MONOPOLISTIC COMPETITION AND SEARCH UNEMPLOYMENT

Thomas Ziesemer*
University of Maastricht
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ABSTRACT

In this paper the monopolistic competition model of Dixit and Stiglitz for the goods market and the search unemployment model of Pissarides are combined. The Pissarides part loses its Walrasian goods market and the Dixit–Stiglitz part loses its Walrasian labour market. Pissarides’ results now depend on the degree of competition. In the Dixit–Stiglitz part the size and number of firms as well as aggregate output now depend on aggregate hiring costs, tightness and unemployment, while real wages are not fixed. Some partial results of comparative static properties of the original models survive. New results concerning the effects of changes in labour (goods) market parameters on goods markets (the labour market) variables are obtained and related to the literature on macroeconomic theory with endogenous unemployment and imperfect competition, empirical results and policy issues.

1. INTRODUCTION

In his theory of search unemployment Pissarides (1990) links the essentials of his theory to the neo-classical production function and the neo-classical growth model. In this paper we link Pissarides’ theory to the monopolistic competition model of Dixit and Stiglitz (1977). The motivation to do so comes from the feeling that many macroeconomic models of unemployment and imperfect competition have unrealistic properties such as fixed real wages, effects of country size on unemployment and number and size of

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1 They are fixed in the sense of ‘constant though endogenous’ and do not depend on anything that is typically discussed as varying over time such as unemployment rates. Wages do vary over time although not as much as desirable from the perspective of a full or high employment objective.
firms independent of unemployment, allowing only for a negative relation
between wages and unemployment rather than positive and negative relations
depending on the type of exogenous changes. This paper presents the model
first and then compares it to the literature in section 5.

Many economists believe that fixed costs and product differentiation are
very attractive features of the Dixit–Stiglitz model, which have made it a
useful tool in new trade and growth theory, macroeconomics and regional
economics. In particular, in economic policy debates the number and size of
firms is often related to the unemployment issue. However, the number and
size of firms are not determined in models using the neo-classical produc-
tion function, and search unemployment does not appear in the Dixit–Stiglitz
model. As many economists argue, the search theory of unemployment has
received much empirical support.² It seems to be a worthwhile effort to link
Pissarides’ theory to the monopolistic competition model, thus joining two
of the workhorse models in economic theory. By implication, the Pissarides
model loses its competitive goods market and the Dixit–Stiglitz model loses
its competitive labour market.

Other results are presented as propositions below and are summarized in
tables 1 and 2 and section 6. Section 5 compares the results of the model to
those of some other macroeconomic models. Which of the results are
better—ours, or those in the literature—is an empirical question that goes
beyond the scope of this paper.

2. THE MODEL

2.1 Trade in the labour market

From the Pissarides (1990) model we use the matching function \( m_L = m(u_L,
v_L) \), where \( L \) is the total number of employed and unemployed workers, \( u \) is
the unemployment rate, \( v \) is the rate of vacancies and \( m_L \) is the number of
matches produced by this function. The function is assumed to be increas-
ing in both arguments, concave and linearly homogenous.³ Defining labour
market tightness as \( \theta = v/u \), division of the matching function by \( vL \) yields

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² See the notes on the literature after every chapter of Pissarides’ book. Also, the whole
book by Franz (1999) is based on the connection of theory and empirics of search theory. Of
course, this does not imply automatically that other theories are empirically wrong, but they
have been investigated less empirically, whereas search theory has been written towards the
empirics.

³ Pissarides (1998, p. 167, footnote 15) refers to estimates of the matching function using a
Cobb–Douglas functional form, which justifies the assumptions made in the text.
\( q(\theta) = m(u/v, 1) \) as the probability of a firm finding a worker for a vacancy with \( q' = \partial q/\partial \theta < 0 \), and \( \theta q(\theta) = m/u = m(1, v/u) \) as the probability of an unemployed worker finding a job in a certain period.\(^4\) A shock is a percentage rate \( s \) at which \((1 - u)L\) employed workers lose their job by assumption in every period. Therefore \( s(1 - u)L\) workers go from a job into unemployment in every period. On the other hand \( \theta q(\theta)uL \) unemployed workers expect to find a job in each period. A labour market steady state equilibrium is defined as a situation where the numbers of workers going into and out of unemployment are equal and expectations\(^5\) turn out to be true, i.e. \( s(1 - u)L = \theta q(\theta)uL \). Solving this equation for \( u \) yields the Beveridge or \( UV \) curve (subscripts referring to variables indicate partial derivatives):

\[
\frac{s}{s + \theta q(\theta)}, \quad u_s > 0, \quad u_\theta < 0
\]  

(1)

Multiplying equation (1) by \( \theta \) yields an equation for the vacancy rate because \( u\theta = uv/u = v \):

\[
\frac{s}{s/\theta + q}, \quad v_s > 0, \quad v_\theta > 0
\]

(1’)

### 2.2 Government and unemployment benefits

The government is assumed to pay unemployment benefits \( z \) to each unemployed worker. Total expenditures of the government for unemployment benefits are \( zuL \). It will turn out that the incentives are ultimately unchanged if both the employed and the unemployed pay a tax or unemployment premium to finance the unemployment benefits. Revenue then is \( tL \). From the balanced budget assumption we make, it follows that \( tL = zuL \) and therefore \( t = zu \). Workers therefore receive \( w - t = w - zu \) and unemployed benefits are \( z - t = z - zu \). As \( z \) is considered to be a policy variable, the budget equation determines the value of \( t \), whereas \( u \) is determined in the general equilibrium part.

\(^4\) The model is formulated in continuous time, which will become clear from the dynamic equations below (see equation (3) and the following). The probabilities are densities, which hold for any point in time. In other words, the time periods for which they hold are infinitely short. In empirical research estimating the matching functions, the length of the period has been chosen in accordance with the available data.

\(^5\) \textit{Ex ante} probabilities coincide with their \textit{ex post} definition of variables on the macro level, e.g. the \textit{ex ante} value of \( q(\theta) \) equals the \textit{ex post} value of \( m/v \).
of the model below. Introducing the taxes explicitly allows us to calculate the effects of comparative static changes of all parameter changes on net wages below. It also makes the system of budgets explicitly consistent.

