogeneity.

3.3. Exogeneity and conditional models

As pointed out by Johansen (1992a), the asymptotics of partial cointegrated models rests on the validity of exogeneity assumptions. Weak exogeneity has been studied recently in a number of papers in the framework of cointegrated error correction models (see inter alia Boswijk, 1992, 1995; Hendry and Mizon, 1993; Johansen 1992a; Urbain, 1992b). To make the discussion clear and to enable a clear link with the empirical analysis performed in this paper we consider various cases that are of interest:

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5 Alternative ways of identifying the short-run structure of system (3) do exist such as restricting $\Gamma$ to be a unit matrix, or considering a recursive system by restricting the matrix $\Gamma$ of contemporaneous effects to be triangular and $\Omega$ to be diagonal (for further discussion see Boswijk, 1992; Johansen and Juselius, 1994).
(1) When the long-run parameters, i.e., the cointegrating vectors and the error correction coefficients, are the parameters of interest, a sufficient and necessary condition for weak exogeneity of $z_t$ is that the conditioning variables are not error-correcting. In that case standard $\chi^2$ distribution applies for inference on the long-run parameters. Weak exogeneity w.r.t. the long-run parameters can be tested in different model representations and amounts to test for the presence of the cointegrating relationship of interest in the generating model of the conditioning variables. For this purpose, one can build a partial model and test for the presence of the cointegrating vector(s) of interest in the marginal models using misspecification $F$-tests (Johansen, 1992a) or Lagrange multiplier tests (Boswijk, 1992). One can also test such a restriction within the full system framework by testing appropriate restrictions on the weighting matrix $a$.

(2) When a reparameterised structural ECM is built and when the short-run dynamic response parameters are considered as parameters of interest too, then a further orthogonality condition can be necessary (see Urbain, 1992b) since the resulting structural model may not correspond to a valid reparameterisation of the conditional expectation. In this case, a variable addition test computed in a limited information set-up allows us to conduct inference jointly on the orthogonality condition and on the cross-equation restrictions implied by cointegration using traditional $\chi^2$ statistics (see Boswijk and Urbain, 1994).

(3) We should also consider the case where the parameters of interest are those related to one of the (potentially) many cointegrating vectors. Consider for example that we have two endogenous variables, $y_t = (y_{1t}, y_{2t})$, and, given our assumptions, two cointegrating vectors $\beta_1$ and $\beta_2$ which we assume have been properly identified. We rewrite (3) as a two-equations system where the indices arise from the partitioning of $y_t$:

$$
\Gamma_{11} \Delta y_{1t} + \Gamma_{12} \Delta y_{2t} = \lambda_1 \beta_1' x_{t-1} + \kappa_1 + \Pi_{10} \Delta z_t
$$

$$
\Gamma_{21} \Delta y_{1t} + \Gamma_{22} \Delta y_{2t} = \lambda_2 \beta_2' x_{t-1} + \kappa_2 + \Pi_{20} \Delta z_t
$$

We assume that the parameters of interest are given by $(\lambda_1, \beta_1)$. Weak exogeneity of $z_t$ for $(\lambda_1, \beta_1)$ still requires the absence of $\beta_1' x_{t-1}$ in the marginal model (system) for $\Delta z_t$. We can however also ask whether the second endogenous variable, $y_2$, is weakly exogenous for the cointegrating parameters of the first equation. As shown in Boswijk (1997, Theorem 5.1)
this amounts to test for the validity of a block-recursive structure, i.e.,
\[ \Gamma_{21} = 0, \quad \Omega_{21} = \text{cov}(\eta_{1t}, \eta_{2t}) = 0. \]
Consequently, weak exogeneity of \( y_2 \) for \((\lambda_1, \beta_1)\) can be tested by means of a \( \chi^2 \) distributed LR test (see Boswijk, 1992).

When weak exogeneity of the conditioning variables is violated, a full system analysis might be preferred, but we can still work within the structural ECM framework and follow the two-step procedure recently proposed by Boswijk (1992, 1995) in order to achieve full efficiency, even in the case of failure of weak exogeneity.

4. Empirical results

In this section we model aggregate imports for the Belgium economy, known for its high degree of openness on the foreign markets. The empirical study reported in this paper is an extension of the work presented in Chapter 5 of Urbain (1993) which was based on the imperfect substitutes model presented in Section 2. Let us briefly summarise the results reported in that study. Using quarterly data for the period 1960–1991 and using the basic imperfect substitutes model as theoretical background it was found that: (i) one long-run relation could be detected from the data, i.e., one cointegrating vector, (ii) a complete SEM could be derived at the expense of using several shift dummies, (iii) no significant price effects were detected in the long run, (iv) the conditional analysis showed some clear advantages over the full system based approach, and this mainly in relation to the uniqueness of the cointegrating relationship.

However, from an economic point of view, the zero long-run price effects might be questioned for its economic implications, especially with respect to the effectiveness of economic policy affecting Belgium's trade balance. In order to check the robustness of the results, we choose to extend this analysis in order to take into account the potential existence of quantum effect in Belgian imports, i.e., the possibility of differential responses of aggregate imports to large and small price changes usually respectively associated to changes in exchange rates and prices. Therefore, one has to replace the import price expressed in domestic currency as used in (1) by two separate terms. If in a bilateral set-up this disaggregation is easily obtained, the total aggregate framework requires the use of terms reflecting the potential changes that can occur with all the different trade partners. For this purpose, we use the effective exchange rate published by the National Bank of Belgium which is calculated as weighted averages across the major trade partners, using current-period import shares as weights. For the construction of the import price in foreign currency we use a similar weighted average, using the trade shares published by the National Bank of Belgium and
the export unit value index of the trade partners expressed in the basis 1975 = 100. Given that effective exchange rates are only calculated since 1970, the analysis reported in this section concerns the sample 1970:02-1990:02.