2.3 Households and workers

Households are assumed to have love-of-variety preferences of the constant elasticity of substitution (CES) type, \( y = \left[ \int_{i=0}^{n} c_i^\alpha \, di \right]^{1/\alpha} \), with \( 0 < \alpha < 1 \), on a continuum of goods with index \( i \), ranging from zero to \( n \), the (integral measure of) the number of firms.\(^6\) The market for goods is assumed to have no search frictions. It is well known that this specification of preferences leads to a constant elasticity of the inverse demand function, \( \alpha - 1 \), and to relative demand of goods being independent of the income earned by employed or unemployed persons. If the temporal utility function is discounted with rate \( \rho \) and integrated we may get an inter-temporal utility function. It is well known from endogenous growth theory or the theory of optimal growth that, in the absence of a rate of permanent productivity growth, the steady-state value of consumption will be stationary and the interest rate \( r \) will equal the discount rate \( \rho \).\(^7\) The problem of a household with an infinite time horizon then is to choose the values of \( c_i \) and of savings such that the choice maximizes

\[
\int_{\tau=0}^{\infty} e^{-\rho \tau} \left[ \int_{i=0}^{n} c_i^\alpha \, di \right]^{\rho/\alpha} d\tau \quad \text{subject to the budget constraint} \quad \dot{W} = I - \int_{i=0}^{n} p_i c_i \, di + r W
\]

and \( W(0) = W_0 \geq 0 \), where \( W \) is current wealth, a dot indicates a time derivative, \( r \) is the interest rate, \( p_i \) is the price of good \( i \), and current non-interest income is \( I = (1 - u) w + uz - t \). The assumption here is that a household gets the wage \( w \) with probability \( (1 - u) \) and is unemployed and gets benefits \( z \) with probability \( u \), but pays taxes \( t \) in both cases. As the utility function exhibits risk neutrality there are no complications from this uncertainty. A second interpretation could be that every household is representative in the

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\(^6\) By implication we only consider the case of a large number of firms in which no strategic behaviour takes place.

\(^7\) The interest rate should be determined in order to close the model. Other approaches, which make the model more complex are to introduce money and financial markets or to introduce capital and international capital movements and determine the interest rate in the international capital market.
sense that the same share $1 - u(u)$ of its members is (un-)employed as in the total labour force of the economy. In the first interpretation the (ex post) employed workers lend money to (ex post) unemployed workers allowing the latter to smooth consumption under the assumption of a perfect capital market. In the second interpretation this happens within the households and lending among identical households must be zero in equilibrium. In the appendix we show that the inverse price elasticity of the demand for a good is $\alpha - 1$ and the interest rate in a steady state with a constant number of firms is $r = \rho$. All results henceforth are steady-state results.

The value function for an unemployed worker is $rU = z - t + \theta q(\theta)(E - U)$ because he obtains $z - t$ if unemployed and he will go from the state of being unemployed to one of being employed with probability $\theta q(\theta)$. This function is independent of the production technology, which uses constant returns to scale in Pissarides’ model and will use increasing returns below. Therefore it carries over unchanged with the minor difference that we use $z - t$ as the net unemployment benefit, which is just $z$ in Pissarides’ notation. The value function for an employed worker is $rE = w - t + s(U - E)$. The worker obtains $w - t$ when employed and has a probability $s$ to move from employment to unemployment. Again this value function is independent of the production technology. The present value, discounted at rate $r$, of the expected income stream of an unemployed and an employed worker, $U$ and $E$, respectively, can be found by solving the two value functions for $E$ and $U$, respectively. Using $t = zu$ the solution is:

$U = [z - zu + \theta q(\theta)(E - U)]/r$ and $E = [w - zu + s(U - E)]/r$

$E - U$ is the income difference an unemployed worker can gain by finding a job with probability $\theta q(\theta)$. $U - E$ is the corresponding loss by a worker from losing his job with probability $s$. These two equations can be solved for $E$ and $U$ explicitly:

$U = \frac{(r + s)z + \theta q(\theta)w}{r + s + \theta q(\theta)} / r - zu / r$, $E = \frac{sz + [r + \theta q(\theta)]w}{r + s + \theta q(\theta)} / r - zu / r$

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See Pissarides (2000), section 3.4 for this interpretation.

Appendices are available from the working paper version http://ideas.repec.org/p/dgr/umamer/2002021.html.
2.4 Firms

The value function of a vacancy is \( rV = [-\gamma + q(\theta)(J - V)] \). It consists of the hiring costs \( \gamma \) and the net return of transferring the vacancy \( V \) into a job with value \( J \) expected with probability \( q(\theta) \). As the value of the vacancy is zero in equilibrium, we get \( J = \gamma / q(\theta) \): the value of a job is equal to the vacant job costs \( \gamma \) multiplied by the expected duration of the vacancy. When considering the firms’ hiring costs we must consider that the occupied job may be separated from the worker again with probability \( s \). The current value of the expected value of a job therefore is \( (r + s)J = (r + s)\gamma / q(\theta) \). These are labour costs that are added to the real wage received by the worker. Labour costs then equal \( w + (r + s)\gamma / q(\theta) \). Pissarides (2000) links the above to the neoclassical production function in his equations (1.8) and (1.14) for the value of a job, \( J \), which we have not used so far. In this paper we will link it to the model by Dixit and Stiglitz (1977). The value functions for \( E \), \( U \) and \( V \) and the results derived so far are independent of the production technology used and independent of the assumption of perfect or imperfect competition in the goods market and therefore carry over from Pissarides’ work. For the value function \( J \) the results below will differ from those of Pissarides because the increasing returns technology introduced next generates a different marginal product of labour.

Technologies are defined by \( l_i = f + ax_i \), with \( a, f, x_i > 0 \). The left side represents demand for labour to produce good \( i \), \( f \) is the fixed part and \( ax_i \) is the variable part of labour demand, where \( x_i \) is the output of a firm. As all goods are assumed to enter into the utility function and in the production technology, their prices and quantities will be the same.

Total labour requirement is \( nl_i = n(f + ax_i) \). Equating this to the employment \( (1 - u)L \) yields \( (1 - u)L = nl_i = n(f + ax_i) \). Solving the latter equation we find the rate of unemployment linked to the number of firms as:

\[
\frac{u}{L} = \frac{(L - nl_i)}{L} = \frac{L - n(f + ax_i)}{L}
\]

There is a partial negative relation between the rate of unemployment and the number of firms. The interesting question to be answered below is whether a higher number of firms induces a lower rate of unemployment or the causality goes the other way around.

The present-discounted value of the firm’s expected profit, which has a current value of zero in every period in free-entry equilibrium, is defined in nominal terms as:

\[
\Pi_i = \int_0^\infty e^{-\rho t} \{ p(x_i)x_i - W(f + ax_i) - p\gamma V \} dt
\]
$W$ is the nominal wage rate and real hiring costs for vacancies, $\gamma V_i$, are made nominal by multiplying their real value with the price of goods. The assumption is that nominal hiring costs are given from the labour market; monopoly pricing then has no impact on the value of hiring costs. The firm maximizes profits as defined in equation (3) through choice of the quantity $x$ and the number of vacancies $V_i$ using the dynamic concept of the large firm from Pissarides (1990, chap. 2). The dynamics come from the fact that the firm can post a number $V_i$ of vacancies, which increase employment with probability $q(\theta)$ and costs $p\gamma V_i$. On the other hand, the firm loses workers $sl_i$. The expected change in employment then is $\dot{l}_i = q(\theta)V_i - sl_i$. From $l_i = f + ax_i$ and $dl_i = adx_i$, we get the corresponding change in quantity as

$$\dot{x} = q(\theta)V_i/a - s(f/a + x)$$

The current-value Hamiltonian for each firm’s decision problem then is:

$$H = p(x_i)x - W(f + ax_i) - p\gamma V_i + \lambda[q(\theta)V_i/a - s(f/a + x)]$$

The first-order condition for the number of vacancies determines the value of the co-state variable as marginal hiring costs:

$$\frac{\partial H}{\partial V_i} = -\gamma p + \lambda q(\theta)/a = 0, \text{or } \lambda = \gamma pa/q(\theta)$$

The other canonical equation is:

$$-\frac{\partial H}{\partial x} = -\{p'x + p - aW - \lambda s\} = \dot{\lambda} - r\lambda$$

Insertion of $\lambda$ from the previous first-order condition and setting its change equal to zero in the steady state, and noticing that the price elasticity is $p'x/p = \alpha - 1$, this latter first-order condition yields:

$$p\alpha = a[W + p(r + s)\gamma/q(\theta)]$$

(4)

In the steady state the change of employment also must be zero and therefore we get the number of vacancies as a function of the quantity produced:

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10 Of course, this implies that the vacancy does not remain filled forever and when the separation has taken place it has to be filled again. The dynamic optimization of the firm takes all this into account, of course, at the cost of assuming perfect information with respect to the probabilities.
The solution for the quantity and the tightness ratio will be derived below.

2.5 Wages

There are two sorts of rents in Pissarides’ model: on occupied jobs, indexed \( j \), employed workers do not have to search and therefore have an income rent of \( E_j - U \) and firms do not have to incur hiring costs and therefore have a rent \( J_j - V \). Bargaining these rents is assumed to determine real wages. This is done by choosing the real wage by maximizing the Nash product, 
\[
(E_j - U)^{\beta} (J_j - V)^{1-\beta},
\]
with \( \beta \) as the bargaining power of workers and \( 1 - \beta \) that of firms, \( V = 0, E_j = [w_j - zu + sU]/(r + s) \), \( U \) according to the explicit solution given above, and \( J_j = \gamma/q(\theta) = (\alpha a - w_j)/(r + s) \) where the last equality stems from the solution of (4) for expected hiring costs. \( E, U \) and \( V \) are as in Pissarides (1990). The value for \( J \) differs from Pissarides’ model because we have replaced the neoclassical production function by elements of the Dixit–Stiglitz model: as we have increasing returns on the firm level, the value of an occupied job is the present-discounted value of the marginal profit from a worker gross of hiring costs. The result of the maximization with respect to the real wage in its general form is identical to that of Pissarides in that workers get a share \( \beta \) of the sum of the rents to be distributed: \( E_j - U = \beta (E_j - U + J_j - V) \). Insertion of the values for \( E_j, U, J_j \) and \( V \) yields the solution for real wages:

\[
\begin{align*}
 w_j &= (1 - \beta)(rU + zu) + \beta \frac{\alpha}{a} \\
 rU &= z - zu + \theta \beta \gamma(1 - \beta).
\end{align*}
\]

The last term of this equation contains parameters from the Dixit–Stiglitz model because \( J_j = (\alpha a - w_j)/(r + s) \) was used in the derivation instead of equation (1.14) in Pissarides (2000). At this step the increasing returns technology matters and changes labour market results. It is the marginal value product \( \alpha a \) that matters here. Imperfect competition is reflected here through the CES parameter \( \alpha < 1 \). By implication, this equation and equation (6) below differ from those in Pissarides (1990, 2000). The fixed costs parameter does not show up here. Insertion of \( E_j - U = \beta (E_j - V)/(1 - \beta) \) from the general form of the bargaining result and \( J = \gamma/q(\theta) \) into \( rU = [z - zu + \theta q(\theta) (E - U)] \) yields \( rU = z - zu + \theta \beta \gamma(1 - \beta) \). Insertion of \( rU \) into the above wage result yields:
\[ w_j = (1 - \beta)z + \beta \left( \frac{\alpha}{a} + \theta \gamma \right) \]  

(6)

The last term indicates that workers participate in the hiring costs saved on occupied jobs compared to vacancies. The second but last term is net revenue per worker. The unemployment tax, \( zu \), has dropped out only in the very last step of the calculation yielding (6). This shows that Pissarides’ approach is consistent with an explicit financing scheme for the unemployment benefit if both unemployed and employed workers have the same reduction of their gross payments \( w \) and \( z \), respectively. Then the difference when going from a status of unemployed to employed workers is unchanged and incentives of benefits are exactly as in Pissarides’ model. This model is kept as simple as the basic workhorse models.