The variables we thus use in the analysis are: import volume denoted by $m_t$, import prices expressed in foreign currency denoted by $p_m$, domestic prices (wholesale price index) denoted by $p_d$, real income (GDP deflated by $pd$) denoted by $y_t$, and effective exchange rate denoted by $ex_t$. These variables are justified by the imperfect substitute model as well as by those derived by Gagnon (1988) and Husted and Kollintzas (1987). Following the discussion in Urbain (1992c) we do not use relative prices. All variables are transformed in natural logarithm. Data sources and figures for both levels and 1st differences are reported in the Appendix. All the empirical calculations have been undertaken with RATS 3.10 and P.C.GIVE 6:01.6

4.1. Multivariate cointegration and simultaneous equation models

We now build a multivariate model enabling us to investigate the cointegration properties of the data. Since we found no evidence in favour of $I(2)$ processes, we conduct the analysis under the assumption that our selected variables are well represented by $I(1)$ processes with potential deterministic components. We first consider a closed model for aggregate imports and follow the approach proposed in Hendry and Mizon (1993). The starting parametric model we use is a cointegrated VAR model which, if congruent with the data evidence, is considered (see Spanos, 1990) as a characterization of the so-called Haavelmo distribution, i.e., a statistical model for the joint dependent variables denoted by the $(5 \times 1)$ vector $x_t$, with $x_t' = (m_t, y_t, p_m, ex_t, pd_t)$. To this end, we use the maximum likelihood estimation procedure developed in Johansen (1988, 1991) and Johansen and Juselius (1990), based on a complete vector autoregressive error correction model such as (2). The model is fitted with three centered deterministic seasonal quarterly dummies. As the data series display

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6 Since the data are unadjusted quarterly times series, the problem of seasonality has to be accounted for. Applying the Hylleberg et al. (1990) procedure, the presence of unit roots at seasonal frequencies is rejected for all series when centered seasonal dummies are introduced. Given the lack of support for the presence of seasonal unit roots, we use deterministic centered seasonal dummies in the subsequent multivariate models. For each series, the presence of a unit root at the zero frequency is not rejected using Augmented Dickey–Fuller tests, Phillips–Perron test and Schmidt–Phillips test. The same battery of tests applied to the first differences of the series leads to a clear rejection of the unit root hypothesis in the first differences. The series seem thus well characterised as $I(1)$ processes displaying a single unit root in their autoregressive part and a nonzero drift. Due to space availability we do not report the results of these unit root tests. These results are available from the author upon request.
linear trend components, we allow an unrestricted nonzero deterministic drift vector $\mu$ to be present in the model.\footnote{This parameterisation of the deterministic part of the model is also formally derived using the sequence of LR test statistics as proposed in Johansen (1992c).}

As pointed out in Boswijk and Franses (1992) and Reimers (1992) the choice of the maximum lag length used in the specification of the vector error correction model (2) can affect the determination of the dimension of the cointegrating space. In particular they find that overfitting implies a loss of power, while underfitting leads to potential spurious cointegration. The problem that we encounter with the modelling of trade flows is the long-delayed effects of prices on imports which may last up to two or three years (see the results reported in Goldstein and Khan, 1985; Urbain, 1990). As a matter of fact, using lag order selection criteria such as BIC or AIC results in low-order VAR models, usually of order 2 or 3. Our problem is here different for we are trying to develop a model that has to be general enough to capture the long-run relations, but also the short-run dynamic which will afterwards be more parsimoniously reparameterised. Consequently, we deliberately use VAR's of relatively high order to be ensured that the short-run dynamic is well captured. Following the device of Boswijk and Franses (1992), we applied Johansen's procedure for different numbers of candidate models and based our final choice of the lag length on two criteria: (i) absence of serial correlation in the residuals (using Box–Pierce tests for 16th-order serial correlation) and (ii) the significance of the parameter estimates of the short-run coefficients (using $F$-tests for the nullity of the corresponding column in the VAR). This resulted in the choice of a lag length of four periods. The results are reported in Table 1.

The trace test and the maximal eigenvalue tests suggest the existence of two cointegrating vectors given by the two first columns of the $\beta$ matrix. The second unrestricted vector looks like a long-run import demand function with theoretically expected signs and an important weight in the import equation. The statistics on the residuals of this model are shown in Table 6. According to Jarque and Bera's normality test, distributed as a $\chi^2(2)$ under the null, normality of the residuals is rejected for the equation determining domestic prices. A within-sample analysis of each equation of this UVAR, using multivariate recursive least squares and sequential Chow tests, shows substantial parameter nonconstancy and outliers around the years 1973, 1979, 1982 and 1986. In analogy to what happens with unit roots test, we can expect this parameter nonconstancy to affect the behaviour of the statistics reported in Table 1. Moreover, for the UVAR to be considered as a valid starting point in the analysis, it should be a congruent data representation with constant parameters. As shown in the Appendix, $\Delta pm_t$ and $\Delta pd_t$ display important values at the end of 1973, end of 1979, as well as around the period 1985–1986, corresponding respectively
Table 1
Multivariate cointegration analysis, $k = 4$

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Trace test</th>
<th>$H_0$</th>
<th>Max. eigenvalue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$r \leq 4$</td>
<td>4.681</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r \leq 3$</td>
<td>8.941</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r \leq 2$</td>
<td>12.995</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r \leq 1$</td>
<td>24.602**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r = 0$</td>
<td>31.579*</td>
</tr>
</tbody>
</table>

Normalized cointegrating vectors: $\beta$ matrix

<table>
<thead>
<tr>
<th>$m$</th>
<th>-0.193</th>
<th>1.000</th>
<th>1.000</th>
<th>1.000</th>
<th>1.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-0.028</td>
<td>-1.225</td>
<td>-4.700</td>
<td>-0.933</td>
<td>-0.610</td>
</tr>
<tr>
<td>$pm$</td>
<td>0.397</td>
<td>0.630</td>
<td>-24.034</td>
<td>-0.954</td>
<td>0.418</td>
</tr>
<tr>
<td>$ex$</td>
<td>1.000</td>
<td>1.046</td>
<td>-22.140</td>
<td>2.168</td>
<td>0.515</td>
</tr>
<tr>
<td>$pd$</td>
<td>-0.383</td>
<td>-1.058</td>
<td>-37.977</td>
<td>1.659</td>
<td>-1.048</td>
</tr>
</tbody>
</table>

Weights: $\alpha$ matrix

<table>
<thead>
<tr>
<th>$m$</th>
<th>-0.046</th>
<th>-0.454</th>
<th>0.000</th>
<th>-0.052</th>
<th>-0.084</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-0.225</td>
<td>-0.016</td>
<td>0.002</td>
<td>0.038</td>
<td>-0.172</td>
</tr>
<tr>
<td>$pm$</td>
<td>0.019</td>
<td>-0.074</td>
<td>0.006</td>
<td>-0.001</td>
<td>0.006</td>
</tr>
<tr>
<td>$ex$</td>
<td>0.184</td>
<td>0.220</td>
<td>0.001</td>
<td>0.035</td>
<td>0.006</td>
</tr>
<tr>
<td>$pd$</td>
<td>-0.109</td>
<td>0.075</td>
<td>0.000</td>
<td>-0.025</td>
<td>0.022</td>
</tr>
</tbody>
</table>

* and ** indicate significance at 5% and 10% levels, respectively.

to the first and the second oil-shocks and to the important dollar rise and fall of the mid-eighties. Similarly, the important outlier in the first quarter of 1982 reflects the devaluation of the Belgium franc. In order to remove these effects from our data set we added four shift dummies denoted by $D_{74,079,082}$, $D_{82,086}$. Notice that the use of these dummies is required due to the specific form of the model that we are maintaining since we marginalise with respect to all the variables not included in the information set, i.e., the variables which are probably the source of the parameter nonconstancy and outliers. As these dummies will enter each equation of the VAR, and hopefully only selectively in the final SEM, the task of encompassing the VAR model will be even more difficult. Once these dummies are introduced, there remains almost no non-normality (see Table 6) and sequential Chow tests revealed no significant

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* $D_{74} = 1$ in 1973:04 and 1974:01, 0 otherwise, $D_{79} = 1$ in 1979:03, $D_{86} = 1$ in 1986:01, $D_{82} = 1$ in 1982:01, 0 otherwise.
Table 2

Multivariate cointegration analysis, with dummies, $k = 4$

Eigenvalues

<table>
<thead>
<tr>
<th></th>
<th>0.402</th>
<th>0.299</th>
<th>0.131</th>
<th>0.111</th>
<th>0.017</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trace test</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r \leq 4$</td>
<td>1.341</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td>10.434</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>21.247</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>48.550*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r = 0$</td>
<td>88.076*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Trace test

$H_0$

|       |       |       |       |       |
|-------|-------|-------|-------|
| Max. eigenvalue | 1.341 | 9.093 | 10.813 | 27.303** |

Trace test $rG4$ Max. eigenvalue $rG3$

21.247 9.093 10.813 27.303**

Normalized cointegrating vectors: $\beta$ matrix

<table>
<thead>
<tr>
<th></th>
<th>$m$</th>
<th>$y$</th>
<th>$pm$</th>
<th>$ex$</th>
<th>$pd$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.000</td>
<td>-1.030</td>
<td>0.310</td>
<td>0.301</td>
<td>-0.654</td>
</tr>
<tr>
<td></td>
<td>0.242</td>
<td>-0.444</td>
<td>0.443</td>
<td>1.000</td>
<td>-0.581</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>-17.443</td>
<td>263.065</td>
<td>305.184</td>
<td>409.612</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>-70.561</td>
<td>59.022</td>
<td>136.194</td>
<td>75.561</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>-0.629</td>
<td>0.463</td>
<td>0.658</td>
<td>-1.265</td>
</tr>
</tbody>
</table>

Weights: $\alpha$ matrix

<table>
<thead>
<tr>
<th></th>
<th>$m$</th>
<th>$y$</th>
<th>$pm$</th>
<th>$ex$</th>
<th>$pd$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.792</td>
<td>-0.392</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>-0.439</td>
<td>-0.611</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>-0.019</td>
<td>-0.045</td>
<td>0.002</td>
<td>0.007</td>
<td>0.015</td>
</tr>
</tbody>
</table>

* and ** indicate significance at 5% and 10% levels, respectively.

within-sample parameter nonconstancy. Given these results, the number of cointegrating vectors and their structure is reanalysed within a fourth-order VAR augmented by the four dummies defined above. The results are reported in Table 2.

Again, the trace test and the maximal eigenvalue test suggest the existence of two cointegrating vectors given by the two first columns of the $\beta$ matrix. These two linear combinations are graphed in Figs. 1 and 2. They are very similar to the vectors found in the previous case, although it is now the first unrestricted vector which looks like a long-run import demand with theoretically expected signs and an important weight in the import equation. The second vector is difficult to interpret without any identifying information.