3. THE EQUILIBRIUM SOLUTION: EXISTENCE AND UNIQUENESS

In equations (1)–(6) the index \( j \) is dropped because we will now consider the general equilibrium with all jobs occupied by the same type of labour receiving the same wage from identical firms. The equations determine the six variables of the model when goods produced serve as numéraire \( (p = 1) \): \( V, u, n, x, \theta \) and \( w \). Inserting (5) for the number of vacancies into (3), the zero-profit version of equations (3), (4) and (6) can be solved for \( x, w \) and \( q \); then (1) determines \( u \) and (2) determines \( n \). Insertion of wages per worker from (4) and the number of vacancies into the current profit function contained in (3) allows us to solve for the zero-profit\(^{11}\)-equilibrium quantity:

\[
x = \frac{\left[ \frac{\alpha}{a} - \frac{r \gamma}{q(\theta)} \right] f}{1 - \alpha + \frac{ar \gamma}{q(\theta)}}, \quad \frac{\partial x}{\partial \theta} < 0
\]  

(7)

Using equation (7) we can calculate the labour demand per firm as:

\[
l_i = f + ax_i = \frac{f}{1 - \alpha + \frac{ar \gamma}{q(\theta)}}, \quad \frac{\partial l_i}{\partial \theta} < 0
\]

\(^{11}\) Note that if the sum of all present-discounted profits is zero, in a steady state with all terms in the profit function constant—except for time in the discount factor—it follows from carrying out the integration that current profits have to be zero.

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Both output and labour demand depend negatively on hiring costs via the probability \( q(\theta) \) because an increase in the tightness ratio increases expected marginal hiring costs. Each firm knows that it will be separated from the worker with probability \( s \), resulting in \( sl_i \) separations, and can fill a vacancy with probability \( q(\theta) \). A flow equilibrium of the firm—allowing the firm to keep the labour demand, which allows it to produce the profit maximizing output level—then requires that expected separations equal expected hirings, \( sl_i = q(\theta)V_i \). The number of vacancies the firm will post to express its labour demand, \( l_i \), then is calculated from this equilibrium flow condition as:

\[
V_i = f[s/q(\theta)]/[1 - \alpha + ar\gamma/q(\theta)]
\]  

(8)

The equilibrium output quantity of the model is directly dependent on the labour-market parameters \( r \) and \( \gamma \) and indirectly on all those having an impact on the tightness ratio stemming from Pissarides’ part of the model (unemployment benefit \( z \), hiring costs \( g \), unemployment rate \( u \), vacancies \( v \), separation rate \( s \), power parameter \( \beta \) and interest \( r \); see below). Clearly, this result is due to the fact that the firm part of the Dixit–Stiglitz model is changed by adding hiring costs (per vacancies actually filled) to the wage rate; these terms, the wage and the expected hiring costs constitute marginal costs and therefore have an impact on the quantity, the employment and the vacancies posted. Note that the fixed costs parameter appears in the solution for the vacancies and therefore has an impact on the labour market results.

Using (7) to replace \( x \) in (2), we get:

\[
n = L(1 - u)\frac{1 - \alpha + ar\gamma/q(\theta)}{f}
\]

(2’)

This is a function \( n(\theta) \). A higher tightness ratio increases expected hiring costs, decreases the firm size and the unemployment rate and therefore increases the number of firms. Aggregate output can be found by multiplying the solutions for the output and the number of firms, equations (7) and (2’):

\[
nx = (1 - u)L[\alpha/a - r\gamma/q(\theta)]
\]

Although there are internal economies of scale on the firm level, aggregate output has constant returns in the size of the economy \( L \), and employment \( L(1 - u) \) for a given tightness ratio. An increase in the marginal value product
of labour, $d\alpha/dz > 0$, increases $n_x$ directly because it appears in the numerator but will be shown below to have an indirect impact on the tightness ratio, hiring costs and the unemployment rate. Using the result for the number of firms from equation (2'), we can calculate the total number of vacancies from equation (8) as $v L = n V = n(s/q)L = (s/q)L(1 - u)$. Cancelling $L$ and dividing by $(1 - u)$ yields $v/(1 - u) = s/q = \theta u/(1 - u)$. This equation can be retransformed into equation (1) by solving for $u$.

**Proposition 1:** The equilibrium quantity and employment of the firm, the total number of firms and vacancies as well as aggregate output of the modified Dixit–Stiglitz model are all dependent on marginal hiring costs, which link it to the labour market variables via the tightness ratio. Vacancies per firm depend on the fixed cost parameter.

To solve the system the next steps serve to get a second equation—besides the wage curve (6)—relating the real wage and the tightness ratio. Dividing (4) by the price and solving for the real wage yields:

$$w = \frac{\alpha}{a} - \frac{r + s}{q(\theta)} \gamma$$

(4')

Larger hiring costs $(r + s)\gamma/q(\theta)$ imply lower wages according to (4') when the interest rate is exogenous. By implication, wages $w$ always move in the opposite direction to hiring costs, $(r + s)\gamma/q(\theta)$. Equation (4') is drawn as a function $w(\theta)$ in the upper right quadrant of figure 1, indicated as the MM curve. The MM curve is rotated downward by increases in $r, s$ and $\gamma$ and shifted downward by decreases of $\alpha a$. It is also drawn in the upper left quadrant of figure 1 with wages as a function of hiring costs.

The intersection of lines BB and MM determines the wage and the tightness rate in the upper right quadrant, and hiring costs in the upper left quadrant. Given the rate of tightness thus determined, the solution for the rates of unemployment and vacancies can be found in the lower right quadrant.\(^{12}\)

Equations (4') and (6) are two functions of the form $w(\theta)$. Equating (4') and (6) we get equation (9) below, which differs from Pissarides in that it contains the degree of competition:

$$\alpha = a \left[ z + \frac{1}{1 - \beta} - \frac{r + s}{q(\theta)} \gamma + \frac{\beta \gamma \theta}{1 - \beta} \right]$$

(9)

---

\(^{12}\) Equation (1) and (1') are drawn in the lower right quadrant of figure 1. Equation (6) is drawn as the BB curve in the upper right quadrant of figure 1.
The left side is marginal revenue and the right side is marginal cost. In figure 2 both functions are drawn. The left side is denoted as MR and the right side as MC in figure 2. MC is increasing in $\theta$ and may have a negative second derivative in $\theta$.\(^{13}\)

\(^{13}\) To get a negative second derivative of the MC curve it is sufficient to assume that the matching function is of the Cobb-Douglas type.
The MC curve starts at $az$ if $\theta = 0$ and $\lim_{\theta \to 0} q = \lim m(1/\theta, 1) = \lim m(\infty, 1) = \infty$,\(^{14}\) otherwise it starts above $az$. As $\theta$ goes to infinity the MC curve also goes to infinity. Thus, the MC curve either intersects once or not at all. Therefore we have a unique equilibrium or no equilibrium at all.