In order to identify the two separate cointegrating vectors, we impose import prices and exchange rates to have identical effects in the long run in order to

---

9 It should be noted that the analysis is performed under the assumption that the presence of these dummies does not affect the limiting behaviour of the trace and maximum eigenvalue tests.
identify the import demand equation. In a first stage we tried to identify the
long-run import demand equation by imposing the theoretically motivated
symmetry of the price effects, but such a restriction resulted in a lack of what
Johansen and Juselius (1994) call empirical identification. For the second cointe-
grating vector, we dropped income in order to identify a long-run relation for
effective exchange rates. Under these restrictions, we obtain the cointegrating
and weights matrices reported in Table 3. Table 4 reports the (justidentified)
long-run reduced form $\hat{\beta}_e = \hat{\beta}(C'\hat{\beta})^{-1}$ with matrix $C$ defined as $C' = (I_2 \ 0)$.
Further restrictions can be tested to reduce the cointegrating space as suggested by Table 3. Note that the price homogeneity assumption in the long-run import demand function, i.e., symmetric effects between domestic and foreign prices, is clearly rejected by the data. The LR test for this hypothesis, computed using the switching algorithm proposed in Johansen and Juselius (1992), distributed as a $\chi^2(2)$ under the null gives a value of 8.102. Proceeding sequentially, we tested further reductions and finally end up with the following description of the cointegrating space: (i) the long-run import demand cointegrating vector only enters the import equation, (ii) unit long-run income effects (restriction on the first cointegrating vector), (iii) symmetry of the price coefficients in the second cointegrating vector, (iv) absence of the second cointegrating vector in the import equation (restriction on the weights). The LR test for these joint hypotheses, distributed as a $\chi^2(10)$ under the null, gives a value of 10.125, and hence we cannot reject these reductions at a 5% level. The restricted cointegrating vectors and weights are given in Table 5.

The results in this table also suggest that $y$, $pm$, $ex$, and $pd$ are likely to be weakly exogenous for the parameters of the long-run import demand equation, but not for the second long-run relationship which we can interpret as a (probably incomplete) long-run effective exchange rate equation suggesting that the Belgian effective exchange rate is positively related to imports and negatively to relative prices.
Table 5
Restricted cointegrating vectors

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>1.000</td>
<td>-0.194</td>
<td>-0.821</td>
</tr>
<tr>
<td>$y$</td>
<td>-1.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$pm$</td>
<td>0.259</td>
<td>0.347</td>
<td>0.000</td>
</tr>
<tr>
<td>$ex$</td>
<td>0.259</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$pd$</td>
<td>-0.587</td>
<td>-0.347</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 6
Statistics on the error processes

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\sigma$</th>
<th>JB</th>
<th>BP</th>
<th>$\sigma$</th>
<th>JB</th>
<th>BP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>0.027</td>
<td>0.161</td>
<td>21.877</td>
<td>0.023</td>
<td>0.431</td>
<td>19.552</td>
</tr>
<tr>
<td>$y$</td>
<td>0.030</td>
<td>1.112</td>
<td>23.324</td>
<td>0.026</td>
<td>4.425</td>
<td>18.138</td>
</tr>
<tr>
<td>$pm$</td>
<td>0.015</td>
<td>0.004</td>
<td>12.441</td>
<td>0.014</td>
<td>0.250</td>
<td>13.017</td>
</tr>
<tr>
<td>$ex$</td>
<td>0.015</td>
<td>1.012</td>
<td>13.432</td>
<td>0.013</td>
<td>2.928</td>
<td>17.528</td>
</tr>
<tr>
<td>$pd$</td>
<td>0.009</td>
<td>8.624</td>
<td>11.030</td>
<td>0.007</td>
<td>4.669</td>
<td>20.776</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\sigma$</th>
<th>JB</th>
<th>BP</th>
<th>$\sigma$</th>
<th>JB</th>
<th>BP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>0.026</td>
<td>0.512</td>
<td>23.520</td>
<td>0.028</td>
<td>0.575</td>
<td>21.825</td>
</tr>
<tr>
<td>$y$</td>
<td>0.029</td>
<td>2.757</td>
<td>22.748</td>
<td>0.030</td>
<td>0.878</td>
<td>21.353</td>
</tr>
<tr>
<td>$pm$</td>
<td>0.016</td>
<td>0.970</td>
<td>13.181</td>
<td>0.017</td>
<td>0.852</td>
<td>9.881</td>
</tr>
<tr>
<td>$ex$</td>
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<td>0.961</td>
<td>16.272</td>
<td>0.013</td>
<td>1.181</td>
<td>12.940</td>
</tr>
<tr>
<td>$pd$</td>
<td>0.007</td>
<td>5.310</td>
<td>23.802</td>
<td>0.010</td>
<td>4.920</td>
<td>18.110</td>
</tr>
</tbody>
</table>

Test for the validity of overidentifying restrictions $\chi^2_{49} = 48.130$

JB: Jarque and Bera's normality test, $\chi^2_2$ under the null.
BP: 16th-order Box-Pierce test, $\chi^2_{16}$ under the null.

The unrestricted model is then reparametrised in the space of $I(0)$ variables. A parsimonious VAR model (PVAR) is specified using first differences of the variables, dropping the variables for which we obtain insignificant $F$-tests for the nullity of the associated column in the VAR and adding two identities defining the cointegrating vectors. The PVAR is checked for parameter constancy, normality, and absence of serial correlation. When these properties are satisfied, the PVAR constitutes the general model against which the encompassing
properties, i.e., the overidentifying restrictions, of the final SEM are tested. The final SEM (estimated by FIML) is presented below and was derived using a general to specific modelling approach. We added as identities two empirically defined error correction terms, denoted by $EC_1$ and $EC_2$, corresponding to the two cointegrating vectors of Table 5. The statistics on the error processes from the different models as well as the outcome of the encompassing test statistic are reported in Table 6.

**FIML estimates of the SEM (asymptotic standard in parentheses)**

\[ \Delta m_t = 1.435 \Delta p_d + 1.757 \Delta p_{d_{-3}} + 0.689 \Delta y_t + 0.404 \Delta y_{t-3} - 1.086 \Delta e_{x_{t-3}} \]
\[ (0.36) \quad (0.42) \quad (0.23) \quad (0.11) \quad (0.29) \]
\[ - 0.475 \Delta p_{m_{-2}} - 0.174 \Delta p_{m_{-4}} - 0.704 \quad EC_{1_{-1}}, \]
\[ (0.18) \quad (0.7) \quad (0.09) \]
\[ \Delta y_t = 0.070 D^{82} + 0.048 D^{86} - 0.333 \Delta y_{t-1} - 0.294 y_{t-3} \]
\[ (0.02) \quad (0.02) \quad (0.09) \quad (0.09) \]
\[ - 1.478 \Delta p_d - 0.260 \quad EC_{2_{-1}}, \]
\[ (0.32) \quad (0.08) \]
\[ \Delta p_m = 0.26 D^{73} + 0.410 \Delta e_x - 1.504 \Delta e_{x_{t-1}} + 1.494 \Delta p_{d_{-1}} \]
\[ (0.01) \quad (0.19) \quad (0.11) \quad (0.14) \]
\[ - 0.080 \Delta m_{t-4} + 0.140 \quad EC_{2_{-1}}, \]
\[ (0.04) \quad (0.06) \]
\[ \Delta e_x = - 0.039 D^{82} - 0.587 \Delta p_d + 0.153 \Delta p_{m_{-1}} - 0.135 \Delta p_{m_{-4}} \]
\[ (0.01) \quad (0.13) \quad (0.04) \quad (0.03) \]
\[ - 0.109 \Delta m_{t-1} + 0.099 \Delta m_{t-4} + 0.223 \Delta e_{x_{t-1}} \]
\[ (0.04) \quad (0.04) \quad (0.08) \]
\[ + 0.183 \Delta e_{x_{t-3}} - 0.195 \quad EC_{2_{-1}}, \]
\[ (0.08) \quad (0.04) \]
\[ \Delta p_d = 0.26 D^{73} - 0.048 D^{86} + 0.449 \Delta p_m + 0.102 \Delta p_{m_{-2}} \]
\[ (0.006) \quad (0.008) \quad (0.06) \quad (0.03) \]
\[ + 0.052 \Delta p_{m_{-4}} + 0.581 \Delta e_{x_{t-1}} \]
\[ (0.02) \quad (0.13) \]
\[ - 0.390 \Delta p_{d_{-3}} - 0.092 \quad EC_{2_{-1}}, \]
\[ (0.11) \quad (0.05) \]
\[ EC_{1_t} = \Delta m_t - \Delta y_t - 0.587 \Delta p_d + 0.259(\Delta p_m + \Delta e_x) + EC_{1_{-1}}, \]
\[ EC_{2_t} = - 0.194 \Delta m_t - 0.347 \Delta p_d + \Delta e_x + 0.347 \Delta p_m + EC_{2_{-1}}. \]
The first equation of the SEM corresponds to what is expected for a short-run import demand equation and takes the form of an error correction model with an important error correction term. Domestic prices have a more immediate and important effect than import prices whose effect may last up to four quarters. The use of a relative price is thus inadequate if a well-specified short-run dynamic model for aggregate imports is to be derived. Note also that the use of separate price and effective exchange rate terms enables us to support the view of some quantum effect as the exchange rate has a more important effect on imports than import prices. The remaining equations of the system are well-behaved (see Table 6). As expected, changes in $ex$ and in $pm$ have a positive effect on domestic prices. These results are interesting as they provide a support to a substantial domestic price feedback effect of import prices, see Goldstein and Khan (1985, pp. 1092–1096). It is also interesting to note that the error correction term associated to a long-run exchange rate equation significantly enters the equation for income in which the major role is played by domestic prices, supporting a (negative) unidirectional causality running from prices (inflation) to income. For all but the import equation the presence of dummies is necessary in order to encompass the PVAR, indicating that our information set is probably insufficient to model correctly the behaviour of these variables. Our final SEM is nevertheless relatively parsimonious and encompasses a densely parametrised fourth-order VAR as shown in Table 6 where the computed encompassing test statistic, asymptotically $\chi^2(49)$ under the null, gives a value of 48.130, well below the 5% level. Note however that the four last equations of the SEM are mainly instrumental in this model since their structural economic content is quite difficult to assess.

4.2. Partial approach

Since our interest lies first of all in modelling aggregate imports and not in deriving empirical models for prices, exchange rates, or real income, a conditional approach could well be suitable if the remaining variables in the system are weakly exogenous for both the short- and long-run parameters of the import model. According to the imperfect substitutes model and given the disaggregation of import prices in two terms, $pm$ and $ex$, we now consider two endogenous variables given by import volume $m$ and effective exchange rates $ex$. In order to identify the long-run import demand relationship we again impose the restriction ($R_1$) that import prices and exchange rates have identical effect on imports in the long run. In order to identify the long-run exchange rate equation, we

---

10 This LR test is computed using Sims’ small-sample correction.

11 The computation of a $F(40, 56)$ test for parameter constancy over the last eight observations yields a value of 1.13.
exclude income from the long-run relation \((R_2)\). We assume that the matrix of structural error correction coefficients is a diagonal matrix so that each equation of this two-equations system contains the jointly dependent variables but only one error correction term.\(^{12}\)

The maximum lag length is fixed at four periods in order to ensure that the model has residuals with the desired innovation properties. Wald tests for cointegration are computed based on unrestricted error correction models with a lag length restricted to four periods. The models are estimated by IV and the results are reported in Table 7. For equation \(i\) the instruments selected are the predetermined variables in the model as well as \(R_{i,t-1}\) which are valid instruments for equation \(i\) since they are absent from the equation \(i\) due to the identification restriction but correlated to the other endogenous variables due to the error correction term in the other equation. The presence of cointegration is tested using a Wald test statistic for the significance of the error correction term.