If $z \geq a/\alpha$, the tightness ratio is zero, there are no vacancies and unemployment is 100% according to equation (2). With no output, $z$ cannot be paid. Therefore this cannot be an equilibrium situation.

**Proposition 2:** The existence of a unique equilibrium is guaranteed if $z < a/\alpha$. This implies a positive equilibrium value for the tightness ratio $v/u = \theta$. The fixed cost parameter $f$ and the size of the economy, $L$, have no impact on the value of $v/u = \theta$.

\(^{14}\) With standard specifications for the matching case such as Cobb-Douglas or other CES functions the $q$ goes to infinity as $1/\theta$ goes to infinity. If there are specifications, which impose limits to keep $q$ from infinity, this may be different.
4. COMPARATIVE STATIC ANALYSIS

All comparative static changes could be done using figure 1. We also use figure 2 because this allows us to save space.

The following changes in the determinants of labour costs drive up the MC curve on the RHS of equation (9), but leave the MR curve unchanged: \( d\beta > 0, d\gamma > 0, dr > 0, dz > 0, ds > 0 \). This decreases the value of the tightness ratio in figure 2. The UV equation (1), drawn as \( u(\theta) \) in the lower right quadrant of figure 1, shifted to higher \( u \) in case of \( ds > 0 \), then implies that the rate of unemployment goes up. When labour market parameters are changed in this model, the causality goes from tightness and employment to the number of firms—not the other way around as is often hoped for in justifications for policies based on the impression that having more firms implies higher employment.\(^{15}\)

For a given \( z \), all increases of unemployment require more unemployment benefits, \( zuL \). As all persons pay the tax \( zu \), this is increased whenever the rate of unemployment is increased. If the benefit \( z \) is increased too, there is a second reason for getting a higher tax rate.

From (1′), which does not contain \( b \) and \( z \), we can conclude that increased bargaining power and unemployment benefits lead to fewer vacancies because of the fall in the tightness ratio. In figure 1 this can be seen as only the BB curve shifts (for \( dz \)) and also rotates upward (for \( d\beta \)) but all other curves do not change their positions. This leads to higher wages and lower hiring costs. The size of firms is increased through lower hiring costs and the number of firms is decreasing because of the higher rate of unemployment and the lower hiring costs.

In the upper right quadrant of figure 1, increasing the rate of interest and the strength of separation shocks rotates the MM curve downward and leaves the BB curve unchanged. We can conclude that increasing the rate of interest decreases wages and the tightness ratio.\(^{16}\) The impact on the size and number of firms remains unclear because the decrease in the tightness ratio decreasing hiring costs increases firm size, but the increase in the interest rate has the direct effect of decreasing firm size.

The impact of the change in hiring costs, \( d\gamma > 0 \), is to decrease tightness according to (9) and vacancies according to (1′). In terms of figure 1 it rotates the MM curve downward and increases the slope for the BB curve. Its

\(^{15}\) See Kirchesch (2001) and the literature cited there for a recent contribution on this issue.

\(^{16}\) The impact on vacancies remains unclear here. Pissarides’ result is achieved if the direct effect of an increase of \( s \) dominates.
effect on wages depends on the strength of these two movements. In (6) $\gamma$ and $\theta$ have a negative effect on wages in (4') and a positive one in (6). In equation (6) the question is whether $\gamma \theta$ is increasing or decreasing. Applying the implicit function rule to (9) and dividing by $\theta$ and multiplying by $\gamma$ yields:

$$\frac{\partial \theta/\theta}{\partial \gamma/\gamma} = \frac{(r+s)(\gamma/q) + \beta \theta \gamma}{(r+s)\gamma(-1)q^{-2}q_{o}\theta + \beta \gamma \theta}$$

The numerator and the denominator have identical terms up to $-q_{o}\theta/q$. This term is smaller than one. By implication the percentage decrease of $\theta$ is larger in absolute terms than the increase of $\gamma$, leading to a fall in wages according to (6). In terms of figure 1 this means that the upward rotation of BB is weaker than the downward rotation of MM.

Proposition 3: With increases in workers’ bargaining power $\beta$, hiring costs $\gamma$, the discount rate $r$ and unemployment benefits $z$ and the size of shocks $s$, we confirm Pissarides’ results of decreasing tightness and expected hiring costs and increasing unemployment. Wage rates go up with bargaining power and benefits and go down when interest and separation rates or hiring costs increase. We add the following results: (i) Lower employment requires an increasing unemployment tax or premium. (ii) There is an increasing size and a decreasing number of firms from increasing bargaining power, benefits and separation rates with unclear effects on aggregate output. (iii) Increases in hiring costs $\gamma$ and interest rates $r$ have direct effects on the number and size of firms and aggregate output, which are opposite to those of the decreasing tightness ratio and lead to no clear results for these variables.

Changes in fixed costs, $df < 0$, which is one of two possible versions of exogenous productivity increases, do not change the tightness ratio and therefore the unemployment and vacancy rates, and marginal labour costs are

17 Using the properties of the matching function presented on the first page and Euler’s theorem we get (with subscripts indicating partial derivatives):

$$-q_{o}\theta/q = \frac{m_{t}/\theta}{m_{t}/\theta + m_{z}} < 1$$
unchanged.\textsuperscript{18} Firm size $x$ is decreased and the number of firms is increased as in Dixit–Stiglitz. Aggregate output, $nx$, is unaffected.

A decrease in marginal costs via $da < 0$, decreases the slope and intercept of the MC curve in figure 2 and shifts up the MM curve in figure 1. The result is a larger value for the tightness ratio, $v/u = \theta$. This reduces the unemployment rate and increases the rate of vacancies according to equation (1) and (1'). The direct impact is to decrease the number of firms and increase their size and aggregate output. However, the indirect effect through larger tightness is to increase hiring costs: this decreases firm size and aggregate output and increases the number of firms. Moreover, a lower unemployment rate increases the number of firms and aggregate output. Wages are increased according to equation (6).