The 5\% critical values are tabulated in Boswijk (1991, 1992) and approximately equal 19.82 for three exogenous variables. As shown in Table 7, we find two significant cointegrating vectors whose interpretation is similar to that given in the previous section. Note that the reported standard errors, calculated using Bardsen's (1989) method, are only valid if weak exogeneity of \(y_t, p_m,\) and \(p_d,\) is verified. The bottom rows of the table, reporting statistics on the residuals of the general models, do not display misspecification. Before conducting inference on the cointegrating vector components in order, for example, to test the hypothesis of a unit income coefficient, we must be assured that the remaining variables are not error-correcting. Following a general to simple analysis, we first reduce the general unrestricted ECM for imports and then test the weak exogeneity hypothesis of the conditioning variables for the parameters of the import demand equation.

In relation to the discussion presented in Section 3.3 and to the generic model (4)--(5), we consider the following exogeneity hypotheses:

\(H_1:\) Weak exogeneity of \(z_t' = (y_t, p_m, p_d)\) for the long-run parameters and the structural error correction coefficients of both equations \((\lambda_1, \beta_1, \lambda_2, \beta_2).\)

\(H_{1a}:\) Weak exogeneity of \(z_t' = (y_t, p_m, p_d)\) for the long-run parameters and the structural error correction coefficient of the import demand equation \((\lambda_1, \beta_1).\)

\(H_{1b}:\) Weak exogeneity of \(z_t' = (y_t, p_m, p_d)\) for the long-run parameters and the structural error correction coefficient of the effective exchange rates equation \((\lambda_2, \beta_2).\)

\(^{12}\)Note that this assumption is not restrictive per se. As far as there is some contemporaneous simultaneity, the cointegrating vector will effectively influence all the endogenous variables in the associated reduced form.
Table 7
Partial system cointegration analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>$m_t$</th>
<th>$e_{xt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error correction coefficients</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_{1,2}$</td>
<td>-0.700</td>
<td>-0.319</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.15)</td>
</tr>
</tbody>
</table>

Cointegrating vectors

<table>
<thead>
<tr>
<th>Variable</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{t-1}$</td>
<td>1.000</td>
<td>-0.127</td>
</tr>
<tr>
<td></td>
<td>(0.32)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>$y_{t-1}$</td>
<td>-0.763</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td></td>
</tr>
<tr>
<td>$p_{m_{t-1}}$</td>
<td></td>
<td>0.543</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.13)</td>
</tr>
<tr>
<td>$e_{xr_{t-1}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(p_{m}+e_{xt})_{t-1}$</td>
<td>0.349</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td></td>
</tr>
<tr>
<td>$p_{d_{t-1}}$</td>
<td>0.556</td>
<td>-0.626</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.18)</td>
</tr>
</tbody>
</table>

Wald statistics for cointegration

<table>
<thead>
<tr>
<th>Test</th>
<th>Value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>0.026</td>
<td>0.016</td>
</tr>
<tr>
<td>$\eta_1 x^2(2)$</td>
<td>1.46</td>
<td>2.45</td>
</tr>
<tr>
<td>$\eta_2 F(5, 38)$</td>
<td>1.06</td>
<td>0.13</td>
</tr>
<tr>
<td>$\eta_3 F(4, 35)$</td>
<td>1.38</td>
<td>0.19</td>
</tr>
<tr>
<td>$\eta_4 x^2(1)$</td>
<td>0.87</td>
<td>0.61</td>
</tr>
<tr>
<td>$\eta_5 x^2(8)/8$</td>
<td>1.41</td>
<td>1.36</td>
</tr>
</tbody>
</table>

$\lambda_{1,2}$ are the structural error correction coefficients from Eqs. (4) and (5). $\eta_1$ = Jarque and Bera's normality test, $\eta_2$ = LM test for fifth-order serial correlation in F-form, $\eta_3$ = LM test for fourth-order ARCH effects in F-form, $\eta_4$ = Sargan's instrument validity test, $\eta_5$ = parameter constancy test of the eight last observations.

* indicates significance at 5% level.

H$_2$: Weak exogeneity of $e_{xt}$ for the long-run parameters and the structural error correction coefficient of the import demand equations i.e., recursiveness of the system.

H$_3$: Weak exogeneity of $z'_t = (y_t, p_{m_t}, p_{d_t})$ for both the long-run and short-run parameters of the import demand equation.

H$_1$, H$_{1a}$, and H$_{1b}$ are tested using LM test statistics in F-form (see Boswijk, 1992) for the presence of the cointegrating vector(s) in a marginal (reduced form) ECM for $z'_t = (y_t, p_{m_t}, p_{d_t})$, respectively distributed (see Judge et al., 1985,
Table 8
Weak exogeneity tests

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Test</th>
<th>χ² Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₁</td>
<td>F (6, 150)</td>
<td>4.762*</td>
</tr>
<tr>
<td>H₁ₐ</td>
<td>F (3, 150)</td>
<td>1.988</td>
</tr>
<tr>
<td>H₁ₜ</td>
<td>F (3, 150)</td>
<td>3.800*</td>
</tr>
<tr>
<td>H₂</td>
<td>χ² (2)</td>
<td>1.002</td>
</tr>
<tr>
<td>H₃</td>
<td>χ² (6)</td>
<td>13.059*</td>
</tr>
</tbody>
</table>

* indicates significance at 5% level.

As shown in Table 8, H₁, H₁ₐ, and H₃ are rejected at the 5% level. The recursiveness of the system composed of the two error correction equations is however not rejected as shown by the outcome of the LR test for H₂, asymptotically distributed as a χ²(2) under the null. Consequently, we cannot reject the hypothesis that ex is weakly exogenous for the parameters of the structural import demand equation, so that the two equations can be analysed separately.

Given that H₁ nests H₁ₐ and H₁ₜ, it appears that the rejection of H₁ is due to the presence of the second cointegrating vector (corresponding to the long-run

13 In terms of the notation used in the two-equations system (4)-(5), we have mᵢ = yᵢᵣ, exᵢ = yᵢᵣ, and zᵢ = (yᵢ, pmᵢ, pdᵢ). The LR test statistic for H₂, asymptotically χ²(2) under the null, is computed as

\[ LR = T \ln \frac{\left| \tilde{A}_{11} \tilde{A}_{22} \right|}{\left| \tilde{A} \right|} - 2T \ln \frac{\left| \tilde{r}_{11} \tilde{r}_{22} \right|}{\left| \tilde{r} \right|}, \]

where : denotes estimation under the null and : denotes the unrestricted 2SLS estimators of (4)-(5).
Table 9
Individual weak exogeneity tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>(Ay_t)</th>
<th>(Apm_t)</th>
<th>(Apd_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significance of the first cointegrating vector in the marginal models</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ECI_{t-1})</td>
<td>0.174</td>
<td>0.076</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(0.11)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>Significance of the residuals in the import ECM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\hat{\nu}_t)</td>
<td>0.082</td>
<td>0.123</td>
<td>-1.087*</td>
</tr>
<tr>
<td></td>
<td>(0.21)</td>
<td>(0.24)</td>
<td>(0.53)</td>
</tr>
<tr>
<td>LM tests for weak exogeneity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(LM_{weak})</td>
<td>0.978</td>
<td>0.975</td>
<td>5.261**</td>
</tr>
<tr>
<td>(\chi^2(2))</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Diagnostics for the marginal models

| \(\sigma\) | 0.038 | 0.019 | 0.012 |
| \(R^2\) | 0.75 | 0.90 | 0.70 |
| \(\eta_1\) | \(\chi^2(1)\) | 0.94 | 0.07 | 9.16* |
| \(\eta_2\) | \(F(5, 43)\) | 1.22 | 0.93 | 0.98 |
| \(\eta_3\) | \(F(4, 40)\) | 0.27 | 0.77 | 0.19 |
| \(\eta_4\) | \(\chi^2(8)/8\) | 2.02 | 1.33 | 1.84 |

\(ECI = m_t - 0.763y_t + 0.349(pm + ex), - 0.556pd_t\). \(\hat{\nu}_t\) are the residuals from the marginal models.

For definitions of \(\eta_t\)'s see Table 7.
* and ** indicate significance at 5% and 10% levels, respectively.

effective exchange rate relationship in the marginal system for \(z_t\). The rejection of \(H_3\) can be indicative of a potential lack of orthogonality. More information on the source of this rejection can be obtained if we test individually the weak exogeneity assumption of each variable in \(z_t\) for both the long- and short-run parameters. The results are reported in Table 9. Notice that we do not report the coefficients of the short-run dynamic. The row \(LM_{weak}\) is the LM test for the weak exogeneity hypothesis of the conditioning variables with respect to both the long- and short-run parameters of the import demand equation; see Boswijk and Urbain (1994).

As shown in this table, the assumption of weak exogeneity for both long- and short-run parameters is rejected at a 10% level for domestic prices. It should however be noted that the joint nullity is rejected due to the significance of the added residual, i.e., lack of orthogonality. Given the outcome of these weak exogeneity tests, we pursued the analysis under the hypothesis that effective exchange rate, income, and import prices are weakly exogenous for both long- and short-run parameters of the import equation, while domestic prices may only be considered as weakly exogenous for the long-run import demand
parameters and the error correction coefficient. Using IV estimation in order to take into account the potential simultaneity between import volume and domestic prices, unit long-run income effects could not be rejected by the data \(F(1, 55) = 2.038\) and our finally selected error correction models for aggregate imports reads as follows (deterministic terms are omitted to save space):

\[
\Delta m_t = 0.473 \Delta y_t + 0.399 \Delta y_{t-3} + 1.642 \Delta pd_t + 1.685 \Delta pd_{t-3} \\
- 0.531 \Delta pm_{t-2} - 0.183 \Delta pm_{t-4} - 1.093 \Delta ex_{t-3} - 0.702 EC_{t-1},
\]

with standard errors in parentheses.

\(\sigma = 0.029, \quad \text{Sample period} = 1970:01-90:02,\)

Additional instruments: \(\Delta pd_{t-1}, \Delta pd_{t-2}, EC2_{t-1},\)

\(\eta_1 \chi^2(2) = 0.56, \quad \eta_2 F(5, 56) = 0.87, \quad \eta_3 F(4, 53) = 1.63,\)

\(\eta_4 \chi^2(2) = 0.13, \quad \eta_5 \chi^2(8)/8 = 1.37.\)

The error correction term \(EC1_t\) is defined as

\[EC1_t = m_t - y_t + 0.214(pm + ex)_t - 0.496 pd_t,\]

while the instrument \(EC2_{t-1}\) is defined as the second cointegrating vector obtained in Table 7. \(\eta_i, i = 1, 2, 3, 4, 5,\) are respectively Jarque and Bera's normality test, a LM test for fifth-order serial correlation in \(F\)-form, an \(F\)-test for fourth-order ARCH effects, Sargan's instrument validity test, and a forecast test over the eight last observations asymptotically distributed as a \(\chi^2(8)/8\) under the null of no structural changes in any parameter between the sample and the forecast periods; see Hendry (1989). No misspecification seems to affect the model which provides point estimates of both long- and short-run income and price effects similar to those obtained in the full system framework although the current domestic price effect is substantially higher. \(\eta_5\) does also not reject the parameter constancy hypothesis over the last eight observations. The within-sample parameter constancy hypothesis of our selected conditional model was then analysed using recursive IV estimation techniques and the same set of instruments.

The recursive IV estimates (± two standard errors) of the error correction coefficient and of the coefficient of current domestic price changes\(^{14}\) are reported in Figs. 3 and 4. These figures do not indicate any sign of parameter nonconstancy although, contrary to the SEM estimated in the preceding section, the conditional model is estimated without impulse dummies.