\textit{Proposition 4:} (i) Productivity increases in the fixed-cost parameter, $df < 0$, when we leave labour market variables unchanged, decrease the size of firms while increasing the number of firms as in the Dixit–Stiglitz model. Aggregate output, however, is unchanged. (ii) Pissarides’ productivity results appear in our model and are caused by a decrease in the variable labour demand parameter, $da < 0$. Wages, tightness and employment increase.\textsuperscript{19} The number and size of firms and aggregate output do not have clear effects anymore as they did in Dixit–Stiglitz because unemployment and hiring costs change with the tightness ratio.

By implication, policies that try to enhance the number of firms in order to decrease unemployment should not—according to this model with a constant elasticity of substitution—try to reduce fixed costs of firms but rather variable costs, or, the \textit{personal} fixed costs of entrepreneurs for setting up a firm according to the model of Fonseca \textit{et al.} (2001).

A decrease in love-of-variety, $d\alpha > 0$, which means that the elasticity of substitution is getting larger in absolute terms and competition is increased, shifts up the MR curve in figure 2 and the MM curve in figure 1. The tightness ratio and expected hiring costs increase, the unemployment rate falls and the number of vacancies goes up.\textsuperscript{20} Wages increase according to (6). Firm size

\textsuperscript{18} Effects of changes in fixed costs may be quite different in endogenous growth models. See de Groot (2000).

\textsuperscript{19} In Pissarides’ model this effect comes from an increase in the total factor productivity of the neoclassical production function.

\textsuperscript{20} This result can also be found in the literature (see Boone, 2000; Aghion and Howitt, 1998). However, Boone’s model is one of a partial equilibrium and in Aghion and Howitt each firm consists of one worker thus fixing firm size making employment equal to the number of firms.
grows through the direct effect but decreases through the increase in expected hiring costs according to (6). The effect on the number of firms is ambiguous because the direct effect is to decrease the number of firms whereas the indirect effects via the rate of unemployment and expected hiring costs are to increase the number of firms. This latter aspect is analysed formally in an appendix for the case of constant hiring costs and summarized at the end of the following proposition.

**Proposition 5:** A preference-induced increase in substitution and competition increases employment, the tightness ratio and wages. Unlike the Dixit–Stiglitz model, the effect on the number of firms is ambiguous even for a negligible change in hiring costs: a decrease in monopoly power $d\alpha > 0$ decreases (increases) the number of firms if the measure of scale economies is larger (smaller) than the elasticity of employment with respect to monopoly power. The size of firms and aggregate output are increased if the increase in hiring costs is sufficiently weak.

For the decrease of unemployment through a higher degree of competition, $d\alpha > 0$, an increase in the number of firms is inessential. What matters are the larger marginal revenue and the larger marginal value product of labour, which increases labour demand.

A transition from monopolistic to perfect competition can be made by setting fixed costs $f = 0$ and removing product differentiation setting $\alpha = 1$. This increases the tightness ratio and vacancies and decreases unemployment. By implication unemployment is lower under perfect competition than under monopolistic competition.\(^{21}\)

Integrating two identical economies doubles $L$. This leaves the tightness ratio, the rates of unemployment and vacancies unchanged. The number of firms and varieties is doubled as in the Dixit–Stiglitz model and so are the number of unemployed workers and vacant jobs, $uL$ and $vL$.

**Proposition 6:** Doubling the size of the market doubles the number of firms, the number of unemployed people and the number of vacant jobs, but leaves the rate of unemployment unchanged. Doubling the number of varieties is a gain from trade and integration of two economies.

All changes in parameters inducing the above comparative static results have to be interpreted as stemming from perfectly non-anticipated shocks

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\(^{21}\) The number and size of firms then become indeterminate according to equations (2') and (7).
that are expected to be permanent with probability of one because the Pissarides part of the model uses steady-state present values.\footnote{Bean and Pissarides (1993) show that it can be adjusted for use on short-run issues as well.}

The economic mechanism of causation (in the sense of finding the solution of the model) of the comparative-static changes, as summarized in figure 1, always goes from the exogenous change to its impact on real wages, hiring costs and the tightness ratio and from there to the rate of unemployment and the number of firms.

Changes in the fixed cost parameter have no impact on aggregate output. The results of this section are summarized in tables 1 and 2.

<table>
<thead>
<tr>
<th>Table 1. Comparative static results of changes in labour market parameters</th>
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<tr>
<td>( \text{Effect on} \rightarrow )</td>
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<td>( \text{from increases of} \downarrow )</td>
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<tr>
<td>Bargaining power ( \beta )</td>
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<tr>
<td>Hiring costs ( \gamma )</td>
</tr>
<tr>
<td>Interest rate ( r )</td>
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<tr>
<td>Benefit ( z )</td>
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<td>Separation rate ( s )</td>
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<table>
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<tr>
<th>Table 2. Comparative static results of changes in goods market parameters</th>
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<tr>
<td>( \text{Effect on} \rightarrow )</td>
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<tr>
<td>( \text{from increases of} \downarrow )</td>
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<tr>
<td>Fixed costs ( f )</td>
</tr>
<tr>
<td>Marginal costs ( a )</td>
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<tr>
<td>Marginal revenue ( \alpha )</td>
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<td>Country size ( L )</td>
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</table>
The first four columns of table 1 clearly show that Pissarides’ results for parameter changes in the labour market carry over to our model. The fifth column adds results concerning the number of firms added by this model: Whenever unemployment is increased, the number of firms is decreased unless there are dominant offsetting effects from interest and expected hiring costs. The sixth column indicates that firm size increases when tightness decreases, but direct effects from increases of interest $r$ and hiring costs $\gamma$ may be stronger. Finally, the change of the unemployment tax or premium has the same sign as the change of the unemployment rate.

In table 2 the column for wages is qualitatively exactly the same as in Dixit–Stiglitz, but real wages now depend also on hiring cost and the effects therefore differ in quantity. The first three columns summarize the effects of goods market parameters from the Dixit–Stiglitz model on the labour market variables. These are the new results: country size and fixed costs have no impact on labour market variables; increasing monopoly power increases unemployment and decreases tightness and the number of vacancies. The results for marginal productivity are the same as those of Pissarides for productivity, and country size also had no impact on the solution of his perfect competition model. The column concerning the number of firms has different results than Dixit–Stiglitz concerning marginal costs and marginal revenue because the impact via the unemployment rate and hiring costs changes these results. This is caused by the direct effect, $d(\omega a) > 0$, which increases the size of the firm. This has a negative effect on the number of firms and increases employment, which has a positive impact on the number of firms. Moreover, hiring costs change. Aggregate output is increased by $d\omega a > 0$ unless hiring costs increase too much. The last column adds results for the unemployment tax or premium $t$, which again follows those of the unemployment rate $u$.

With regard to net wages, $w - zu$, we can say that they will be increased whenever gross wages are increased and unemployment is decreased, that is, by lower hiring costs $\gamma$, interest rates $r$, separation rates $s$ and marginal costs $a$, and by higher $\alpha$, i.e. lower monopoly power. These results follow directly from the definition of net wages and the results in the tables. The impact of changes in bargaining power $\beta$ and benefits $z$ are more difficult to get. These results are shown in an appendix.

**Proposition 7:** An increase in bargaining power $\beta$ increases (decreases) net wages if the effect of the induced decrease of the tightness ratio increases unemployment (multiplied by constant benefits) less than it decreases rents from expected hiring costs. An increase in benefits $z$ decreases net wages if (sufficient) the induced decrease of the tightness ratio (multiplied by initial
benefits) increases unemployment more than it decreases rents from expected hiring costs.

The intuition for this result is as follows. Increases in bargaining power or benefits increase gross wages and unemployment. The first effect increases net wages and the second decreases net wages. Which one is larger? The tightness ratio is decreased. The more hiring costs or rents decrease, the higher the positive impact on gross wages (according to equation (4')) and the larger the increase in the rate of unemployment the larger is the unemployment premium to be paid. In short, if the increase in the tightness ratio affects hiring costs more strongly the net wage increases but if they affect unemployment more strongly the net wages decrease. With regard to the effect of benefits there is one additional term stemming from the direct effect of $z$ on $w - zu$.

5. A COMPARISON WITH SOME MACROECONOMIC MODELS


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23 The papers by Vogt and Blanchard and Kiyotaki are of course monetary but the monetary part is essential for our purposes although not for theirs.
federation. Unions maximize total labour income, which is enhanced by distorting the allocation in favour of the R&D sector. In Bräuninger (2000) bargaining is about profits and rents from being employed. Unlike the Pissarides model there are no hiring costs creating rents for firms but rather—unlike the Dixit–Stiglitz model—profits are positive because the model has no entry but rather a large, exogenous number of firms and no fixed costs. Blanchard and Giavazzi (2003) in a similar set up have fixed costs and also consider entry. Lingens (2003) adds union wage bargaining to the Aghion and Howitt (1992) model. The firm, which achieves the latest and highest quality improvement, has a monopoly position in the intermediate sector. Therefore the number of firms is fixed to unity by the assumption of having only one best intermediate good and its size is endogenous. The union maximizes the difference between the bargained and the competitive wage. These papers differ from that of Pissarides and ours in the assumptions concerning the source of rents or profits.

There have been other attempts to relate search unemployment and monopolistic competition in the literature. One group of attempts is related to the Aghion and Howitt (1994, 1998 chap. 4) quality ladders approach. In that approach there is vertical product differentiation. Each firm has the exogenous size of one worker and because unemployment is endogenous, the number of firms is also endogenous, whereas in our approach both the number and the size of firms are endogenous. We use the Dixit–Stiglitz approach of horizontal differentiation because it has an endogenous number and size of firms. Bean and Pissarides (1993) adjust the Pissarides (1990) model to an overlapping-generations growth model by fixing the employment duration to last one period. Bean and Pissarides then go on to analyse the effect of savings shocks on employment and growth; this topic is not considered here. In their model, firms play Cournot in each of the markets for differentiated products. From this discussion of the literature it may become clear which gap in the literature this paper fills: combining the search model of the labour market with the goods market model of Dixit–Stiglitz.

In this paper, we did investigate to what degree the solution of the models and the comparative static results of Pissarides (1990) and Dixit–Stiglitz (1977) are unchanged and which results on the number and size of firms and real wages can be added. We limit our comparison with the literature below to other static models. Results from literature that use either a different theory of unemployment or a different model of monopolistic competition can be compared with our results more straightforwardly.

There are two qualifications concerning our result that decreasing fixed costs has no impact on the rate of tightness and unemployment. First, Blanchard and Giavazzi (2003) show in a framework without matching
imperfections and hiring costs, that the increase in the (integral measure of the) number of firms obtained from the fixed costs reduction decreases unemployment if the CES parameter, which is also the degree of competition, changes with the number of firms. However, if that number of firms is large—as is the assumption built into the continuum of goods in the Dixit–Stiglitz model—the model does not have this effect. The question then is, which model best represents the macro economy. Second, fixed personal start-up costs of people becoming entrepreneurs may have an impact on the unemployment rate according to Fonseca et al. (2001). Policy therefore should focus on these fixed personal start-up costs or on variable costs.

The positive cross-country correlation of employment changes and start-ups as shown in Fonseca et al. (2001, fig. 4), has a two-way causality: on the one hand it is plausible that countries with a lower start-up cost index have more start-ups and therefore lower unemployment; on the other hand it may also be the case that changes in productivity and competition, $d\alpha / da$, and in labour market parameters reduced the rate of unemployment and thereby increased the number of firms.

Liu and Yang (1999) observe that there is declining average firm size (employment), growth of per capita income and productivity in the data of some countries. If recent shifts of parameters are assumed to be $d\alpha > 0$, because of an increase in competition, and $da < 0$ and $df < 0$ both because of productivity improvements, our model generates exactly these observed changes if the change in the fixed costs dominates the others in regard to the number and size of firms.