\(^{14}\)Due to space availability we do only report the figure of the recursive estimates of the error correction term and that of \(Apd_t\). Figures for the remaining coefficients do not provide any evidence against the parameter constancy hypothesis and are available from the author upon request.
5. Conclusions and discussion

In this paper, we have tried to illustrate the apparent dichotomy between partial and full system approaches the applied researcher is facing when modelling cointegrated systems. Full system based approaches, while theoretically and asymptotically appealing, have their own practical problems: problems of the size of the system when an important number of variables is introduced,
problems of interpretation, lack of economic theory to build 'complete' specifications, ... This makes the use of partial (conditional) systems attractive for empirical research. The validity of these however relies on the exogeneity status of the variables on which we condition.

Our empirical analysis leads us to formulate a comment on the nature of the SEM we have derived in our practical application of the approach advocated by Hendry and Mizon (1993). The first question is whether all the various equations from the SEM, which emerge from the reduction/reparametrisation process, can be labelled as structural equations (see also Sims, 1991). Although, technically, the analysis is performed in closed system by full information maximum likelihood, this approach could be interpreted as one of limited information type and hence the underlying SEM as an example of incomplete simultaneous equation model. In particular, if we consider the empirical analysis of Hendry and Mizon (1993) as well as the one contained in this paper, it is noted that in both applications a number of equations of the SEMs are best interpreted as (restricted) reduced form equations. In agreement with the discussion made in Malinvaud (1981), it should again be stressed that the search for a richer modelling of the dynamic structure of the variables under study has an unavoidable implication in terms of the restrictiveness of the underlying information set. Hence, as in usual incomplete SEMs, exogeneity assumptions (if satisfied) can be useful to reduce the size of the model. This does certainly not limit the attractiveness of the framework advocated by Hendry and Mizon (1993) which, as discussed by the authors, can be applied to open systems so that small (possibly incomplete) SEMs can easily be checked against rival models. Clearly, there is a trade-off between detailed description, in terms of number of economic explanatory factors, and detailed dynamic analysis in terms of detailed dynamic structure. Since the (closed) SEM we have considered here\(^\text{15}\) arises as a reduction/reparameterisation of a congruent VAR, we either have the possibility of leaving additional variables out or include them up to the maximum lag. This results in the well-known explosion of the number of parameters to estimate. Exogeneity hypotheses can be useful in avoiding this problem, since they allow, when valid, the number of equations and hence the number of parameters to remain manageable. A drawback however is that, in order to test the validity of exogeneity assumptions using LM test statistics, marginal models have to be built for each potential conditioning variable.

We have illustrated these two approaches in a study of Belgian aggregate imports over the sample period 1970–1990. Using quarterly data, we followed

\(^{15}\) Obviously, this need not be always the case since a SEM may be derived entirely from a priori economic theory.
both a complete system based approach as well as a partial system approach. In both cases we have allowed for the introduction of identifying information on the cointegrating vectors. After a thorough exogeneity analysis it appears that a single-equation ECM for aggregate imports allows us to capture most of the important information contained in the data, and is moreover very similar to the import equation derived within the complete set-up. Since weak exogeneity is not rejected for the long-run response parameters, it allows reliable inference on the crucial problem of long-run price and income effects in international trade. However, recognising the short-run dynamic structure of interest leads to the rejection of the weak exogeneity of domestic prices, illustrating once again the need of clearly specifying the parameters of interest in any exogeneity analysis. In comparison with the system based analysis, the (partial) structural ECM seems more suited for our problem since we are first of all interested in modelling the import volume behaviour. The closed system approach shows sensitivity to the use of dummies while the partial system approach did not necessitate any shift dummies to model statistically well the behaviour of import in Belgium. The derived price and income effects have correct signs and plausible magnitudes. In particular, the symmetry of price effects, often imposed in empirical model for aggregate imports, is clearly inadequate to describe the short-run response of imports to changes in import and domestic prices.

Given our empirical analysis, we want to favour a progressive empirical modelling strategy where full systems and partial systems emerge as complementary in the modelling process and not as substitutes. Conditional (partial) models have a number of practical advantages which make them quite appealing for empirical purposes. Moreover, economic theory is often of little guide for complete 'structural specifications'. Nonetheless, a complete system is an appealing and convenient tool for having a general and reliable statistical description of the data, often more informative for the derivation of econometric models than simple univariate statistics such as unit root tests. The Johansen procedure allows us to discover and identify the number of long-run relationships (cointegrating vectors), while the unrestricted (or restricted) estimates of the factor loadings provide an indication on the presence of the cointegrating vectors in the various equations of the systems, i.e., and indication of the exogeneity status of the corresponding variable for the long-run and the adjustment parameters. Finally, a first check on the validity of meaningful restrictions on the long-run relationships is easily obtained in this framework. When more structure is imposed, especially on the short-run structure, a partial approach has the advantage of minimising the risk of misspecification of the model for the conditioning variables. Since weak exogeneity is also testable in this framework, one may conduct a more detailed specification and still check for the efficiency of the resulting estimates and for the validity of the inference in due course.
Appendix: The data

The following data, available from the author upon request, are used in this paper:

\[ m_t = \text{import volume (source: I.M.F. financial statistics)}, \]
\[ pm_t = \text{import prices expressed in foreign currency (source: I.M.F. financial statistics and Banque Nationale de Belgique)}, \]
\[ pd_t = \text{wholesale price index (source: I.M.F. financial statistics)}, \]
\[ y_t = \text{GDP deflated by } pd = \text{real income (source: various publications of 'Les Cahiers Economiques de Bruxelles')}, \]
\[ ex_t = \text{effective exchange rate (source: Banque Nationale de Belgique).} \]
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