Matusz (1998) has linked the Dixit–Stiglitz model to efficiency-wage unemployment of the Shapiro–Stiglitz type. In the non-shirking constraint the number of varieties appears. Therefore a change in country size or international trade, which increases the market size and yields more variety, relaxes the non-shirking constraint. By implication the unemployment needed to deter shirking is lower. In Matusz (1996) an increase in market size increases productivity because of an increased number of intermediate products, which allows for higher wages and less equilibrium unemployment again via a non-shirking constraint. In this paper we find no market size effects of international trade on the unemployment percentage rate for several reasons. First, 24

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24 Loveman and Sengenberger (1991, section V) also report the decrease in the trend concerning the size of firms. The decrease in fixed costs in our model can be associated as a short cut for "... the new bread of ‘flexible’ capital ... and R&D ... less costly ... to small firms". The authors present many more complementary arguments to explain the phenomenon. Liu and Yang emphasize transaction costs and Lordon (1997) emphasizes changes in taste to explain these phenomena.
with differentiated consumer goods in our model, the productivity effect of Matusz (1996) is absent in our set up. Second, having no non-shirking constraint in the Pissarides part of our model, variety cannot have such a prominent role in relation to unemployment as it has in Matusz (1998). Two other channels—discussed verbally by Matusz (1996)—that might in principal affect the rate of unemployment, are also absent here. First, the price elasticity of demand in his model as in ours is independent of the number of firms. If it were dependent on the number of firms as it is in models with strategic interaction, entry induced by larger market size might affect firms’ size and decrease average cost, which might be (similar to) a productivity effect. However, it needs a formal proof to see the interaction of market size, firms’ size, productivity and unemployment. Second, if there were an impact of more variety on the search intensity, the rate of unemployment might be affected. In our set up, this effect is absent because the bargaining is on income and not on utility, and the utility function and the Nash-product are specified independently of each other. These channels are also not modelled here because there is hardly any empirical indication that they are relevant. I agree with Matusz that it is an empirical question whether or not these channels matter quantitatively.

In the models by Matusz the size of the firms equals that of the Dixit–Stiglitz model and therefore is independent of the rate of unemployment whereas our model has larger hiring costs when steady-state unemployment is lower and therefore the size of firms is lower.

Concerning an increase in worker bargaining power Dutt and Sen (1997) find a different result in a demand-constrained model of monopolistic competition cum unemployment of the Weitzman (1985) type. In their model (as in ours) bargaining power raises wage rates. As workers save less than profit earners, demand is increased and so is employment. The authors show that the result can neither be proven nor rejected if a decreasing marginal product of labour exists. Unlike Dutt and Sen (1997) we do not find that increased bargaining power of unions increases demand and therefore employment. The reason is that our model, unlike the Weitzman (1985) type of model used by Dutt and Sen, is not demand constrained via pessimistic expectations and allows for real-wage flexibility and entry.

In the model by Pissarides (1990) the neo-classical way to get full employment with decreasing real wage costs is blocked by search frictions. In most monopolistic competition models without hiring costs this road is absent anyway because real wages are fixed as in (4) (see Weitzman, 1985; Blanchard and Kiyotaki, 1987; Matusz, 1996, 1998). The Dixit–Stiglitz causal relation to full employment used to be entry whenever profits are positive, according to equation (2) with \( u = 0 \). In Weitzman (1982) \( u > 0 \) comes in exogenously
via the assumption that firms expect aggregate demand to be (in symbols of our model) $u'wL$, where all the three variables are given exogenously. With this expected demand, a self-fulfilling prophecy yields the unemployment rate $u = u' > 0$. In our model, however, the Dixit–Stiglitz road to full-employment is present but limited by Pissarides’ search friction, resulting in an endogenous value of unemployment $u > 0$ incorporated in equation (2).

In Weitzman (1982), the rate of unemployment is negatively related to real wages: if the (expected) rate of unemployment goes up, real wages decrease because lower expected demand moves the firm up its average cost curve. This is interpreted as a pro-cyclical movement of real wages. In our model a positive or a negative relation may exist depending on the reason that drives up wages: if the exogenous change in question increases (decreases) tightness, the unemployment rate goes down (up). For example, an increase in the rate of interest or the separation rate, which decreases tightness and wages and increases unemployment, yields a negative relation as in Weitzman’s model. On the other hand, an increase in unemployment benefit $z$ or bargaining power $\beta$ increases wages, decreases the tightness ratio and increases unemployment. A similar negative relationship $w(u)$ can be found in Weinrich (1993) based on a linear transformation of a Cobb–Douglas effort function that generates constant effort. In this relationship $u$ is determined through the use of an equation similar to our equation (4), which fixes the real wage in the absence of hiring costs (see Weinrich, 1993, p. 545). Moving the latter equation along the former would result in a negative relation between $w$ and $u$. Specific combinations of parameter shifts could also generate a positive relation between $w$ and $u$ in the comparative static manner in our model.25

In our model there is no fixed wage.

With the exception of Vogt (1996) all the other static monopolistic competition models mentioned above do not take hiring costs into account. Vogt has criticised monopolistic competition models because of their property of translating wage increases completely into price increases (see Weitzman, 1985; Blanchard and Kiyotaki, 1987; Matusz, 1996, 1998) for empirical reasons. He develops a potential competition model in which sunk costs deter entry of a potential Bertrand competitor resulting in a demand curve with a kink at a limit price. This ensures that prices, quantity and employment are independent of cost increases within certain limits. In this paper, the problem is avoided by introducing search unemployment into a model of monopolistic competition. Our combination of the models by Pissarides and

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25 Of course, one could imagine generating both directions through movements along U-shaped costs curves. However, to the best of our knowledge this has not yet been elaborated in connection with the use of explicit theories of unemployment.
Dixit–Stiglitz generates another, though less strict, separation of wages from prices because costs, price and employment are still interdependent. Wage increases can either increase prices or decrease hiring costs.

Our model has higher employment under perfect than under monopolistic competition because the marginal value product of labour is larger. In Blanchard and Kiyotaki (1987) this result is achieved by an aggregate demand externality based on real balances in the utility function and endogenous labour supply.

6. CONCLUSION

Linking Pissarides’ (1990) search theory of unemployment to the Dixit–Stiglitz (1977) model rather than to the neo-classical production function yields results that are collected in the propositions. In the previous section we compared the model extensively to the literature. It is simple enough to allow for extensions of the model in many directions—in this sense we speak of ‘workhorse models’. One of the possible extensions is to introduce energy-input coefficients and treat the issues of environmental policy under endogenous unemployment. A second possible extension is to treat information and communication technology as technical progress in the matching and production functions. A third one is to derive the central planner’s optimum without and with acceptance of the bargained-wage relation and derive optimal policies.

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